1992 International Conference on Plasma Physics

Joint Conference of the

9th Kiev International Conference on Plasma Theory

and the

9th International Conference on Waves and Instabilities in Plasmas

combined with the

19th EPS Conference on Controlled Fusion and Plasma Physics

Innsbruck, 29 June - 3 July 1992
Editors: W. Freysinger, K. Lackner, R. Schrittwieser, W. Lindinger

Contributed Papers, Part III

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PREFACE

The 1992 International Conference on Plasma Physics is held as a joint conference of the 9th Kiev International Conference on Plasma Theory and the 9th International Congress on Waves and Instabilities in Plasmas with the 19th EPS Conference on Controlled Fusion and Plasma Physics in Innsbruck, Austria, from 29 June to 3 July 1992. The conference is organized by the Institute of Ion Physics of the University of Innsbruck.

The programme, format and schedule of the Conference were determined by the International Programme Committee, which also selected the plenary and topical invited lectures based on suggestions from the International Advisory Committee of the ICPP and from other members of the scientific community. Some topical invited lectures were selected from among the contributed papers. The programme included 12 plenary, 35 topical invited lectures presented orally, and 532 poster contributions.

Innsbruck, May 1992

The Editors

Acknowledgement:

The conference organizers acknowledge the financial support of the following agencies and institutions:
- The Austrian Federal Ministry of Science and Research
- The State Government of Tyrol
- The City of Innsbruck
- The International Union of Pure and Applied Physics (IUPAP)
- The Kongresshaus Innsbruck
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**PART II**

**Topic 4: Plasma Edge Physics**

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**Topic 8: General Plasma Theory**

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Topic 9:

Space and Astrophysical Plasmas
MODULATIONAL INSTABILITY OF LOWER HYBRID WAVE IN A CYLINDRICAL PLASMA

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I. Introduction: A number of workers/1-5/ have studied the Modulational instability (M.I) because of its relevance to plasma heating, laser plasma interaction and ionosphere experiments. Recently Konar et al/6/ have investigated M.I. of lower hybrid wave (LHW) in a plasma slab taking into account non-local effects. For the purpose of direct experimental verification, however, it is more useful to study M.I. of LHW in a realistic geometry. In this paper, we develop a non local theory of modulational instability in a cylindrical plasma.

To calculate the electron density fluctuation, it is most convenient to use the ponderomotive force. The non-linear currents produced by the coupling of the pump to the density perturbation generates mixed electromagnetic-electrostatic modes at \( \omega \pm \omega_o \) and \( k \pm k_o \). These side bands in turn, beat with the pump field to generate a slowly varying envelope of the electric field which gives rise to a ponderomotive bunching force that amplifies the original perturbation.

In Section II, we derive the mode structure equation of the LHW pump and the side-bands and obtain their solution. In Section III, we derive the non-linear dispersion relation for modulational instability and then use it to obtain an analytic expression for its growth rate. The explicit expression for \( \gamma \) is used to study its variation with \( \omega \) and \( \omega_o \) for a typical cylindrical plasma.

II. Mode Structure for LHW pump and side bands

We consider a cylindrical plasma which is infinite in the z-direction and confined within a radius 'a'. The plasma is immersed in a static magnetic field \( B = B_0 z \). Following Liu and Tripathi/5/, the mode structure equation for the LHW is given by the equation

\[
\nabla^2 \phi_o + K^2_o \phi_o = 0 \quad (1) \text{ where } K_o = k_o \sqrt{\frac{(\omega_{pe}^2 + \omega_{pi}^2) / \omega_o^2 - 1}{1 + \omega_{pe}^2 / \omega_{ce}^2 - \omega_{pi}^2 / (\omega_o^2 - \Omega_e^2)}}^{1/2} \quad (2)
\]

and \( \omega_{pi} \), \( \omega_{pe} \) and \( \omega_{ce} \) are the ion plasma, electron plasma and electron cyclotron frequencies respectively. In a cylindrical plasma, \( \phi_o (z,r) \) may be written as

\[
\phi_o = \phi_o(r) e^{-i\omega_o t - ik_o z}
\]

Substituting the above expression for \( \phi_o \) in Eq(1) and introducing a new variable \( R = K_dr \), we
obtain:

$$R^2 \frac{d^2 \phi_o}{dR^2} + R \frac{d \phi_o}{dR} + R^2 \phi_o = 0 \quad (3)$$

Eq(3) is a standard equation whose solution is $J_o(R)$ - a zeroth order Bessel function of the first kind and may be written as

$$\phi_o = \phi_m = A_m J_o(R) = A_m J_o(rK_o) \quad (4)$$

where $K_o = j_{o,m} / a$ and $j_{o,m}$ represents the $m$th zero of the zeroth order Bessel function of the first kind. Similarly, the mode structure equation for the lower and upper side bands can be written as

$$R^2 \frac{d^2 \phi_j}{dR^2} + R \frac{d \phi_j}{dR} + R^2 \phi_j = 0 \quad (5)$$

The solution to Eq(5) is given by

$$\phi_j = \phi_{nj} = A_n J_o(rK_j) \quad (6)$$

Equation for $K_j$ may be obtained from $K_o$ i.e. eqn(1) by substituting $\omega_j$ for $\omega_o$.

III. Nonlinear coupling and Dispersion Relation

The linear response of the electron to the high frequency modes i.e. $\omega_o$, $\omega_1$ and $\omega_2$ is given by the EXB drift and may be written as

$$\vec{v}_j = e \vec{v}_j X \delta_{co} / m_e \omega_c^2 ; \quad \vec{v}_{oz} = - e k_{oz} \frac{\phi_o}{m_e \omega_c} \quad (7)$$

(Where subscript $j = 0, 1, 2$.)

Using Eq (7), one can write the expression for the ponderomotive potential $\phi_p$ as

$$\phi_p = - \left[ e k_{oz}^2 / 2m_e \omega_c^2 \right] \left( \phi_o^2 \phi_2 + \phi_o \phi_1 \right) \quad (8)$$

The non-linear response of the electrons is given by

$$n_{\omega} = \left[ \omega_{pe}^2 / 4 \pi e \omega_c \right] \nabla^2 (\phi + \phi_p) \quad (9)$$

Obtaining the side band density perturbation from equations of continuity and substituting them in the poisson equation we obtain the following mode structure equations for the two side bands:

$$\nabla^2 \phi_1 + K_1^2 \phi_1 = - \left[ e^2 k_{oz}^2 k^2 / 4m_e^2 \omega_o^4 e P \right] \chi_o (1 + \chi_d) \phi_o^2 (\phi_o \phi_2 + \phi_o \phi_1) \quad (10a)$$
and $\nabla^2 \phi_2 + K_2^2 \phi_2 = -\left[ e^2 k_{oz}^4 k^2 / 4m_e^2 \omega_e^4 \varepsilon \right] \chi_0 (1 + \chi_i) \phi_0 (\phi_2^* \phi_2 + \phi_1 \phi_1) \quad (10b)$

where $\varepsilon = 1 + \chi_0 + \chi_i$ and $\varepsilon = 1 + \omega_{pe}^2 / \omega_{ce}^2 - \omega_{pl}^2 / (\omega_0^2 - \Omega_c^2)$

To find the solution to Eq. (10a) and Eq. (10b), we expand $\phi_1$ and $\phi_2$ in terms of a complete set of orthonormal functions in terms of $\phi_{in}$ and $\phi_{2m}$ respectively as

$$\phi_1 = \sum_n A_n^{(i)} \phi_{in} (K_1 r) \quad \text{and} \quad \phi_2 = \sum_m A_m^{(2)} \phi_{2m} (K_2 r) \quad (11)$$

As indicated in Eq. (4) and (6), these orthonormal functions are zeroth order Bessel functions of the first kind. Subtracting Eq. (10a) from corresponding linear mode structure equation and multiplying the resulting equation by $r \phi_1$ and integrating over $r$ we obtain:

$$\left( K_1^2 - K_{in} - \eta \int r \phi_0 \phi_1^* \phi_{in} \phi_{in}^* dr \right) A_n^{(1)} = \eta \sum_m A^{(2)}_m \int r \phi_0 \phi_0^* \phi_{2m} \phi_{in}^* dr \quad (12)$$

where $\eta = -\left[ e^2 k_{oz}^4 k^2 / 4m_e^2 \omega_e^4 \varepsilon \right] \chi_0 (1 + \chi_i)$

Similarly for $\phi_2$, we have

$$\left( K_2^2 - K_{2in} - \eta \int r \phi_0 \phi_2^* \phi_{2m} \phi_{2m}^* dr \right) A_m^{(2)} = \eta \sum_n A_n^{(1)} \int r \phi_0 \phi_0^* \phi_{2m} \phi_{in}^* dr \quad (14)$$

Multiplying Eq. (12) and Eq. (14), and taking the mode numbers for the upper and lower sidebands to be equal ($m = n$), we obtain the following dispersion relation

$$\left[ K_1^2 - K_{in}^2 - \Delta_1 \right] \left[ K_2^2 - K_{2in}^2 - \Delta_2 \right] = \mu \quad (15)$$

where $\Delta_1 = \eta \int \phi_0 \phi_0^* \phi_{2n} \phi_{2n}^* dr \quad \Delta_2 = \eta \int \phi_0 \phi_0^* \phi_{in} \phi_{in}^* dr \quad (16)$

and $\mu = \eta^2 \int \phi_0 \phi_0^* \phi_{2n} \phi_{2n}^* dr \int \phi_0 \phi_0^* \phi_{in} \phi_{in}^* dr \quad (17)$

Using Taylor's expansion for $K_1^2$ and $K_2^2$ and assuming $\omega$ to be complex we obtain from equations (15) to (17) the following expression for $\omega_r$ and growth rate $\gamma$:

$$\omega_r = \left[ k_x^2 \omega_o^2 / k_{oz} A \right] \left[ A / \omega_o^2 - 1 \right] \quad (18)$$

$$\gamma = 1/D \left\{ \mu - \Delta_1 \Delta_2 + (\Delta_1 + \Delta_2) \left[ 2 \omega_r A k_{oz}^2 / \omega_o^3 + 3 \omega_r^2 A k_{oz}^2 / \omega_o^4 + k_x^2 (A / \omega_o^2 - 1) \right] \right\}$$

$$+ 2 k_x k_{oz} (A / \omega_o^2 - 1) (\Delta_1 - \Delta_2) - 8 \omega_r^4 A k_{oz}^4 / \omega_o^8 + 4 \omega_r^2 A^2 k_{oz}^4 / \omega_o^6 - (A / \omega_o^2 - 1)^2 \left[ k_x^4 - 4 k_x^2 k_{oz}^2 \right]$$

$$- (A / \omega_o^2 - 1) \left[ 6 \omega_r^2 A k_{oz}^2 k_x^2 / \omega_o^4 + 8 \omega_r A k_x k_{oz}^2 / \omega_o^4 \right]^{1/2} \quad (19)$$
where \( D = \left\{ 4A^2 k_{ox}^4 / \omega_o^6 - 36 \omega_r^2 \omega_o^2 A^2 k_{ox}^4 / \omega_o^8 \right\}^{1/2} \) and \( A = \omega_{pe}^2 + \omega_{pi}^2 \)

In evaluating the integrals occurring in \( \Delta_1 \) and \( \Delta_2 \) we have taken \( n = 7 \). We have also assumed in our calculations that \( \mu = \Delta_1 \Delta_2 \).

**IV. RESULTS AND DISCUSSION**

Equation (19) gives the expression for growth rate of modulation instability of a lower hybrid pump in a cylindrical plasma. To obtain numerical appreciation of the growth rate of \( M > 1 \) of lower hybrid wave, we assume the following values for a typical cylindrical plasma

\[
\begin{align*}
\omega_{ce} &= 1 \times 10^{12} \text{ s}^{-1}, \quad k_z = 0.025 \text{ cm}^{-1}, \quad k_{oz} = 50k_z, \quad \delta = 8 \times 10^4, \\
\omega_{pe} &= 2.5 \times \omega_o; \quad \text{radius of cylinder } "a" = 3 \text{ cm}.
\end{align*}
\]

Fig (1) and Fig (2) shows the variation of \( \gamma \) of M.I. in a cylindrical plasma as a function of perturbation frequency \( \omega_r \) and the pump wave of frequency \( \omega_o \), respectively. It is seen from Fig.1 and Fig.2 that \( \gamma \) increases monotonically with increasing values of \( \omega_r \) and \( \omega_o \).

![Graph 1](image1)

1. Variation of \( \gamma/\omega_{pe} \) with \( \omega_r \)

![Graph 2](image2)

2. Variation of \( \gamma/\omega_{pe} \) with \( \omega_o \)

**References**

The interaction between a magnetized plasma and a localized ensemble of 'heavy' ions in relative motion arises in a variety of situations in space plasmas (Critical ionization velocity experiments /1/; neutral gas releases in the solar wind: experiment AMPTE /2/; interaction of the solar wind with a charged cloud of dust particles /3/; encounter of the solar wind with the ionosphere of an unmagnetized body or its interaction with the atmosphere of a comet). Common features of these situations are (1) an unspecified orientation of the magnetic field to the flow direction; (2) both electron cyclotron radius and Debye length are small compared to interesting scale lengths, while the ion gyroradius is in general not; (3) the relative kinetic energy of flow ions in the cloud frame is large compared to their thermal energy; (4) the cloud ion mass is large compared to the mass of the ions in the plasma stream; and (5) collisions between ions are largely negligible. Conventional MHD theory does not apply here, but a different treatment of electrons and ions and the inclusion of a second ion species is required. Several simulations in this context (mainly to model the AMPTE experiment) have been carried out, most of them based on hybrid code simulations /4-7/. We use a lower level of description, and our interest is primarily in basic phenomena of the collisionless coupling that survive even when the plasma model is rigorously simplified /8/.

Characteristic features of the plasma model are (1) 1-D geometry, i.e., the ion cloud degenerates to a plane-layered slab (Fig. 1); (2) Cloud ions are assumed immobile, i.e., mass loading, cloud expansion and ion production are ignored; (3) electrons are treated as massless, strongly magnetized, isothermal flow; (4) ions are considered as weakly magnetized cold fluid, (5) quasineutrality is assumed throughout. Thus, Debye length and lower hybrid frequency are excluded, but the model produces ion dominated low frequency phenomena (ion acoustic waves, compressive and shear Alfvén waves). The basic
The equations are

\[ \frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} (n_i v_{ix}) = 0 \]

\[ \frac{\partial v_i}{\partial t} + v_{ix} \frac{\partial}{\partial x} v_i = \frac{e}{m_i} (E + v_i \times B) \]

\[ E = E_1 + E_2 = -k_B T_e \frac{\partial n_e}{\partial x} (x \cdot \mathbf{b}) \mathbf{b} - (v_e \times B) \]

\[ \frac{\partial E}{\partial t} = -\frac{e}{\varepsilon_0} (n_i v_i - n_e v_e) + c^2 \frac{\partial}{\partial x} (0, -B_z, B_y) \]

\[ \frac{\partial B}{\partial t} = -\frac{\partial}{\partial x} (0, -E_z, E_y) \]

\[ n_e = n_i + n_c \]

Subscripts \( e, i, c \) denote electrons, flow ions and cloud ions, respectively. Our general strategy is to look for the steady-state response of the plasma flow to the ion layer. To this end we solve the ordinary differential equations which result from the basic system by cancelling time derivatives.

(a) Magnetic field aligned to the flow: Fig. 2 illustrates the response of the plasma stream to an ion layer with Gaussian shape and a peak density \( n_{c,\text{max}} = 0.2 n_0 \) for two ion acoustic Mach numbers. A fairly supersonic flow penetrates the layer with almost no effects on the ions (layer charge balanced by electrons, weak electric field). In a fairly subsonic flow, however, charge neutrality at the layer is maintained by an ion density depletion and a much stronger electric DC field with opposite sign is established. Electrons are too ‘stiff’ to react and are almost unaffected. In gasdynamic terms, the model obstacle acts on the plasma flow like a nozzle. If the peak layer density exceeds a (Mach number-dependent) critical value,
the plasma stream cannot longer penetrate the layer. In a three-dimensional situation, the ion cloud would then act like a 'rigid' obstacle to the flow. The critical density tends to zero when the flow becomes sonic.

(b) Magnetic field perpendicular to the flow: In this case, the electron temperature does not enter the analysis and the Alfvén Mach number is the relevant quantity. Unlike the former case, however, the layer size comes into play. One can distinguish between a 'small' and a 'large' column density case. In the first one typical features of the magnetic field-free interaction are reproduced. In the latter case, the flow distortion is no longer confined to the area covered by the obstacle but a super-Alfvénic (sub-Alfvénic) flow undergoes a pronounced compression (expansion) when passing the ion layer. In both cases the flow is significantly deflected in y-direction. Consequently, even a low-density ion ensemble may become impenetrable in case of a large enough size. These results are largely consistent with hybrid code simulations /7/.

(c) Magnetic field inclined to the flow: The most interesting feature in this case is the excitation of stationary waves (waves with $\omega=0$ in the layer rest frame, i.e., phase velocity in the plasma frame equal but opposite to the flow velocity). They are absent for $B_0V_0$, $B_0V_0$ because of the acoustic-like dispersion of the relevant modes. Stationary waves develop either downstream or upstream of the layer according to whether
Fig. 3  (a) $M_a = M_A = 0.2$  (b) $M_a = M_A = 5$
Magnetic field 45° inclined to the flow direction

the component of the associated group velocity in flow direction is smaller or greater than $v_o$ (Fig. 3). While the downstream wave is basically electrostatic in nature, the upstream waves is a fast Alfvén mode. The layer does not admit a steady-state penetration of the plasma stream for flow velocities ranging between the smallest and greatest group velocity of the three low-frequency modes.

/1/ P.T.Newell, Rev. Geophys., 23, 93, 1985
Rayleigh–Taylor Instability favoured Drift Turbulence in a Spread F Simulation Experiment
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In bottom side of F–region of the ionosphere, the density gradient (∇n) and electric field are antiparallel to gravity (g), the ∇n is perpendicular to magnetic field (B), ions and electrons are magnetised (νin/Ωt < 1, νen/Ωe < 1) and the crossfield term (En/Bn) is less effective than gravity i.e., En/Bn < g/νin. In this region the long wavelength modes (kρi < 0.1) are found to have power spectral form of S(k) ∝ k^−n with n = −2 to −2.5. Rayleigh – Taylor instability mechanism has been invoked to explain the k^−2 spectrum (Chaturvedi and Kaw, 1976; Sudan and Keskinen, 1984; Zargham et al., 1987).

In the short wavelength regimes (0.1 < kρi < 1) the experimental observations exhibit a spectral index n = −4 to −5 in density fluctuations and electric field fluctuations have n = −2 to −3 (Singh and Szuszcwiesic 1984, LaBelle et al., 1986). Drift waves, with spectral index in range of −4.5 to −5.0, are favoured as good candidate for the generation of short wavelength fluctuations (Hasegawa and Mima, 1978; Fyfe and Montgomery, 1979; Hasegawa et al., 1979; Tchen et al., 1980; Pécel et al., 1980; LaBelle et al., 1986). The theory of Chaturvedi and Kaw (1977) predicts R–T instability as the primary source of irregularity which is effective at wavelengths of the order of vertical density gradient scale lengths, and the growth of R–T instability is followed by generation of short scale length drift waves.

We have recently reported, for the first time, experiments on low frequency instabilities in laboratory plasma simulating conditions obtained in the equatorial spread–F (Prasad et al. 1992 a,b). The experiments satisfy the conditions on the dimensionless parameters νin/Ωt < νen/Ωe < 1 for the F–region. The equilibrium plasma density and potential profiles satisfy En || ∇n⊥ B, and gravity is simulated by the curvature of the magnetic field. In this paper we report a study of spectral characteristics of density and potential fluctuations of plasma as a function of νin/Ωt keeping νen/Ωe < 1 in all cases. The k-spectra obtained indicate dominance of R–T instability in long wavelength regime and existence of R–T favoured Drift turbulence in shorter wavelength regime. The results are compared with theoretical predictions, numerical simulations, experimental observations of F region and laboratory studies.

The experiment is conducted in a toroidal device BETA, having toroidal magnetic field, a major radius (R) of 45 cm and minor radius of 15 cm. A radially movable cylindrical Langmuir probe with exposed tip of 2.5 mm in length and 0.75 mm in diameter insulated with glass is used to measure equilibrium profiles of plasma density, floating potential and electron temperature. A pair of probes placed at radial location of 6 cm with angle of separation 5° is used to get wave number and frequency spectra using the technique outlined by Beall et al. (1985) when the fluctuations are turbulent. The data is acquired on dual channel Lecroy oscilloscope with a sampling time of 20μsec. of 8 bit resolution with 32 kbyte of samples per channel.

The plasma has peak density ~ 10^11/cm^3, with electron temperature in the region where experiment is performed is ~ 4eV with T_e = 0.23 ± 0.06eV. The effect of gravity is simulated by curved magnetic field, g = B^2/2R ≈ 2 × 10^9 cm/sec^2 for Argon plasma. Typical density and floating potential profiles measured are shown in figure 1. The fluctuations have δn/n ~ eφ/T_e. The density gradient scale length L_n = n/∇n ~ 18 cm. During the experiment we varied the neutral Argon gas pressure from 10^−4 Torr to 4×10^−4 Torr. Under these operating conditions νen/Ωe < 1 is satisfied and hence electrons
Figure 1: Radial profiles for (a) density and (b) floating potential.

are magnetised. The k - spectra in density and potential fluctuations are obtained at different magnetic fields and at different pressure. Under most of these conditions the power spectrum indicates turbulence. The density fluctuation spectrum (figure 2) has a shallower slope of \( n = -2.2 \pm 0.2 \) for \( k \rho_i < 0.5 \) and has steeper slope of \( n = -4.2 \pm 0.5 \) for \( 0.5 < k \rho_i < 2 \) whereas potential spectrum (figure 3) has only one slope \( n = -4.5 \) for the variation of \( \nu_{in}/\Omega_i \) from 0.04 to 0.1. The variation of k - spectral indices in density and potential as a function of \( \nu_{in}/\Omega_i \) is shown in figure 4 and indicates that the density spectrum has dual slope where as the potential spectrum has only one slope for the variation of \( \nu_{in}/\Omega_i \) from 0.04 to 0.1. It is also clear from figure 4 that the difference between the shallower slope of density spectrum and potential spectrum is ~ 2 and steeper slope of density spectrum and potential spectrum are nearly similar. One of the important tool used in identifying the nature of the instability is the spectral relation between density and electric field (or potential) in different scale length regimes. The relation between density and potential for R – T instability is given (Kelley, 1989) by

\[
E(k) = - \left( \frac{\mu B}{\nu_{in}} \right) \frac{n}{n^2}
\]  

(1)

The spectral index for electric field fluctuations should, for this case, be similar to that of density fluctuation implying that the density and potential spectral indices should differ by ~ 2. The relation between density and potential spectral indices for drift waves for which Boltzmann relation is valid and is given by

\[
\tilde{E}(k) = - i k \left( \frac{K_B T_e}{e} \right) \frac{\tilde{n}}{\tilde{n}}
\]  

(2)

where \( K_B \) is the Boltzmann constant. Electric field and density fluctuation spectra should have spectral indices differing by ~ 2 or the potential and density fluctuation spectra should have a similar form. In our experiment, the observed spectral indices, of \( n = -2.4 \pm 0.2 \), and \( n = -4.2 \pm 0.3 \), for density and potential fluctuations respectively for the case of \( k \rho_i < 0.5 \) is in agreement with equation (1), and the measurements of LaBelle et al. (1986) in night time F – region. The theory of Chaturvedi and Kaw (1976) predicts
Figure 2: $k$-Spectrum of density fluctuations. The power law least square fits are shown by solid lines and experimental data points are shown by dots.

Figure 3: $k$-Spectrum of potential fluctuations. The symbols are same as in figure 2.
similar spectral form. Sudan and Keskinen (1984) predict a spectral index of $n = -5/3$ for one dimensional case; however many numerical simulation studies and experimental observations differ from this theoretical value. This theory is applicable when inertial ranges are well developed and turbulence is strong, and is not applicable in present case. The observed spectral indices of $n = -4.5 \pm 0.3$ and $n = -4.2 \pm 0.3$ for density and potential fluctuations respectively, in the range $0.5 < k_{pF} < 1$, are in close agreement with equation (2), the drift wave spectral index measured in laboratory (Pécseli et al., 1980) and F-region (LaBelle et al., 1986) experiments.

In conclusion we have observed density and potential fluctuations at low frequency in a toroidal magnetised plasma, simulating conditions of equatorial spread-F conditions. The observed spectra are turbulent and indicate Rayleigh-Taylor instability as dominant mechanism for production of long wavelength fluctuations while R-T induced Drift waves appear to be responsible for the shorter wavelength turbulence.

References
SIMULATION OF ELECTRON BEAM-DRIVEN INSTABILITIES BY A 3-D ELECTROMAGNETIC PARTICLE CODE

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Abstract

We have developed a 3-D electromagnetic and relativistic particle code in which the fields are directly updated through current deposition of the particles, eliminating Poisson’s equation and its time-consuming solution /1/, /2/. We have used this code to investigate waves excited by a localized electron beam along the magnetic field (in the $z$-direction). Periodic boundary conditions are applied along this direction, while in the $y$- and $z$-directions we apply boundary conditions which allow radiation to escape freely out of the simulation domain without reflection /3/. Particles which hit the radiating boundaries are arrested there and redistributed more uniformly by having the boundaries slightly conducting: this allows electrons to recombine with ions and provides a realistic way of eliminating escaping particles from the simulation system. The simulation results using a mesh of dimension $85 \times 85 \times 160$ with a few million particles show whistler emissions and beam modes like those evidenced by wave data from the Spacelab 2 mission. The modulation of the parallel current density, due to bunching of the beam electrons, is responsible for the generation of whistler waves. The waves are saturated due to the slowdown and heating of the beam electrons. The hodogram of the magnetic field in the $x - y$ plane shows a right-hand polarization, consistent with whistler mode propagation. In the initialisation of the beam-plasma system, “quiet start” conditions were approached by including the poloidal magnetic field of the beam: thermal noise was the only cause of the excitation of the growing modes.

Introduction

During the Spacelab 2 flight, which was launched on July 29, 1985, a spacecraft called the Plasma Diagnostics Package (PDP) performed a fly-around of the shuttle at distance of up to 300 meters while an electron beam was being ejected from the shuttle in a steady (d.c.) mode. Gurnett et al. /4/ have investigated the plasma waves observed during a magnetic conjunction between the PDP and the shuttle while the electron gun was being operated. A spectrogram of the plasma wave electric field intensities during a 30-minute period around the magnetic conjunction is shown in Figure 2 of the work by Gurnett et al. /4/. At higher frequencies a very well-defined funnel-shaped feature can be seen extending up to about the electron cyclotron frequency, $f_c \approx 1$ MHz, spreading out at higher frequencies. At even higher frequencies, the intense narrowband emission at about 3.1 MHz is believed to be either the electron plasma frequency, $f_p$, or the upper
In this work we use a newly developed 3-d relativistic electromagnetic particle code \cite{1}. The mixed boundary conditions are developed in order to investigate the localized electron beam. The periodic condition is used along the magnetic field (z-direction). The radiating boundary conditions are used for x- and y-directions. A larger system than the previous work \cite{5} is used with all components of the magnetic fields and the relativistic effects.

Simulation model and results

The simulations are performed on the Cray-2 at National Center for Supercomputing Applications (NCSA), The University of Illinois at Urbana-Champaign. The localized electron beam is located in the middle of the simulation domain. The system size used for simulations is $L_x = L_y = 85\Delta$, $L_z = 160\Delta$, where $L_x$, $L_y$, and $L_z$ are the lengths of the system in three directions and $\Delta (= 1)$ is the grid size. The magnetic field is in the z-direction. Along the z-direction, the fields and particles are periodic. On the other hand, “radiating” boundaries, which allow radiation to escape freely out of the simulation domain without reflection, are used for the boundaries perpendicular to the magnetic field.

We consider the column of electron beam in the middle of the system along the z-direction, whose radius, $r_{eb}$ is 4.47$\Delta$ (its center is located at $x_{\text{cnt}} = y_{\text{cnt}} = 43$). The beam electrons are randomly chosen in the column. The beam density in the column is about the half (0.502) of the total electron density. The total number of beam electrons in the column, $N_{eb}$ is 4899. The drift velocity of beam electrons, $V_d = 3.5\upsilon_{st}$, where $\upsilon_{st}$ is the electron thermal velocity. The temperature of the beam electrons, $T_{eb}$ is 0.09$T_e$, where $T_e$ is the temperature of the background electrons. In this study the ion drift velocity, $V_i$ is zero.

Based on the observed data, we chose a set of parameters. The parameters used for the simulations are the following: $m_i/m_e = 64$, $T_e/T_i = 1$, $\Omega_e/\omega_{pe} = 0.4$, $c/\upsilon_{st} = 10.67$, $\omega_{pe}\Delta t = 0.10$, $\beta = 0.111$, where $m_i$, $m_e$, $T_i$, $\Omega_e$, $\omega_{pe}$, $c$, are the ion and electron mass, the temperatures of the background ions, the electron cyclotron and plasma frequencies, and the speed of light, respectively. The Debye length, $\lambda_D$ is 0.469$\Delta$. The simulation plasma is quasi-neutral all over the system. Other parameters used for the simulation are the electron and ion gyro-radius, $\rho_e = 1.17$, and $\rho_i = 9.38$, skin depth, $c/\omega_{pe} = 5.00$.

Several simulations have been performed with different parameters. The most important development in this study is including the initial poloidal magnetic field (IPMP) generated by the beam electrons which corresponds to a quiet start. In this way the transient stage due to the instant onset of the parallel current was greatly reduced. For the detailed simulation results see \cite{6}. The hodogram shown in Fig. 1 is one of the new unique results obtained by this 3-D electromagnetic code.

In order to examine the excited electromagnetic waves, the local magnetic fields were sampled at the two different distance (a) at $x = 16$, (b) $x = 43$ from the electron beam (at $y = 43$, $z = 55$). Figure 1 shows hodograms of the magnetic fields ($B_x - B_y$) in the $x - y$ plane. The value is plotted for every 5 time steps ($\omega_{pe}\Delta t = 0.5$). Figure 1a show the hodogram at $x = 16$ for the time period 10 $\leq \omega_{pe}t \leq 30$. The ambient magnetic field is coming out of the surface. Due to the larger distance from the electron beam the signal is mixed with thermal noise, however, the direction of the rotation is clearly seen and the same direction as electron cyclotron motion, therefore the field is right-hand polarized.
Fig 1: Hodograms of the magnetic field amplitude (relative) $B_x - B_y$ at $y = 43, z = 55$.
(a) at $z = 16$
(b) at $z = 43$ (at the center of the beam)

At the middle of the electron beam (Fig. 1b) the hodogram $(30 < \omega_{pe} t < 50)$ shows a clear rotation with larger amplitudes. This confirms that the whistler waves are emitted from the electron beam and propagate obliquely.

Conclusion

We have investigated whistler and electrostatic waves driven by the electron beam ejected from the shuttle. The simulations have been performed by using a newly developed 3-D electromagnetic and relativistic particle code.

The simulation results show a whistler emission and beam mode as shown by wave data obtained from the Spacelab 2 mission. The wave analysis of the local electric and magnetic fields show the growing whistler waves whose frequencies are less than the electron cyclotron frequency. The simulation results show that the frequencies of whistler waves are depend on the distance from the electron beam. The more distant from the beam, the frequencies become closer to the electron cyclotron frequency which is consistent with the characteristics of the funnel shaped spectrum observed the PDP /7/. Besides whistler waves electron plasma waves and/or upper hybrid waves are also excited. As we expect, these simulation results are very similar to the previous results /8/, /9/, /5/, /10/, /11/.

The detail analysis of the generation mechanism of the whistler wave conclude the previous results by Pritchett et al. /11/. The perturbed parallel current density due to the bunching of beam electrons is responsible to the generation mechanism of the whistler waves /12/, /13/, /14/. The electrostatic waves are saturated due to the slowdown and heating of the beam electrons around $\omega_{pe} t = 25$. It should be noted that even at the stage (after $\omega_{pe} t = 25$) where the electrostatic waves are decaying, the whistler waves are still being excited. At this stage the bunching of beam electrons disappears in the phase space $z - v_z$. The 3-dimensional spiral bunching structure of the beam electrons which is not
well shown in the 2-dimensional display may be responsible for this prolonged excitation of whistler waves. The hodogram of the magnetic field amplitude (relative) in the $x - y$ plane shows a right-hand polarized wave which is consistent with a whistler wave.

Night Time Equatorial Spread F Simulation in a Laboratory Plasma

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The night time equatorial F-region plasma has two distinct regions: The region I is well known as bottom side of F region in which the density gradient ($\nabla n$) is antiparallel to gravity ($g$). In region II (above the F-region peak density) the density gradient parallel, to gravity. The numerical simulation and theoretical predictions suggest that Rayleigh-Taylor ($R-T$) instability plays crucial role in the generation of irregularities of the order of density gradient scale length in the region I where cross field ($E_n/B$) and velocity shear forces are less dominant. The in-situ observations exhibit power spectrum, $S(k) \propto k^{-n}$ with index $n = -2.0$ for $k\rho_i \ll 1$ (Singh and Suszczewicz 1984). In-situ and rocket observations (Kelley et al., 1982) in the region II exhibit similar features to that of the region I. The theoretical predictions cannot explain this because gravity is ineffective in good curvature region. The numerical simulations of $R-T$ instability by Scannapieco and Ossakow (1976) clearly exhibit the occurrence of similar fluctuations in either sides. In order to clarify some of these features of F-region plasma we performed a laboratory experiment in which the plasma has both regions similar to F-region (region I and region II) with $\nu_{in}/\Omega_i \ll 1, \nu_{cm}/\Omega_e \ll 1$ at low operating pressures (1~5 $10^{-4}$ torr).

The experiment is performed in device BETA (Bora 1984), which is toroidal in shape having a major radius ($R$) of 45 cms and a minor radius of 15 cms. A variable toroidal field upto 1 kG can be applied. A tungsten filament 2 mm in diameter and 21 cm in length, is placed vertically at major radius of 45 cm. The plasma is produced by striking a discharge between the cathode (incandescent tungsten filament) and the anode (vessel wall). Gravity is simulated by the curvature of the toroidal magnetic field ($g = C_s^2/R$ where $C_s$ is ion acoustic speed and $R$ is major radius of the system). The density is measured by ion saturation currents drawn by a low-impedance probe together with current to voltage converter. The floating potential is measured by a high impedance probe terminated with a voltage follower. The plasma produced has peak density $\sim 10^{11}\text{cm}^{-3}$. The electron temperature at the radial locations where these instability measurements are made ($\pm 5$ to $\pm 8$) varies from $5\text{eV}$ to $3\text{eV}$, ion-temperature $T_i \approx 0.2eV$. The typical density and potential profiles are shown in figure 1 which exhibits two regions i.e. between radial locations $9\text{cm}$ to $1\text{cm}$, the $g$ is antiparallel to $V_n$ and between $-1\text{cm}$ to $-9\text{cm}$, $g$ is parallel to $V_n$. The value of local $E_n \approx 1.0\text{ V/cm}$ in the region $\pm 4$ to $\pm 8$ suggests that effects of gravity is relatively strong.

The fluctuations in density and floating potential are picked up by Langmuir probes at different radial and azimuthal locations. Low pass filter with upper band limit of 25 kHz is used to avoid aliasing. The data is acquired on a PC based dual channel Lecroy oscilloscope with a sampling time of 20 $\mu$s. The signals are picked up by a pair of probes whose distance of separation is $\sim 0.5\text{cm}$. The local wave number and frequency spectrum $S(k, \omega)$ are computed using probe pair technique (Beall et al., 1984). A pair of probes which are separated by $0.5\text{cm}$ at radial locations $\pm 6\text{cm}$ in the azimuthal direction is used to obtain $S(k, \omega)$ spectrum both in region I and region II.

In the region I the gravity effects are relatively strong at $\nu_{in}/\Omega_i \ll 1$. The $k$-spectrum of density and potential shown in figure 2 is obtained at $\nu_{in}/\Omega_i \approx 0.04$. The density spectrum has clearly two breaks with a spectral index $n \approx -2$ for $k\rho_i \leq 0.6$ and $n = -4.2$ for $k\rho_i \geq 0.6$. The potential spectrum has only on spectral index with $n = -4.2$. 
Figure 1: Density and Potential profiles of the plasma simulating Region I and Region II of night time F-region of equatorial Ionosphere.

Figure 2: $k$-Spectra of density and potential in region I.
We also observe that the difference between density and potential spectral index $\sim 2$ for $\kappa \rho_i \leq 0.6$ have similar spectral nature for $\kappa \rho_i \geq 0.6$. The $k$-spectrum of density and potential fluctuations obtained in region II are shown in figure 3 indicates that the spectral nature is similar to that of the region I. The spectral index of density and electric field fluctuations in both region I and region II for $\kappa \rho_i \leq 0.6$ is $\sim 2$, which is consistent with spectral relation for R - T instability (Kelley 1989). However the good curvature region, in principle, is not conducive for excitation of R - T instability. As we have already mentioned, the numerical simulation studies of R - T instability (Scannapieco and Ossakow, 1976) showed that the irregularities first excited, in the region of the plasma where $g$ is antiparallel to $\nabla n$, by R - T instability then bubble to the top side of the F region density peak by $E \times B$ polarization motion. The in-situ observation (Kelley et al 1982) have confirmed the observation of similar spectra in density fluctuation above the F region peak density. We observed similar spectra in region I and II. However we observe the fluctuations simultaneously in both regions. The observation of lag in the onset of these fluctuations in region II as in the case of numerical simulation or in-situ experiments is not possible in our case due to large growth rate and $E_x\times B$ motion of the instabilities and geometrical constraint of the system. The polarisation drift in our experiment is dominated by $E_y\times B$ drift. The steeper part of the $k$-spectrum ($n = -4.5$) in density and potential $k$-spectrum satisfy conditions of drift wave (Kelley 1989) similar to other laboratory experiments on drift wave turbulence (e.g. Pecseli et al. 1983).

Parametric studies have been conducted in the region I and II. The ratio $\nu_{in}/\Omega_i$ is varied in both regions. This is achieved by varying either the neutral reasure or the ambient toroidal magnetic field. Thus we have varied the value $\nu_{in}/\Omega_i$ from 0.04 to 0.1 which is still much smaller than unity. Therefore we have confined ourselves to the regime when the effective gravity term is dominant over the cross-field term in the bad curvature region. The variation of spectral index in both region I and II with respect to $\nu_{in}/\Omega_i$ is shown in figure 4. It is clear from figure 4 that the spectral index in density in both region I and II have dual slope of $n \approx -2$ and $n \approx -4.5$ where as the potential fluctuation

![Figure 3: k-Spectra of density and potential in region II.](image-url)
Figure 4: Variation of Spectral index with $\nu_m/\Omega_i$ for density and potential in region I and II.

has only one spectral index of $n = -4.5$.

Thus in conclusion in a laboratory plasma, simulating the characteristics of nighttime equatorial F region, electrostatic fluctuations are observed. The characteristics of the fluctuations in density and potential are as follows:

i). the $k$ - spectra of $\tilde{n}$ exhibit two slopes with values of $n \approx -2$ and $-4.5$, both in region I and in region II;

ii). the potential fluctuation exhibits only one slope with index $n \approx -4.5$ in these regions.

The observed fluctuations satisfy the power spectral relation between $\tilde{n}$ and $\tilde{\phi}$ for R - T and drift wave instabilities in different scale length regimes. The observations of similar power spectral characteristics in the region of plasma where $g$ is parallel to $\nabla n$ is attributed to the fact that the fluctuations excited by R - T instability in the region of plasma where $g$ is antiparallel to $\nabla n$ is carried by $E_o \times B$ drift which is consistent with direction of propagation of these waves. The results are in good agreement with in - situ experiments and numerical simulation studies.

References:
Powerful radio waves can be used for artificial modification of the Earth’s ionosphere [e.g. 1,2]. It is then well-known that the nonlinear interaction of an intense electromagnetic wave with the background plasma can give rise to parametric instabilities involving resonant three-wave decay interactions, as well as modulational and filamentation instabilities. In the three-wave interaction processes, the radio wave pump decays in most cases into a high-frequency daughter wave and a low-frequency electrostatic wave, whereas the modulational and filamentational instabilities involve both the upper- and lower-radio sidebands as well as non-resonant static (or finite frequency) density fluctuations. In a fully ionized collisional plasma, the latter are basically reinforced by the radiation pressure and the differential Joule heating of the electrons.

The plasma in the lower part of the Earth’s ionosphere (viz. in the mesosphere or in the E-region) is partially ionized and highly collisional. Here, the number density of the neutrals is so high that it becomes essential to account for the collisions of the charged particles with the neutral atoms, and to pay particular attention to electron impact ionization and recombination processes that depend on the radio wave energy, as well as to accommodate the electron energy dependence of the frequency of the attachment of the charged particles on the oxygen molecules. Some of these effects have been included in a recent study [3] which, however, did not treat the differential heating nonlinearity in the Earth’s ionosphere in a correct manner. Here we present an improved investigation of the stimulated attachment instability of radio waves in the ionosphere.

Let us consider the propagation of a high-frequency radio wave $\omega_0, k_0$ in the lower part (heights below 130 km) of the Earth’s ionosphere whose constituents electrons, ions, and neutrals form a partially ionized plasma. The nonlinear interaction between this pump wave and the ionospheric electron number density perturbation $n_1$ gives rise to radio wave sidebands which are governed by the equation

$$2i\omega_0 (\partial_t + v_g \cdot \nabla)E + c^2 \nabla^2 E = \frac{\omega_{pe}^2}{n_o} n_1 E,$$  \hspace{1cm} \text{(1)}$$

where $E$ is the high-frequency electric field amplitude, $v_g = k_0 c^2/\omega_o$ is the group velocity of the pump, $c$ is the speed of light, $n_o$ is the equilibrium density, and $\omega_{pe}$ is the electron plasma frequency. For simplicity, we have assumed that $\omega_o \approx (k_0 c^2 + \omega_{pe}^2)^{1/2}$ is much larger than the electron gyrofrequency $\omega_{ce}$. The right-hand side of (1) represents the
usual nonlinear electron current density arising from the coupling of the rapidly varying electron quiver velocity of the pump electric field with the electron density perturbations associated with the plasma slow motion.

We shall now derive the expressions for $n_1$ in the presence of a strong radio wave in a partially ionized collisional magnetoplasma. Combining the electron continuity, momentum and energy equations [e.g. 2], assuming that $|\partial_t| << (v_{te}^2/\nu_{en}) |\partial_x|^2 << 1/\tau_e$, (where $v_{te}$ is the electron thermal velocity, $\nu_{en}$ is the electron-neutral collision frequency which is much larger than the electron-ion collision frequency, the $x-$ axis is defined by the direction of the geomagnetic field $B_0\hat{z}$, and $\tau_e$ is the electron energy relaxation time) and noting that the ponderomotive force as well as heat diffusion can be neglected because $\tau_e\nu_{en} >> 1$, we obtain the approximate relation

$$\frac{e\phi}{m_e} - \frac{v_{te}^2}{\nu_{en}} \frac{n_1}{n_0} - \frac{4}{3} \frac{e^2 \tau_e \nu_{en}}{m_e^2 \omega_0^2} |E|^2 \approx 0,$$

(2)

where $\phi$ is the electrostatic potential, $-e/m_e$ is the electron charge to mass ratio, and $n_0(x) + n_1$ is the electron number density.

Next, combining the ion continuity and momentum equations, assuming that $|\partial_t| << \omega_i$ and $\nu_i$, where $\omega_i$ is the ion gyrofrequency which is assumed to be much larger than the ion-neutral collision frequency $\nu_{in}$, and where $\nu_i$ is the attachment frequency which is a function of the radio wave intensity $|E|^2$ [1], assuming quasi-neutrality, we obtain

$$\frac{v_{in}^2}{\omega_i} \nabla^2 \phi + \frac{\nu_{in}}{v_{in}^2} \partial_z^2 \phi \approx \frac{B_0}{c} \frac{\nu_{in}}{n_0} \frac{n_1}{n_0} + (\partial \nu_a/\partial E_0 |E_0|^2) |E|^2,$$

(3)

where $|E_0|^2$ is the mean value of $|E|^2$. As we consider heights lower than 130 km, it has here been possible to neglect ionization and recombination processes [3]. Eliminating $\phi$ from (2) and (3), we have

$$(D_\perp \nabla^2_\perp + D_z \partial_z^2 - \nu_i) \frac{n_1}{n_0} = \left[ \frac{\partial \nu_a}{\partial |E_0|^2} - \frac{4 e^2 \tau_e \nu_{en}}{3 v_{te}^2 m_e^2 \omega_0^2} (D_\perp \nabla^2_\perp + D_z \partial_z^2) \right] |E|^2,$$

(4)

where $D_\perp = \nu_{in} c_s^2/\omega_i^2$ and $D_z = c_s^2/\nu_{in}$, and where $c_s = (T_e/m_e)^{1/2}$ is the ion sound velocity.

We note that (4) is significantly different from the corresponding result in Ref. [3] because $\tau_e \nu_{en}$ generally is much larger than unity. In order to estimate the importance of the $\partial \nu_a/\partial |E_0|^2$ term in (4), we now insert some numerical values in order to compare the magnitudes of the two terms on the right-hand side of (4). Thus, choosing $E_0 \approx 1V/m, \tau_e \nu_{en} \approx 10^3, m_e/m_i \approx 3 \times 10^{-5}, \omega_e = 2 \times 10^7s^{-1}, \nu_{en} \approx 700s^{-1}$ and $|\partial \nu_a/\partial \omega| \approx 10^{-6}cm^{-1}$, we find that the attachment nonlinearity dominates the Joule heating nonlinearity as $(\partial \nu_a/\partial |E_0|^2) |E_0|^2 > 5 \times 10^{-9}s^{-1}$ at the height 110 km [3]. Clearly, (1) and (4) are basic for studies of the filamentation and self-focusing of intense radio waves in the ionosphere.

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RADAR COLLECTIVE BACKSCATTERRING AND THE HIGH LATITUDE PLASMA TURBULENCE

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Long wavelength radar collective scattering is a continuous observation means of the terrestrial plasma environment, especially in the northern auroral zone. The radar pulsed emission, combined with programmed delayed echo reception, provides several key informations: The (small scale) delay time defines the echoing plasma distance. The signal mean square amplitude is proportional to the plasma non-uniformity at that distance. While the long time scale signal fluctuations provide a measurement of the plasma motion.

Interpreting these fluctuations in term of plasma motion is far from obvious. The most commonly accepted view is that of Doppler frequencies. Indeed, a plasma collective scattering device is known to define a virtual network of interference planes, characterized by a “scattering wavevector” \( \mathbf{k} \), through which the spatial electron density distribution is analyzed. This k-vector depends on the electromagnetic waves “incident” \( \mathbf{k}_i \), and “scattered” \( \mathbf{k}_s \), wavevectors, as

\[
\mathbf{k} = \mathbf{k}_s - \mathbf{k}_i
\]

Any density irregularity moving at a velocity \( \mathbf{v} \) through this analysing network, provokes a scattered signal fluctuation at the Doppler frequency

\[
\omega = \mathbf{k} \cdot \mathbf{v}
\]

Here we like to point out, that in the case of convection motion, the resulting frequency spectrum is not the velocity probability distribution, but a more elaborate, and more informative, statistical transform of the plasma turbulent motion.

This point will be brought forwards by a brief theoretical analysis, and an examination of significant auroral plasma scattered signal time correlations.

1/ Collective scattering out of a non-uniform, unsteady fluid.

The collectively scattered signal collected at time \( t \), is a spatial Fourier transform, at the scattering wavenumber \( k \), of the plasma density distribution \( n(r, t) \).

The signal time correlation is

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\[
\langle s(t) s^*(t + \tau) \rangle = \frac{1}{n^2} \int_V \int_V \left( e^{-i \mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} [n(\mathbf{r}_1, t) \, d\mathbf{r}_1] [n(\mathbf{r}_2, t + \tau) \, d\mathbf{r}_2] \right)
\]

Since plasma motion is particle conserving, the integration over position \( r_2 \) at time \(( t + \tau)\) can be changed into an integration over the position \(( r'_2)\) the same volume element occupied at time \( t \). In the transformation, this given volume element is moved by a vector \( \Delta(\mathbf{r}'_2, \tau) \),

\[
\Delta(\mathbf{r}'_2, \tau) = \mathbf{r}'_2 - \mathbf{r}_2
\]

while its number of particles is kept constant.

Furthermore, in case of “frozen-in” turbulence, the plasma motion \( \Delta(\mathbf{r}'_2, \tau) \) is statistically independent of the local density \( n(\mathbf{r}'_2, t) \). This hypothesis is especially justified for incompressible fluid turbulent flows, including the two-dimensional MHD plasma motion.

In those cases, the averaging operator can be factorized, providing two different statistical moments

\[
\langle s(t) s^*(t + \tau) \rangle = \Sigma(k) \left( e^{i \mathbf{k} \cdot \Delta(\mathbf{r}'_2, \tau)} \right)
\]

Where \( \Sigma(k) \) is the “Form factor”, a mean square of the electron density space Fourier transform at wavenumber \( k \). For statistically stationary fluctuations, the form factor is independent of time.

Let be \( P(\Delta, \tau) \) the probability distribution of the plasma displacement \( \Delta \) during time \( \tau \). The averaged term in Eq.(4) is the statistical “characteristic” of this probability distribution. That is, the Fourier transform (of argument the scattering wave vector \( k \)) of this probability distribution.

The scattered signal intensity is thus proportional to the form factor, while

"The collectively scattered signal time correlation is the statistical characteristics of the fluctuating fluid motion probability distribution at the same time." The characteristic function argument is the scattering wave vector.

A non-trivial relation thus exists between plasma motion probability and signal time correlation. Its analysis requires some extent \( 3 \). Its result can be sketched in Figure 1. It depends on the plasma motion correlation length \( L_c \) compared to the scattering wavelength:

For long correlation length, \( k L_c >> 1 \), the plasma motion through the scattering network is seen along almost straight lines. The plasma displacement \( \Delta(t) \) is proportional to the fluid velocity \( v \), and its probability distribution is a straightforward transform of the fluid velocity probability distribution. The "characteristics" is (most likely, although not necessarily) a gaussian function of time (Fig. 1 trace "a", where \( k L_c = 2 \)). Whatever is this characteristics, the frequency spectrum is readily shown to be exactly the familiar "Doppler spectrum" of the fluid velocity probability distribution.
- For short correlation length instead \((k L_c << 1)\), the plasma motion is seen as a random, brownian wandering through the scattering grid. The plasma motion is of a (turbulent) diffusiv type, its “characteristics” is a damped exponential function of time (Fig. 1 trace “c”, where \(k L_c = 0.5\)). Its time Fourier transform, the signal frequency spectrum, then becomes a lorentzian.

- For intermediate scales, when \(k L_c = 1\), the correlation time argument is to be compared to the plasma motion correlation time \(T_c\). At short times, the plasma motion is almost straight, the correlation function is nearly gaussian. At long times, the plasma motion is random, the correlation is exponential (Fig. 1 trace “b”, with \(k L_c =1\)).

2/ Auroral plasma scattered signals.

The “SHERPA” HF radar is observing the high latitude ionosphere on the North-eastern Canada. Backscattered echoes are obtained from regions where the scattering wavevector (along the line-of-sight) is strictly perpendicular to the earth magnetic field: the plasma densities irregularities are “field aligned”. The scattered signal time correlations have been systematicaly monitored over long period of time, and recently analyzed\(^3\) in order to find their dominant shape, whether gaussian, or damped exponential.

A majority of these correlations were found of the exponential type, corresponding to lorentzian frequency spectra, and a significant minority, of the gaussian type. Some of them shows an interesting transition between these two canonical shapes. An exemple of these transition type of correlations, is shown in Figure 2.

Figure 2 is an analysis of the backscattered signal coming from the ionospheric E-region, on May 22, 1991 at 06:07:53 UT. The probed plasma distance was 600 Km, and its depth 30 Km. The radar frequency was 9.4 MHz, corresponding to a scattering wavelength of 16 meters.
The left part is the modulus of the time correlation function, obtained on thirteen different delay times (open circles), from 0 to 48 ms. The data integration time was 12 seconds.

The right part is the frequency spectrum, shown between -400 to +400 Hz. It is calculated from the full (complex) correlation by a time Fourier transform. The peak frequency is -167 Hz (corresponding to a mean plasma velocity of 2673 m/s, directed away from radar), and its standard deviation 20 Hz.

The two full lines shown in the correlation figure, are the best gaussian and exponential fits. The gaussian is seen to fit the early times, while the exponential fits the later times. This is a common experimental feature, in agreement with our analysis and Fig. 1. From this observation, we are able to obtain two significant parameters of the plasma fluctuations in the plasma mean frame:

- the transition time, between “Gauss” and “exponential”, is the correlation time $T_c$
- the correlation amplitude at transition is $\exp[-(k L_c)^2 / 2]$, thus a measurement of the correlation length $L_c$.

From the data of Fig. 2 we find successively $T_c = 22.5$ ms, and $L_c = 3$ metres. To our knowledge, these are the first experimental direct measurements. To get some understanding or the auroral plasma fluctuations, these parameters can then be compared to other characteristics, like the ion gyroperiod and radius, or the ion-neutral collision time and length, respectively.

References:


I. INTRODUCTION

Recently Guha and Asthana/1/ and Sharma and Sudarshan/2/ have studied stimulated scattering of electromagnetic waves by hybrid waves in a two electron temperature plasma. In this paper, we extend our previous work (Sharma and Sudarshan/2/) to derive the dispersion relation of the electromagnetic wave scattered by the low frequency ion cyclotron wave perturbation using the hydrodynamic equations for a plasma having three species - hot electrons, cold electrons and ions. The dispersion relation is used to obtain an explicit expression for growth rate using standard procedures and study the numerical variation of the growth rate with $n_{oh}/n_{oc}$.

II. DISPERSION RELATION

We assume that the ion-cyclotron wave propagates in plasma in a direction making a very small angle with the x-direction (so that $k_x << k_z$) in a plasma in which magnetic field is present in the z-direction. We consider a plasma having three species - hot electrons, cold electrons and ions. We assume that both electrons and ions are magnetized. The equation which described the behaviour of the plasma are:

\[ m_j \frac{\delta \vec{v}_j}{\delta t} = -e\vec{E} - e(\vec{v}_j \times \vec{B}) + e\vec{\phi}_{pe} - m_j c_j^2 \nabla^2 (n_j / n_{oj}) \]  
\[ m_i \frac{\delta \vec{v}_i}{\delta t} = e(\vec{E} + \vec{v}_i \times \vec{B}) - e\vec{\phi}_{pi} - m_i c_i^2 \nabla^2 (n_i / n_{oi}) \]  
\[ \frac{\delta n_j}{\delta t} + \nabla.(n_{oj} \vec{v}_j) = 0 \]  
\[ \frac{\delta n_i}{\delta t} + \nabla.(n_i \vec{v}_i) = 0 \]  
\[ \nabla^2 \phi = 4\pi e(n_e + n_h - n_i) \]

Where the subscript $j = h,c$ stands for hot and cold electrons respectively, $\phi_{pe}$ and $\phi_{pi}$ are the ponderomotive potentials for electrons and ions respectively and $c_j$ is the velocity of the electrons.
Linearizing eqns. (1) to (5) one can easily obtain the x and z components of drift velocities of electrons. Substituting these expressions in eqn (1) we get:

\[
n_e = -\left(\frac{e}{m_c} / \Delta \frac{m_c}{m_c} \right) \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) \left( \phi + \phi_{pe} \right)
\]  

(6)

where \( \Delta = \left( \omega^2 - \omega_{ce}^2 \right) \); \( \Delta_c = 1 - c^2_k k_x^2 / \left( \omega^2 - \omega_{ce}^2 \right) - c^2_k k_z^2 / \omega^2 \)  

(7)

The corresponding expression for the hot electrons is obtained by replacing the subscript 'c' by 'h'. The expression for \( n_i \) may be obtained by proceeding exactly in the same manner as for cold electrons.

Thus one obtains the following expression for \( n_i \):

\[
n_i = \left( \frac{e}{m_i} / m \Delta_i \right) \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) \left( \phi + \phi_{pi} \right)
\]  

(8)

where \( \Delta_i = 1 - k_x^2 c_i^2 / \Delta - k_z^2 c_i^2 / \omega^2 \); \( \Delta' = \omega^2 - \Omega_c^2 \) and \( \Omega_c \) is the ion-cyclotron frequency.

Using Eq.(6) and Eq.(8) in the linearized expression for Poisson's equation, we get:

\[
\phi + \phi_{pe} = \frac{k^2 - (1 + \delta) \omega_{pi}^2 / \Delta_i \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right)}{k^2 - \left( \frac{\omega_{pe}^2}{\Delta_c + \omega_{ph}^2 / \Delta_b} \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) - \omega_{pi}^2 / \Delta_i \left( \frac{k_x^2}{\Delta'} + \frac{k_z^2}{\omega^2} \right) \right)} \phi_{pe}
\]  

(9)

and \( \phi + \phi_{pi} = \phi - \delta \phi_{pe} \)

\[
= \frac{\left( \frac{\omega_{pe}^2}{\Delta_c + \omega_{ph}^2 / \Delta_b} \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) \left( 1 + \delta \right) - k^2_\delta \right)}{k^2 - \left( \frac{\omega_{pe}^2}{\Delta_c + \omega_{ph}^2 / \Delta_b} \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) - \omega_{pi}^2 / \Delta_i \left( \frac{k_x^2}{\Delta'} + \frac{k_z^2}{\omega^2} \right) \right)} \phi_{pe}
\]  

(10)

Now we can express the number densities of cold electrons, hot electrons and ions in terms of the ponderomotive potentials as

\[
n_j = - \left[ \frac{\omega_{pi}^2}{(4\pi e c \Delta_p)} \right] \left[ \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right] [A / G] \phi_{pe}
\]  

(11)

\[
n_i = \left[ \frac{\omega_{pi}^2}{(4\pi e c \Delta_p)} \right] \left[ \frac{k_x^2}{\Delta'} + \frac{k_z^2}{\omega^2} \right] [B / G] \phi_{pe}
\]  

(12)

where \( A = k^2 - \omega_{pi}^2 / \Delta_i [1 + \delta] \left[ \frac{k_x^2}{\Delta'} + \frac{k_z^2}{\omega^2} \right] \)

(13a)

\( B = \left( \frac{\omega_{pe}^2}{\Delta_c + \omega_{ph}^2 / \Delta_b} \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) \left( 1 + \delta \right) - k^2_\delta \right) \)

(13b)

and \( G = k^2 - \left( \frac{\omega_{pe}^2}{\Delta_c + \omega_{ph}^2 / \Delta_b} \left( \frac{k_x^2}{\Delta} + \frac{k_z^2}{\omega^2} \right) - \omega_{pi}^2 / \Delta_i \left( \frac{k_x^2}{\Delta'} + \frac{k_z^2}{\omega^2} \right) \) \)

(13c)
The electron and ion velocities varying as the pump wave (of frequency $\omega_o$) are given by

$$v_{ej} = \frac{ie \omega_o E_0}{m_e (\omega_o^2 - \omega_e^2)} \quad (14)$$
$$v_{oi} = \frac{-ie \omega_o E_0}{m_i (\omega_o^2 - \Omega_i^2)} \quad (15)$$

Maxwell's equation gives the following relation between nonlinear current density and the electric vector of the waves:

$$(\omega_1^2 - k_1^2 c^2) \vec{E}_1 = 4\pi i \omega_1 \vec{j}_1 \quad (16)$$

where $\vec{j}_1$ is the nonlinear current density and is given by

$$\vec{j}_1 = -en_{oc} \vec{v}_{oc} - en_e \vec{v}_{ce} - en_{oh} \vec{v}_{lh} - en_h \vec{v}_{eh} + en_o \vec{v}_l + en_i \vec{v}_{ci} \quad (17)$$

Substituting for $v_{ej}, v_{oi}$ etc. from Eqs. (14) and (15) into Eq. (16) we get the following dispersion relation

$$\omega_1^2 - k_1^2 c^2 - \frac{\omega_{pe}^2 \omega_e^2}{(\omega_1^2 - \omega_e^2)} - \frac{\omega_{pi}^2 \omega_1^2}{(\omega_1^2 - \Omega_i^2)} = \frac{-e^2 E_0^2 A}{4m_e^2 (\omega_o^2 - \omega_e^2) G} \times \left[ \left( \frac{\omega_{pe}}{\Delta_c} + \frac{\omega_{ph}}{\Delta_b} \right) \left( k_x^2 / \Delta + k_z^2 / \omega_e^2 \right) \right] + \frac{e^2 E_0^2 \omega_{pi}^2 B}{4m_i m_i (\omega_o^2 - \Omega_i^2) G} \left[ k_x^2 / \Delta' + k_z^2 / \omega_o^2 \right] \quad (18)$$

The growth rate $\gamma$ of the coupled modes can now be obtained from Eq. (18) by substituting $\omega = \omega_1 + \gamma i$ and solving the resulting equation for $\gamma$. Thus we get the following expression for normalized growth rate $\gamma / \gamma_o$:

$$\frac{\gamma^2}{\gamma_o^2} = \left[ \frac{n_{oh} (\omega_r^2 - c_e^2 k_x^2)}{n_{oc} (c_e^2 k_x^2 - \omega_r)} - 1 \right]$$

$$\gamma_o^2 = \frac{4e^2 E_0^2 k_x^2 (\omega_r^2 - c_e^2 k_x^2)}{4m_e^2 \omega_o^2 (\omega_o - \omega_r) \omega_r} \quad (19)$$

where $\gamma_o$ is the normalized growth rate.

III DISCUSSION

Equation (19) gives the dimensionless growth rate $\gamma / \gamma_o$ of the coupled waves i.e. side-band and the ion-cyclotron wave as a consequence of the three wave interaction in a plasma having two distinct species of electrons i.e. hot electrons and cold electrons. To have a numerical
appreciation of our analytical results, we consider a typical Q-machine plasma having the following parameters

\[ \omega_r = 10^7 \text{s}^{-1}, \quad c_c^2 = 1.75 \times 10^{15} \text{cm}^2\text{s}^{-2}, \quad c_h^2 = 1.75 \times 10^{17} \text{cm}^2\text{s}^{-2}, \quad k = 10 \text{ cm}^{-1} \]

\[ k_z = 0.1 \text{ cm}^{-1}, \quad T_{ec} = 1 \text{ eV}, \quad T_{eh} = 100 \text{ eV}, \quad \omega_{pe}^2 = 10^{20} \text{s}^{-2}, \quad \omega_c^2 = 2 \times 10^{11} \text{s}^{-1}, \quad k_e = 6 \text{ cm}^{-1} \]

Using the above given parameters in Eq(19), we have calculated the dimensionless growth rate \( \gamma / \gamma_0 \) of the coupled waves as a function of \( n_{oh}/n_{oc} \). Fig.(1) shows the variation of dimensionless growth rate \( \gamma / \gamma_0 \) with \( n_{oh}/n_{oc} \). It is seen from Fig.(1) that the growth rate increases with increasing population of hot electrons which is qualitatively similar to that of the growth rate of lower and upper hybrid waves investigated by the authors in a recent paper/2/.

\[ \text{Fig. 1 Variation of } \gamma / \gamma_0 \text{ with } n_{oh}/n_{oc}. \]

REFERENCES

Binding Energy and Triplet-Singlet Splitting for the Hydrogen Molecule in Ultrahigh Magnetic Fields

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The properties of the simplest hydrogen systems or hydrogen-like systems (exitons) are changed dramatically in the strong magnetic field which exceeds the atomic magnitude:

\[ H \gg H_c = \frac{m_e^2 e^3 c}{\hbar^5} \text{ for hydrogen atom, and } H >> H_{ex} = \frac{m_e^2 e^3 c}{\hbar^5 e^2} \text{ for exitons.} \]

We perform analytical solution for the interaction of the hydrogen atoms in a strong magnetic field \( H >> H_c \), and calculate the energy difference of the singlet and triplet states, the binding energy and wave functions for the hydrogen molecule.

The Schrödinger equation for the electrons is (all quantities are expressed in atomic units \( \hbar = m = e = c = 1 \))

\[
(H(1,2) + \frac{1}{2} \mathbf{\hat{\sigma}_1} \cdot \mathbf{H} + \frac{1}{2} \mathbf{\hat{\sigma}_2} \cdot \mathbf{H}) \Psi = E \Psi, \tag{1}
\]

where

\[
(H(1,2) + \frac{1}{2} \mathbf{\hat{\sigma}_1} \cdot \mathbf{H} + \frac{1}{2} \mathbf{\hat{\sigma}_2} \cdot \mathbf{H}) \Psi = E \Psi,
\]

\[ \rho_1^2 = y_1^2 + (b + z_1)^2, \rho_2^2 = y_2^2 + (b - z_2)^2 \]

are the coordinates of motion for the electrons 1 and 2 in the plane perpendicular to \( \mathbf{H} \); the vector potential is \( A = \frac{1}{2} \mathbf{H} \times \mathbf{r} \).

\( \mathbf{\hat{\sigma}_1} \) and \( \mathbf{\hat{\sigma}_2} \) are Pauli matrices, \( \lambda = \sqrt{1/H} \).

We will consider two energy levels, which are the singlet term and the triplet term. Let \( \Psi_s \) be the exact solution of the two-electron coordinate wave equation that is symmetrical in the coordinates of the two electrons. The singlet wave function is thus a product of \( \Psi_s \) and an appropriate spin function for the sum of electron spins \( S = 0 \). Thus, for the singlet wave function \( \Psi_s \), the Schrödinger equation is

\[
H(1,2) \Psi_s = E_s \Psi_s. \tag{2}
\]

The triplet wave function is the product of the exact solution of the two-electron wave function that is antisymmetrical in the coordinates of the two electrons and an appropriate spin function for the sum of the electron spins \( S = 1 \). The lowest energy level corresponds to spin projection \( -1 \). Thus, the Schrödinger equation for the triplet term is

\[
(H(1,2) - \frac{1}{2\lambda^2} - \frac{1}{2\lambda^2}) \Psi_a = E_a \Psi_a. \tag{3}
\]

Using the new variable \( E_{s1} = E_s - \frac{1}{\lambda^2} \), one can reduce Eq.(2) to the form...
Let us consider the functions \( \Psi_1 = (\Psi_1 + \Psi_2)/2 \), and \( \Psi_2 = (\Psi_1 - \Psi_2)/2 \) which are large only when electron 1 is localized near proton a and electron 2 is near proton b. If the phases of \( \Psi_1 \) and \( \Psi_2 \) are properly chosen, the function \( \Psi_1 (r_1,r_2) \) for \( r_1 \to -R/2 \), and \( r_2 \to R/2 \) and \( \Psi_2 (r_1,r_2) \) for \( r_1 \to R/2 \) and \( r_2 \to -R/2 \) are the product of the two hydrogen single-atom wave functions in an ultrahigh magnetic field.

We seek the wave function \( \Psi_1 \) as the product of the two hydrogen-atom wave functions \( \psi_{1,2} \), for the first electron being near the first proton and the second electron near the second proton.

\[
\Psi_1 = B^2 \chi_1 \exp \left( -\frac{p_1^2 + p_2^2}{4\lambda^2} \right) W_{\alpha,1/2} \left[ \frac{2}{\alpha} (a + x_1 + \lambda) \right] W_{\alpha,1/2} \left[ \frac{2}{\alpha} (a - x_2 + \lambda) \right]
\]

where \( B^2 = (1/\alpha) (1/2\pi\lambda^2) \), and \( \chi_1 \) is a slowly varying function compared with the exponential decay. Substituting expression (5) in Eq.(3) we obtain for \( \chi_1 \):

\[
\left( \frac{1}{\alpha} \frac{\partial}{\partial x_1} - \frac{1}{\alpha} \frac{\partial}{\partial x_2} - \frac{2}{R_{12}} + \frac{1}{R} \right) \chi_1 = 0.
\]

The singlet-triplet energy splitting due to the exchange coupling of the spins for the hydrogen molecule can be represented with the exponential accuracy in the following form \( /3,4,5/ \):

\[
E_{s1} - E_a = 2 \int_S \left( \Psi_2 \nabla_1 \Psi_1 - \Psi_1 \nabla_1 \Psi_2 \right) \cdot \text{d}S,
\]

where \( S \) is the hyperplane \( (x_1 = x_2) \) in the six-dimensional space \( \{r_1, r_2\} \).

Substituting the expression \( \Psi_1 \) and \( \Psi_2 \) and using the condition \( 0 < \alpha << 1 \), we obtain for the part of the energy difference \( \Delta E \) between the singlet and triplet states due to the exchange coupling of the spins of two atoms

\[
\Delta E = (E_s - H) - E_a = -\frac{2 R \ln^2(H)}{\cos^2(\theta)} \left[ 2 \ln(H) \cos(\theta) + \frac{1}{2} R \sin^2(\theta) \right] \\
\times \exp \left[ -R \left( 2 \ln(H) \cos(\theta) + \frac{1}{4} R \sin^2(\theta) \right) \right].
\]

For large distances \( R \) between the two hydrogen atoms in an ultrahigh magnetic field, the atoms interact like two quadrupoles. Since the electron density distribution is \( \psi^2(x) \), and the
quadrupole moment for the atoms is \( Q = 2 \bar{x^2} = \alpha^2/2 \), thus the energy of interaction of the two quadrupoles at distance \( R \) is \( U_{\text{QQ}} = (9/8) (\ln^4 H) R^5 P_4(\cos(\theta)) \), where \( P_4(\cos(\theta)) = (1/8) (35 \cos^4(\theta) - 30 \cos^2(\theta) + 3) \) is the Legendre polynomial.

Taking into account the quadrupole interaction as well as the exchange coupling of spins as the singlet-triplet splitting, we can write the final expression for the energy levels of the hydrogen molecule with the condition \( R \gg -\alpha \). For the triplet term:

\[
U_S = E_S - E_0 = -\frac{1}{2} \Delta E + H + U_{\text{QQ}},
\]

and for triplet term:

\[
U_A = E_A - E_0 = +\frac{1}{2} \Delta E + U_{\text{QQ}},
\]

where \( E_0 \) is the ground-state energy of the two isolated hydrogen atoms.

Since the energy of the quadrupole interaction has a deep negative minimum for \( \theta = 49^\circ \), binding states are possible for both singlet and triplet states. However, for the condition \( H >> 1 \), the triplet energy level lies much deeper than the singlet level, which means that the triplet is the ground state for the hydrogen molecule in an ultrahigh magnetic field.

The results of the numerical calculation /6/ according to Eqs. (9) and (10) are shown in Fig. The curves in the figure correspond to the singlet level (top) and the triplet level (bottom), respectively; the numbering 1, 2, 3, on the curves corresponds to the values of the magnetic field, 20, 50, 100, respectively. At a very small distance \( R \leq \alpha \) the quadrupole interaction is replaced by the Coulomb repulsion. For the distance \( \alpha << R \approx 1 \) the interaction energy has a deep minimum for both singlet and triplet terms, which may be the ground or exited states of the hydrogen molecule. The depths of the potential wells for the two terms are increasing with increasing magnetic fields. With increasing values of the magnetic field the molecular size decreases. As long as the molecular size remains comparable to or larger than the atomic size in an ultrahigh magnetic field, the solutions obtained above are asymptotically exact. This result is correct up to a value for magnetic field of about 1000, where the atomic size become
approximately equal to the molecular size.
Thus, a substantial change in the physical properties of matter occurs in the presence of an ultrahigh magnetic field. The interaction of the two hydrogen or hydrogen-like atoms means that a hydrogen molecule or biexiton can form with a ground state that is a triplet state but not a singlet state, as it is for a H₂ molecule in the absence of a magnetic field. If the pair interaction occurs according to the triplet state than the transition into the superfluid state of the hydrogen gas in the strong magnetic field is possible due to the very shallow potential well in this state which means a weak pair interaction /7/. From the other hand the depth of the potential well in the singlet state is extremely large. It is of order of a few hundred eV for the hydrogen atoms. It means that the creation of spin-oriented molecules, long polymeric chain or structures of the liquid crystal type in the metastable state would be possible.
Magnetic fields of the scale of order $10^{10} - 10^{12}$G exist on the surface of neutron stars and pulsars and are significant interest for astrophysics. The characteristic value of the "atomic" magnetic field for a hydrogen-like systems (exitons) is available in laboratories. For example, the value of a few kOe is an "ultrahigh" field for InSb.

1. INTRODUCTION

In this paper we have made attempts to study the effects of ion temperature, plasma density and streaming velocity of the ions on the propagation of a two-dimensional ion acoustic soliton in a collisionless relativistic plasma. For this purpose we consider a two-dimensional magnetic field-free, collisionless and inhomogeneous plasma. It is assumed that the velocity of ions is relativistic in only the x direction, and that the spatial and temporal variations of the physical quantities associated with the nonlinear evolution of the ion acoustic soliton are larger in x direction than in y direction. Since the results of this theory are applied to the radiation belt plasma, we choose in it a region where electron temperature $kT_e = 120$ keV. The effect of terrestrial field ($B = 10^{-7}$ T) on the plasma particles is assumed to be negligible. The two-dimensional Korteweg-de Vries (KdV) equation is obtained and a few limiting cases are presented.

2. THE KdV EQUATION AND ITS SOLUTION

The two-dimensional normalized fluid equations can be written as

$$n_t + n u_x + n v_y + u n_x + v n_y = 0, \quad (1)$$

$$(\gamma u)_t + u(\gamma u)_x + v(\gamma u)_y + \phi_x + p_x/n = 0, \quad (2)$$

$$v_t + u v_x + v v_y + \phi_y + p_y/n = 0, \quad (3)$$

$$p_t + u p_x + v p_y + 3p u_x + 3pv_y = 0, \quad (4)$$

$$(\phi_{xx} + \phi_{yy}) - n_e + n = 0, \quad (5)$$

where: $n_e = \exp(\phi); \gamma = (1 + u^2/2c^2)$.
In the above equations \( n \) and \( n_e \) are respectively the ion and electron density, \( u \) and \( v \) are respectively the streaming velocities in \( x \) and \( y \) directions. \( c \) is the speed of light. \( \phi \) and \( p \) are respectively the electrostatic potential and ionic pressure. Now we expand the physical quantities about equilibrium as \(^2_3\)

\[
\psi = \sum_j \varepsilon^j \psi_j, \quad j = 0,1,2,\ldots ; \quad \nu = \sum_k \varepsilon^{k+1/2} \nu_{k+1}, \quad k = 0,1,2,\ldots
\]

where \( \psi = (n,u,\phi,p) \). The stretched co-ordinates can be written as under \(^1_3\)

\[
\xi = \varepsilon^{1/2} \left[ \int dx'/\lambda_0(x') - t \right] ; \quad X = \varepsilon^{3/2} x ; \quad \tau = \varepsilon y,
\]

where \( \lambda_0 \) is the phase velocity and \( \varepsilon \) is a small \((0 < \varepsilon \ll 1)\) expansion parameter. The use of eqs. (6) and (7) in eqs. (1)-(5) gives the following first order equations in \( \varepsilon \)

\[
- n_1 \xi + (u_0/\lambda_0) n_1 \xi + (n_0/\lambda_0) u_1 \xi + n_0 v_{11} + (n_0 u_0/\lambda_0) \chi = 0,
\]

\[
- n_0 u_1 \xi + n_0 u_0 u_0 \chi + (n_0 u_0/\lambda_0) u_1 \xi + p_0 \chi + (1/\lambda_0) p_1 \xi + n_0 \phi_0 \chi + (n_0/\lambda_0) \phi_1 \xi
\]

\[
-(3n_0 u_0^2/2c^2) u_1 \xi + (3n_0 u_0^2/2\lambda_0 c^2) u_1 \xi + (3n_0 u_0^2/2c^2) u_0 \chi = 0,
\]

\[
- p_1 \xi + (u_0/\lambda_0) p_1 \xi + (3p_0/\lambda_0) u_1 \xi + u_0 p_0 \chi + 3p_0 u_0 \chi + 3p_0 v_{11} = 0,
\]

\[
- n_0 \phi_1 + n_1 = 0.
\]

Integration of eqs. (8)-(11) with appropriate boundary conditions \((n_0 = 1, u_0 = 0, \phi_0 = \phi, p_0 = 1\) as \( \xi \rightarrow \infty \)) yields the following relation for \( \lambda_0 \), the phase velocity:

\[
\lambda_0 = u_0 + \mu_0/\gamma_1,
\]

where: \( \mu_0 = (1 + 2p_0/n_0)^{1/2}; \gamma_1 = (1 + 3u_0^2/2c^2)^{1/2}. \)

Now we write the second-order equations in \( \varepsilon \) as under:

\[
- n_2 \xi + (1/\lambda_0) (n_0 u_2 + n_1 u_1 + u_0 n_2) \xi + (n_0 u_1 + n_1 u_0) \chi
\]

\[
+ n_0 \nu_{21} + n_1 \nu_{11} + v_{11} n_{11} = 0,
\]

\[
- n_0 u_2 \xi - n_1 u_1 \xi + (n_0 u_0/\lambda_0) u_2 \xi + n_0 u_0 u_0 \chi + (n_0 u_1 + n_1 u_0)/\lambda_0 \phi_1 \xi
\]

\[
+ (n_0 u_1 + n_1 u_0) u_0 \chi + (1/\lambda_0) (n_0 \phi_{21} + n_1 \phi_{11}) + (n_0 \phi_1 + n_1 \phi_0 \chi)
\]
Using eqs. (12), (13) and eqs. (8)-(11) in the second-order equations, one can obtain the following two-dimensional KdV equation:

\[ W_x + \left( \frac{1}{\lambda_0^2} \right) \left[ \left( \frac{2}{\nu_0^2} - 1 \right) \nu_0^2 \lambda_0^2 + \nu_0 \left( 1 - \gamma_1 \right) / \nu_0 \lambda_0^2 \right] W \frac{\partial^3 W}{\partial x^3} + \left( \frac{1}{2 \nu_0^2} \right) \lambda_0^2 \frac{\partial^3 W}{\partial x^3} \frac{\partial^3 W}{\partial t^3} + \nu_0 \Delta_1 W = 0, \]

where \( W = n_1^{1/2} \lambda_0 u_1 \) and \( \Delta_1 = 2 \gamma_1 \lambda_0 n_0 v_1 \). The one-soliton solution of eq. (18) is:

\[ W = W_0 \text{ sech}^2 \left( \frac{\xi}{\zeta} \right), \]

where \( \xi = \xi + (B/A) \xi - (C/A) X \). B/A and C/A are the velocities of the stationary wave in two dimensions. The peak soliton amplitude \( W_0 \) and the soliton width \( \zeta \) are:

\[ W_0 = \left( 6 \nu_0 \gamma_1^2 \lambda_0 n_0 / \alpha \right); \zeta = \left( 2 \delta \nu_0^2 \gamma_1^2 \lambda_0 n_0 / \beta \right)^{1/2}. \]

Here, \( \delta \) = constant and \( \alpha \) and \( \beta \) are respectively the coefficients of nonlinearity and dispersion. Using \( W = n_1^{1/2} \lambda_0 u_1 \) in eq. (19) we can obtain the following expressions:\n
\[ n_m = \left( 6 \gamma_1^3 n_0^3 / \alpha \right); \phi_m = \left( 6 \gamma_1^3 n_0^{1/2} \delta / \alpha \right); p_m = \left( 6 \gamma_1^3 (\nu_0^2 - 1) n_0^3 / \delta \alpha \right), \]

where \( n_m \), \( \phi_m \) and \( p_m \) are respectively the peak ion density, peak potential and peak pressure of the two-dimensional soliton.

3. APPLICATION TO RADIATION BELTS AND LIMITING CASES

We choose a typical region in the radiation belt plasma with \( kT_e = 120 \) keV. The effect of earth's magnetic field \( (B = 10^{-7} \) T) is assumed to be negligible on the motion of plasma particles. Three values of plasma densi-
ty $n_0 = 0.1, 0.5$ and $0.9$ are chosen. We fix the temperature ratio $T_i/T_e = T_0 = 0.03$. For $kT_e = 120$ keV, $kT_i = 3.6$ keV. In addition to this three values for the streaming velocity of ions, i.e., $u_0/c = 0.06, 0.08$ and $0.10$ are chosen.

It is evident from the phase velocity relation [eq. (12)] that $\lambda_0$ increases continuously with $u_0/c$ for fixed $T_0$. The effect of ion temperature on $\lambda_0$ is, however, negligible as is evident from eq. (12). This is because the ratio of the ion thermal speed and speed of light $u_{th}/c (=0.002$ for $T_0 = 0.03$) is much smaller compared with even the minimum value $u_0/c (=0.06)$. The propagation of the soliton is influenced by the plasma density also. A careful observation of eqs. (20) and (21) reveals that while $N_0, \eta_m, \phi_m$ and $p_m$ increase, soliton width $\gamma$ decreases with increasing $u_0/c$ for all the values of $n_0$. In brief we can say that propagation of two-dimensional soliton is influenced both by the plasma density and streaming velocity of ions. However, since $u_{th}/c \ll u_0/c$, the thermal effects are almost negligible.

For a one-dimensional non-relativistic plasma ($u_0/c \rightarrow 0, T_0 = \text{finite}$), our KdV equation reduces to equation (44) of Singh and Dahiya. And for one-dimensional non-relativistic cold plasma ($u_0/c, T_0 \rightarrow 0$), our equation attains the form of the KdV equation of Kuehl. Thus, we see that our two-dimensional KdV equation has few interesting limiting cases.

4. REFERENCES

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THE RAYLEIGH-TAYLOR INSTABILITY IN A SUPER-DENSE IGNITING PLASMA

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1. Introduction

The Rayleigh-Taylor (RT) instability arises when two fluids of different density are in contact, and the denser fluid is accelerated towards the lighter. Such a situation is present in a variety of physical scenarios and has important implications. In Inertial Confinement Fusion applications the RT instability, which appears due to inhomogeneous heating of the target pellet, leads to a deformation of the imploding shock front and to the degradation of its convergence. In stellar evolution, the RT instability plays a crucial role in the propagation of the combustion wave in Type Ia supernovae (Woosley 1986). It is therefore important to account for this effect in simulations in order to explain observations.

Combustion flames in supernovae are thought to propagate as deflagration waves especially in the early phases of combustion near the center of the star. Through the subsonic deflagration front the pressure is continuous and thus the incinerated hot material must be less dense than the cold fuel on top of it. The density inversion in the strong gravitational field of the star produces RT instability. The propagation of the combustion passes through at least two distinct phases. Near the center the instability hasn’t enough time to develop and the combustion advances exclusively by conduction (conductive propagation of the burning front). After a while, the instability is capable of mixing the hot material in the front with the fuel, and thus increases the speed of the burning wave.

To date numerical simulations of supernovae explosion have relied on analytical parametrizations of the mixing or of the deflagration velocity, and results therefore depend largely on the approximation adopted (Woosley 1986, Nomoto et al 1984, Bravo et al 1992). Much effort has been devoted to the calculation of the conductive velocity of the laminar flame (Woosley 1986, Timmes and Woosley 1992, Garcia et al 1990, Khokhlov 1992). Timmes and Woosley have fitted an analytical expression to the conductive velocity in a white dwarf star composed of $^{12}C$ and $^{16}O$:

$$v_{\text{cond}} = 80.2 \left(\frac{\rho}{2 \times 10^9}\right)^{0.839} \left(\frac{X(^{12}C)}{0.5}\right)^{1.12} \text{ km/s}$$

where $\rho$ is the density in $\text{g cm}^{-3}$ (typical values are $10^8 - 10^{10} \text{ g cm}^{-3}$), and $X(^{12}C)$ is the relative abundance of $^{12}C$ by number. The same authors have proposed a treatment of the propagation in the turbulent phase. The deformation of the front surface produced by RT instability is responsible for the increasing efficiency of heat transport through the burning front. In this approximation the rate of the turbulent velocity of the burning front to the conductive velocity is the same as the rate of the front surface to the non-deformed spherical surface at the position of the
Various authors have pointed out that the RT front has a fractal geometry and therefore can be described by only 3 parameters, the fractal dimension $\nu$, and the two length scales $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$, between which the flow is self-similar (Peters and Franke 1990, and references therein). The effective velocity of the deflagration is then given by (Woosley 1990)

$$v_{\text{eff}} = v_{\text{cond}} \left( \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \right)^{\nu - 2}$$

The aim of this study is to propose a method of analysis of the unknown parameters of the structure of the burning front that determine the velocity of the deflagration wave, i.e., $\nu$, $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$, for the physical conditions characteristic of supernovae arising from white dwarf stars. In this astrophysical context $\lambda_{\text{min}}$ is the minimum flame deformation compatible with the conductive advance of the front, and $\lambda_{\text{max}}$ is of the order of the size of the burnt zone. The fractal dimension has been found on the basis of theoretical arguments for turbulence, in the limit of large Reynolds numbers, to be $\nu = 7/3$ (Peters and Franke 1990, and references therein). The approach proposed here is to simulate RT instability with a hydrodynamical particle code, SPH, and to study the scaling properties of the frontier between ashes and fuel. In section 2 the numerical treatment of the hydrodynamic equations as well as the physics involved is described. In section 3 the processing procedure of the outputs in order to obtain the fractal parameters of the front surface is presented.

2. Method of calculation

The growth of RT instabilities in the star plasma is an inherent tridimensional phenomenon. To follow its evolution and to extract the main parameters without an excessive use of too much computer time a simple, but robust hydrocode is needed. Smooth Particle Hydrodynamics (SPH) is a Lagrangian particle method which has been specially devised for astrophysical problems when large departures from spherical geometry are expected. In this method fluid properties are concentrated in a set of imaginary particles with mass $m$ which move according to the hydrodynamical equations. An accurate interpolation procedure allows the reconstruction of the continuous properties of the fluid from the calculated particle positions. SPH has the advantage of not requiring a grid.

As a first approximation in order to evaluate $\lambda_{\text{min}}$, $\lambda_{\text{max}}$, and $\nu$ values neither nuclear nor transport processes are included and a relatively simple equation of state is used. With this hypothesis the equations which govern the evolution are (Monaghan 1985):

$$\rho_i = \sum_{j=1}^{N} W_{ij} m_j$$

$$\frac{d \vec{v}_i}{dt} = - \sum_{j=1}^{N} m_j \left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \left[ 1 - \frac{\alpha h \sigma_{ij}}{c} + \beta \left( \frac{h \sigma_{ij}}{c} \right)^2 \right] \nabla_j W_{ij} - \vec{g}_i$$
\[
\frac{d \varepsilon_i}{dt} = \frac{P_i}{\rho_i^2} \sum_{j=1}^{N} m_j (\mathbf{\ddot{u}}_i - \mathbf{\ddot{u}}_j) \cdot \mathbf{\nabla}_i W_{ij}
\]

where \( N \) is the number of particles which transport mechanical as well as thermodynamical properties, \( W_{ij} \) is a smoothing interpolant kernel, \( \mathbf{\ddot{u}}_i \) is the gravitational acceleration, \( \sigma_{ij} \) is the artificial viscosity, and \( \alpha \), \( \beta \) are constants of order unity. The other quantities have their usual meaning.

The equation of state which is used is that of an ideal gas of ions plus radiation plus totally degenerate electrons with a first order temperature correction (Chandrasekhar, 1939).

We start the calculation by constructing an initial model in hydrostatic equilibrium with a central density according to that expected for a Type Ia supernova explosion. The equilibrium configuration is reached after letting the initial random particle distribution evolve with a dissipative term. Then, a selected quantity of mass is incinerated and a set of perturbations, of different size and wavelength is introduced which modifies the initial equilibrium configuration. The subsequent instability evolution will be followed with the SPH code, and the results analyzed as described in the next section.

3. Analysis of the results

The results of the hydrodynamic computation provide a distribution of particles of both the incinerated fluid and the fuel fluid. The following information is extracted from this distribution, fractal dimension of the front surface \( \nu \), maximum wavelength of the perturbation \( \lambda_{\text{max}} \), and minimum wavelength \( \lambda_{\text{min}} \). The fractal nature of the front surface implies that it is self-similar, at least over some significant length scales. A method of computation of the fractal dimension \( \nu \) from Viccelli (1988) is adopted. In this method, the fractal dimension is approximated by the correlation dimension of the sample of points defining the solution (Grassberger and Procacci 1983). First, one must choose randomly distributed points as centers for sets of spherical surfaces of decreasing radius. Then one must count the number of particles inside each sphere for a given radius and sum over all the centers. The correlation dimension is then obtained as

\[
\nu = \lim_{N \to \infty} \lim_{r \to 0} \frac{\ln n(r)}{\ln r}
\]

When plotted on a graph of \( n(r) \) vs. \( r \) the data are distributed on a straight line, where self-similarity is present. The maximum length scale \( \lambda_{\text{max}} \) can be derived from this plot, as the larger scale at which self-similarity is lost.

The correlation dimension is strictly a lower bound to the fractal dimension, and one of its various possible estimators. The correlation dimension has been shown to provide a good determination of the fractal dimension, within the error limits, for some dynamical systems (Grassberger and Procacci 1983). It is also a relatively straightforward method to use. In this case, the results of the hydrocode must be sorted in order to extract the front surface geometry. The particle-counting algorithm is restricted to
particles near the front surface, to avoid calculating the correlation dimension of the whole figure inside the front instead of the front itself.

By limiting the calculation to the frontier another problem is avoided that would have arisen in the interpretation of the results. The correlation dimension of a dynamical system measures the geometry of a sample of points weighting them according to the frequency at which the different parts of the figure are visited as the system evolves. It is important to note that a geometrical measure of the front is only taken into account. If this measure were carried out on the whole figure, then the weighting of the different points would have taken place as a result of the different density of the fluid, caused by a variable accumulation of particles. For instance, a lower density near the front surface would have produced a steeper dependence of $n(r)$ on $r$, and thus an artificially enhanced correlation dimension.

As to the minimum length scale of self-similarity $\lambda_{\text{min}}$, it will be the greatest of various quantities: $r_{\text{min}}$, $d$, $\lambda_c$. $r_{\text{min}}$ is the lower length scale of self-similarity in the plot of $n(r)$ vs. $r$. $\lambda_c$ is the minimum wavelength that grows without being absorbed by the advance of the conductive front, and is determined from the results of the hydrocode in a straightforward way.

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References

NEW ELECTROSTATIC INSTABILITIES IN NONUNIFORM PARTIALLY IONIZED MAGNETOPLASMAS

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The electron-temperature-gradient drift wave instability involving low-frequency long wavelength electrostatic waves in fully ionized collisional magnetoplasmas with equilibrium density and electron temperature gradients has been investigated by many authors /1-3/. Here consideration of the electron energy equation is very essential. In the long mean free path limit in which the wave motion is sufficiently slower than the electron diffusion along the external magnetic field lines, there exists a phase difference between the wave potential and the density and electron temperature perturbations. We thus have the possibility of the collisional temperature gradient drift instability.

However, most of the plasma in radio-frequency discharges, in the E-region of the Earth’s ionosphere, in cometary tails, and in the ionosphere of the Venus are weakly ionized and inhomogeneous. In partially ionized plasmas, collisions between charged particles and neutrals play a dominant role and wave activities can occur provided that there exists free source of energy in the system. For example, during strong auroral activities in the E-region of the Earth’s ionosphere, one /4/ encounters low-frequency electrostatic turbulence and strong \( \eta_e \) due to the heating of the background plasma electrons. Furthermore, recent experiments /5/ in flybys of comet Giacobini and comet Halley have observed small scale density and temperature fluctuations in the transition and sheath regions around the comets. Drift-like activities have also been seen in a partially ionized laboratory plasma /6/. Since fluctuations are seen to exist in the regions where there are equilibrium density and electron temperature gradients, it is of practical interest to enquire whether free energy stored in plasma inhomogeneities could be coupled to low-frequency fluctuations in a weakly ionized magnetoplasma, which is dominated by neutrals.

In this paper, we present new instabilities of low-frequency \((\omega << \nu_{in})\) and long wavelength \((\lambda >> \rho_j, \lambda_{Dj})\) electrostatic modes in a weakly ionized magnetoplasma with equilibrium density and electron temperature inhomogeneities. Here, \( \nu_{in} \) denotes the ion-neutral collision frequency, which is usually much smaller than \( \omega_{ce} \); and \( \rho_j \) and \( \lambda_{Dj} \) are, respectively, the gyroradius and the Debye length of the particle species \( j \) (equals \( e \) for electrons and \( i \) for ions). We consider the dynamics of both the electron and ion fluids, but assume the neutral fluid to be at rest. For wave frequencies of our interest, the charged particles are magnetized and collisional.

Consider a nonuniform partially ionized magnetoplasma consisting of electrons \((e)\), ions \((i)\), and neutrals \((n)\). The plasma is embedded in a uniform magnetic field \( B_0 \) which is along the \( z \) axis, and has the equilibrium density and electron temperature gradients along the \( x \) axis: i.e., \( n_{ei} = n_0(x) \) and \( T_e = T_e(x) \). In the presence of low-frequency \((vix. |\partial_x| << \nu_j << \omega_{ce} = eB_0/m_e c)\) electrostatic potential \( \phi \), the perpendicular (to \( B_0 \)) component of the electron fluid velocity is given by

\[
 v_{e\perp} \approx v_E + v_D, \tag{1}
\]
where \( \mathbf{v}_E = (c/B_0) \hat{z} \times \nabla \phi \) and \( \mathbf{v}_D = -(c/eB_0n_e) \hat{z} \times \nabla (n_eT_e) \) are the \( \mathbf{E} \times \mathbf{B}_0 \) and the diamagnetic drift velocities, respectively. The parallel component of the electron fluid velocity is determined by

\[
\mathbf{v}_{e\parallel} \approx (e/m_e\nu_e) \partial_\parallel [\phi - (T_\parallel/e n_0) n_e - T_{e\parallel}/e],
\]

where \( \nu_e = \nu_{e\parallel} \approx \nu_e, \ n_e = n_e - n_0(x) \ll n_0, \) and \( T_{e\parallel} = T_e - T_0(x) \ll T_0. \)

For \( m_e\nu_{e\parallel} / m_i \ll \nu_i, \) the perpendicular component of the ion fluid velocity is

\[
\mathbf{v}_{i\perp} \approx [\omega_i^2/(\omega_i^2 + \nu_i^2)](c/B_0) [\hat{z} \times \nabla \phi - (\nu_i/\omega_i) \nabla \phi],
\]

where \( \omega_i = eB_0/m_i c \) is the ion gyrofrequency. The parallel component of the ion fluid velocity is given by

\[
\mathbf{v}_{i\parallel} \approx -e/(m_i\nu_i) \partial_\parallel \phi.
\]

Letting \( n_i = n_0(x) + n_{i\perp}, \) and \( T_z = T_0(x) + T_{i\parallel}, \) assuming the perturbations (denoted by the subscript 1) to be small in comparison with the equilibrium quantities (denoted by the subscript 0), and inserting (1) and (2) into the continuity and the energy equations of the electron fluid, we obtain, respectively,

\[
(\partial_t + \nu_R - D\partial_\parallel^2) n_e - (c/B_0) \hat{z} \times \nabla n_0 \cdot \nabla \phi + (e\nu_0 D/T_0) \partial_\parallel [\phi - T_{e\parallel}/e] = 0,
\]

and

\[
(\partial_t - (1/\tau_e) + (2/3) \gamma_{ex} - D \parallel \partial_\parallel^2 - D_\perp \nabla_\perp^2) T_{e\parallel} - (c/B_0) \hat{z} \times \nabla T_0 \cdot \nabla \phi + (2cT_0/3B_0) \hat{z} \times \nabla \ln n_0 \cdot \nabla \phi - (2T_0/3n_0) \partial_\parallel T_{e\parallel} = 0,
\]

where \( D = \omega_i^2/\nu_i, D_\parallel = 2C_\parallel \omega_i^2/3 \nu_i, D_\perp = 2C_\perp \omega_i^2/3 \nu_i, \tau_e = n_0 m_n/2 n_m m_e \nu_e, \) and \( \nu_{i\parallel} = (T_i/m_i)^{1/2} \) is the electron thermal velocity. The recombination rate is denoted by \( \nu_R. \)

On the other hand, substituting for \( \mathbf{v}_i \) from (3) and (4) into the ion continuity equation, we obtain

\[
(\partial_t + \nu_R) n_{i\perp} - [\omega_i^2/(B_0 (\omega_i^2 + \nu_i^2))] [\hat{z} \times \nabla n_0 \cdot \nabla \phi + (\nu_i/\omega_i) \nabla n_0 \cdot \nabla \phi]

= -(e\nu_0 B_0) [(\nu_i/\omega_i) (\omega_i^2 + \nu_i^2)] \partial_\parallel^2 \phi = 0,
\]

Thus, (5), (6), and (7) together with \( n_{e\parallel} = n_{i\perp} \) constitute a coupled set for studying electrostatic instabilities in nonuniform partially ionized magnetoplasmas.

We now derive a general local dispersion relation assuming that the wavelength of the electrostatic waves is much smaller than the scale lengths of the density and electron temperature gradients. Supposing that the perturbed quantities \( \phi \) and \( T_{e\parallel} \) are proportional to \( \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)], \) where \( \omega \) and \( \mathbf{k} \) are the frequency and the wave vector, we can Fourier analyze (5), (6), and (7) and combine them assuming \( n_{e\parallel} = n_{i\perp}. \) The result is

\[
(A_r + iA_i) \omega^2 + (B_r + iB_i) \omega + G_r + iG_i = 0,
\]

where \( A_r = \omega R [1 + f \omega_i^2/(\omega_i^2 + \nu_i^2)], A_i = D k^2 - [1 + \nu_i^2 k_\perp^2/(\omega_i^2 + \nu_i^2)] (k_\perp^2 + \nu_i^2)] k^2 \equiv \omega R - \Gamma, B_r = \omega D (\omega_\parallel + \nu_R) + (\omega_\parallel + \omega_\perp + 2 \omega_D/3) \Gamma, B_i = (\omega_\parallel + \omega_\perp + 2 \omega_D/3) (A_r - \omega_\parallel) - \omega_\parallel (\nu_R + \omega_\parallel + \omega_D(\eta_e - 2/3)), G_r = \nu_R \omega_\parallel (\omega_\parallel - \omega_D(\eta_e - 2/3)) - (A_r - \omega_\parallel) \nu_R \omega_\parallel, G_i = \omega_D (\nu_R \nu_D + \Gamma \omega_R). \) We have denoted \( \omega_R = \nu_R + \omega_D \) and \( \omega_L = (1/\tau_e) + 2 \gamma_{ex}/3 + D \parallel k_\parallel^2 + D_\perp k_\perp^2. \)
Some useful analytical results can be obtained in three limiting cases. Firstly, for $|\omega| >> \nu_R, \omega_L, \omega_D$, the relevant dispersion relation is solved letting $\omega = \omega_r + i\gamma$. We obtain for the real part of the frequency

$$\omega_r = -(\omega_D \omega_{ci} \nu_{in}) [(2a/3)(\kappa_n' \cdot k)(ak_1^2 + k_3^2) + (\kappa_T - 2a \kappa_n'/3) \cdot k\times(a^2(\kappa_n' \cdot k)^2 + (\omega_{ci}/\nu_{in})^2(ak_1^2 + bk_3^2)^2),$$

and an expression for the growth rate

$$\gamma = \omega_D [a(\kappa_n' \cdot k)(\kappa_T - 2a \kappa_n'/3) \cdot k - (2/3)(\omega_{ci}/\nu_{in})^2(ak_1^2 + k_3^2)\times(a^2(\kappa_n' \cdot k)^2 + (\omega_{ci}/\nu_{in})^2(ak_1^2 + bk_3^2)^2)],$$

where $\kappa_n' = \kappa_n - (\omega_{ci}/\nu_{in}) \nabla \ln n_0$, $\kappa_n = \hat{z} \times \nabla \ln n_0$, $\kappa_T = \hat{z} \times \nabla \ln T_0$, $a = \nu_{ci}^2/(\omega_{ci}^2 + \nu_{in}^2)$, and $b = 1 + \text{min}_{\nu_{in}} \nu_{ci}.$

For $\omega_{ci}/\nu_{in} << k_y/k_z$, it is seen that there is a growth when $\eta_e = -d \ln T_0/d \ln n_0 \equiv \nu_{ce}/L_n/L_T$ is greater than $2a/3$. However, it is of particular interest to note that for $\omega_{ci}/\nu_{in} >> k_y/k_z$ the instability occurs when $\eta_e$ is negative and $|\eta_e| > (2a/3)(\omega_{ci}/\nu_{in})/(k_z/k_y)$, otherwise for $d \ln T_0 > 0$ and $d \ln n_0 > 0$ there is no instability. The resistive instability in a partially ionized plasma arises provided that

$$(\kappa_n' \cdot k)(\kappa_T - 2a \kappa_n'/3) \cdot k > (2/3a)(\omega_{ci}/\nu_{in})^2(ak_1^2 + bk_3^2)(ak_1^2 + k_3^2).$$

Secondly, we consider the limit $\nu_R << |\omega| << \omega_D$ and assume that the magnetic field-aligned electron thermal diffusion rate dominates over cross-field diffusion rate, the electron energy relaxation rate, as well as the heat exchange loss. This case, which is of significant interest to a recent laboratory experiment/6/ puts a restriction on the parallel wavelength $\lambda_p = 2\pi/k_z$, viz. $4\pi^2 D_{\parallel}/\lambda_p^2 >> D_{\perp}k_3^2 + (1/\tau_e) + (2/3)\gamma_e$. Here, the dispersion relation is of the form

$$\omega + i(\omega/\omega_D)\{(1 + D/D_{\parallel})\omega - \omega_s[1 + (D/D_{\parallel}) - (3D/2D_{\parallel})\eta_e]\}
- [\omega_s(1 + k_z\nu_{in}/k_y\omega_{ci})/(1 + \nu_{in}^2/\omega_{ci}^2)] + i\Gamma = 0.$$

(12)

Letting $\omega = \omega_r + i\gamma$ in (12), and assuming $\gamma << \omega_r$, we obtain for the real part of the frequency

$$\omega_r \approx \omega_s(1 + k_z\nu_{in}/k_y\omega_{ci})/(1 + \nu_{in}^2/\omega_{ci}^2),$$

(13)

and an expression for the growth rate

$$\gamma \approx [\omega_s^2(1 + k_z\nu_{in}/k_y\omega_{ci})/\omega_D(1 + \nu_{in}^2/\omega_{ci}^2)]a + (D/D_{\parallel})(a - 3\eta_e/2) - k_z\nu_{in}/k_y\omega_{ci}(1 + \nu_{in}^2/\omega_{ci}^2) - \Gamma.$$

(14)

Thirdly, in order to illuminate the effect of recombinational damping, we consider a simple case in which $|\omega|\sim \nu_R << \omega_D, \omega_L$. Here the growth rate is found to be

$$\gamma = [\Omega(A_r - \omega_s)/(1 + \Omega^2)] - \nu_R - \Gamma/(1 + \Omega^2).$$

(15)

where $\Omega = (\omega_s/\omega_L)[\eta_e - (2/3) - \omega_L/\omega_D]$. The growth rate is positive provided that $\Omega > 0$ and the first term is larger than the last two terms on the right-hand side of (15). The effect of recombination is stabilizing.
As an illustration, we apply our result to the E-region of the Earth's ionosphere at heights below 130 km where during strong auroral activities a strong $\eta_e (\geq 7)$ has been observed /4/, and where the recombinational effect is negligibly small. Accordingly, we take some typical plasma parameters: $n_e = 10^6 \text{cm}^{-3}$, $B_0 = 0.5 \text{T}$, $T_{\text{ex}} = 600 \text{K}$, $m_i/m_e = 16 \times 1836$, $L_n = 36 \text{ km}$, and $L_T = 5 \text{ km}$. We thus have $\omega_{pe}/\omega_{ce} \approx 2$, $\omega_{ci} \approx 300 \text{ s}^{-1}$, $c_s = 4.72 \times 10^4 \text{ cm/s}$, $v_{te} = 8 \times 10^6 \text{ cm/s}$, $\nu_R = 0.001 \text{ s}^{-1}$, $\nu_{ci} = 2300 \text{ s}^{-1}$, $\nu_{en} = 9.2 \times 10^4 \text{ s}^{-1}$, $L_n = 36 \text{ km}$, and $L_T = 5 \text{ km}$. For $k_x/k_y = 0.001$, $k_x/k_y = 0.1$, and $k_y = 10^{-3} \text{ cm}^{-1}$, we analyze (10) and (11) numerically. Our results show that the present instability, which involves modes with $k \parallel L_n$, $k \perp L_T >> 1$, can rapidly generate the auroral E-region irregularities whose transverse scale size ($\text{viz.} \lambda_y = 2\pi/k_y$) is of the order of 63 m.

To summarize, we have presented new electrostatic instabilities that are driven by the combined effect of the equilibrium density and electron temperature inhomogeneities in partially ionized magnetoplasmas in which collisions of charged particles with neutrals play an important role. Physically, plasma nonuniformities in conjunction with charged particles-neutral atom collisions provide a proper phase difference between the electrostatic potential, the density and electron temperature perturbations, so that electrostatic perturbations grow. The growth rates of the instabilities critically depend on $\eta_e$ as well as electron-neutral and ion-neutral collision frequencies. The unstable modes may cause anomalous resistivity, thereby changing the transport properties of the weakly ionized plasma.

Our results could have relevance to the understanding of the origin of nonthermal electrostatic fluctuations in space and laboratory plasmas. For example, the European Incoherent Scattering (EISCAT) observations show a strong enhancement of the electron temperature and a strong electron temperature gradient during strong auroral activities in the E-region of the Earth’s ionosphere. The free energy stored in the equilibrium density and electron temperature inhomogeneities is then coupled to electrostatic fluctuations by the instability mechanism discussed here.

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SPONTANEOUS FORMATION OF INTERMEDIATE SHOCKS

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(I) Introduction:

Evolutionary arguments given by Kantrowitz and Petschek (1967) purport that the intermediate shock is an extraneous solution of the Rankine-Hugoniot relation and hence cannot exist in nature. In the pioneer work of C.C. Wu (1987), the intermediate shock was found to form under a special initial condition. A large-amplitude slow wave was initially set up to let evolve. The leading edges of the wave steepen and finally a train of intermediate shocks form. As the transverse magnetic field, in Wu's calculation, has already reversed the signs in the initial slow wave, it does not strike as such a great surprise to find that the transverse magnetic field has different signs across the shock in the final configuration. (Note that the sign change of the transverse magnetic field is an important characteristic of the intermediate shock.) The suspicion raised by Kantrowitz and Petschek that the presence of an initial intermediate wave may make the intermediate shock unstable is found, in Wu's simulation, not to occur. Despite that Wu's result shows the intermediate shock appearing stable, this model can be vulnerable to a criticism that the intermediate shock results merely from a special arrangement of the initial condition. That is, as the intermediate shock has already been demonstrated to exist, at issue is not whether the intermediate shock can exist, but whether the intermediate shock can appear in a more natural setting and a more natural initial condition.

We confront this issue with the following line of consideration. If a slow shock propagates into a medium of increasing density, for which both the Alfvén speed and sound speed decrease, the shock can be slowed down (in response to the decreasing characteristic speeds of the background plasma). However, the shock speed may not decrease as rapidly as the Alfvén speed. Therefore, the longitudinal Alfvén Mach number may increase as the shock evolves, to such an extent that it exceeds unity, and the slow shock can become an intermediate shock. During this process, a rotational discontinuity should appear downstream of the intermediate shock. The appearance of the rotational discontinuity is to re-reverse the downstream transverse magnetic field to attain the original sign in the region far downstream.

It is known that switch-off shocks occur only for the slow shocks, where a finite upstream transverse magnetic field dissappears in the downstream region. As far as the parameter space, such as the plasma $\beta$ or the upstream field angle, is concerned, the switch-off shock marks the boundary between the slow shock and the intermediate shock, where the downstream transverse magnetic field just begins to change sign. We find that for those slow shocks that are near the switch-off shock in the parameter space, it is easier to convert them into intermediate shocks. In fact, there exists a second boundary in the parameter space, outside which the slow shocks can no longer be converted into intermediate shocks (c.f. Table 1).
(II) Numerical Simulation: We computed the following results with a one-dimensional resistive/viscous MHD code, which uses the two-step Lax-Wendroff scheme to advance the calculation in time. The background plasma has a density ramp within a finite region of the computation domain. At $t=0$, we send in a slow shock from the left boundary toward the ramp. The shock will eventually come out of the ramp, either with or without its character changed to become an intermediate shock. Figure 1 shows a time sequence of the evolution, when a slow shock runs into a density ramp. The physical quantities shown from the top are the density, the normal component of velocity, the transverse component of magnetic field and the gas pressure, respectively. The upstream parameters for the initial slow shock are: the slow-wave Mach number $M_s = 1.76$, the upstream magnetic field angle $\theta = 45$, $\beta = 0.42$ and the density ratio across the ramp $= 3$. The width of the ramp can be much larger than that in Figure 1, but the outcome has no noticeable change. Although it is not so clearly shown in Figure 1, we note that the reversal of the transverse magnetic field has already occurred while the shock is still within the ramp. Upon coming out of the ramp, the intermediate shock is not steady, but can gradually settle into a steady one, whose Mach number can, at this stage, be accurately determined. The steadiness of the intermediate shock depends on the width of the ramp; the smoother the ramp is the more steady the shock is. As is obvious from Figure 1, the by-products in company with the intermediate shock are a 180 degree rotational discontinuity, where $B_z$ changes sign but the total pressure is continuous, a fast shock which leads the intermediate shock and a rebounced fast shock of small upstream field angle, moving backwards into the high $\beta$ plasma and behaving like a gas dynamic shock.

For a given longitudinal Alfvén Mach number $M_A$, the success of the conversion depends on the plasma $\beta$, the incident angle with respect to the magnetic field $\theta$, the amount of dissipation and the height of the density ramp. The latter two factors are in fact intercorrelated. For a shock conversion to occur, we find that the larger the resistivity is the higher the ramp must be; however, with a fixed resistivity, the height of the density ramp can hardly affect the final Alfvén Mach number. This implies that whether the conversion can successfully occur depends weakly on the height of the ramp, as long as the height exceeds some value. This also suggests that some kind of dynamical conservation law prevails during the interaction, in such a way that the final Alfvén Mach number remains unchanged regardless of the height and the width of the density ramp.

Another interesting finding of our results is that when measured by the range of the upstream plasma $\beta$, a slow shock with a larger slow-wave Mach number has a narrower range of $\beta$ for a successful conversion to occur. This is shown in Table 1, where several slow-wave Mach numbers are examined for three different upstream field angles separately. The upper limit of $\beta$ marks the initial shock’s being a switch-off shock and the lower limit marks the boundary, below which no slow shock can be converted into intermediate shocks. The tendancy of the outcome clearly depends on the strength of the slow shock. The details of our results will be published elsewhere.

Reference:
Figure 1: A time sequence of evolution for quantities such as the plasma density, the normal component of velocity, the transverse component of magnetic field and the pressure.
Table 1: Limits of the plasma beta for various upstream field angles and slow-wave Mach numbers.

### θ = 20°

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<tr>
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<td>0.41</td>
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<td>Min β</td>
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</thead>
<tbody>
<tr>
<td>Max β</td>
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<td>Min β</td>
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SPECTRUM OF MHD WAVES IN COMETARY TAILS

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Abstract. The wave structure of rays from cometary plasma tails is modeled using the MHD formulation. Since cometary rays are essentially of cylindrical shape, the model will be constructed using the simple cylindrical geometry. The observational data concerning the dynamical behaviour of rays are very scarce in certain cases there is an ambiguity between wave phenomena and the motion of matter. However, we have searched the available data in the literature, comprising ground based and satellite observations to show that the spectrum of MHD waves might explain some features of the plasma dynamics of cometary rays. Based on our theoretical results, we have also proposed cometary observations combining morphological and spectroscopic techniques attempting to disentangle wave phenomena from mass motions.

Introduction. Usually the Type I (or plasma) tail of comets exhibits time varying fine structures in the light of ions $CO^+ (\lambda = 3500 – 5500\AA)$ and $H_2O^+ (\lambda = 5500 – 7500\AA)$. Such fine structures consist on rays, kinks, helical structures and bright condensations/1/. Rays are narrow, long and straight structures with typical thickness $\leq 2000 km$ and length usually reaching over $10^7 km$. What will be modeled using the MHD approach is the individual structure of one ray. The time evolution of rays can be found in references /2,3,4/. Helical features, kinks, undulations and bright knots appear affecting the rays. All these affections can exhibit apparent downstreaming velocity ranging from 20 to 300 $km/s$. This velocity tends to be larger far away from the nucleus, and far from the axis of the

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plasma tail /5,6/. It is still unclear weather such apparent motions correspond to motions or simply to MHD perturbations/7/. An ion number density of $10^4$ cm$^{-3}$ can be assumed according to /8,9,10/, but the abundance ratio $H_2O^+/CO^+$ is highly variable from one comet to other, and even for one same comet at different times/11/. The ICE data /12/ from the tail of the Comet P/ Giacobini-Zinner indicated that two lateral lobes of high intensity magnetic field ($6 \times 10^{-4} G$) were confining in between a plasma sheet with a field of only $5 \times 10^{-5} G$. This last number will be assumed for the magnetic field of the rays. This means that rays are magnetic struture with high $\beta$ value ($\geq 4$). As a consequence the MHD wave spectrum will show the usually property to be described below. In the following sections we will present the model, the results and the conclusions.

The Model. From the first order perturbation MHD equation we obtain a second order differential equation (Hain-Lüst equation) on $\xi_r$ (r-component of the plasma displacement) derived in /13/ by taking the perturbation quantities as $f(r) \exp(-i\omega t + ik_z z + im\theta)$. In /14/ a stability code has been developed to solve the Hain-Lüst equation using the shooting method and the discrete slow wave modes have been obtained in the region of stable spectrum.

The eigenfrequency $\omega$ and the corresponding eigenmode $\xi_r$, for a given and $m$, can be obtained by solving the Hain-Lüst equation with the following boundary conditions: $r \xi_r = 0$ at $r = 0$ and $r = a/14$.

The profiles for $p$, $B_z$, and $B_\theta$ obey the equilibrium equation for the total pressure

$$\frac{d}{dr}(p + \frac{B^2}{2}) + \frac{B^2}{r} = 0$$

where $B^2 = B_z^2 + B_\theta^2$. From three, two profiles can be chosen arbitrarily. We assume that the gas pressure profile is constant. The $B_\theta$ profile is obtained by solving the Ampère's law with a given $J_z$ profile. For the sake of simplicity, we choose profile of the equilibrium current as $J_z(r) = J_0 \{1 - (r/a)^2\}^2$ and density profile $\rho(r) = \rho_0 \{1 - 0.95(r/a)^2\}$. The constant $J_0$ is determined from the condition that the safety factor at the axis, $q(0)$ ($q = 2\pi a B_z/L B_\theta$, where $L$ denotes the length of the tube), is greater than 1. The shooting method is used to obtain the normalized eigenvalue $\omega^2$ which satisfies the boundary conditions
at $r = 0$ and $r = a$. We choose the normalized quantities such that, $B_{2N} = 1$, $\rho_{0N} = 1$, and $a_N = 1$. Also we choose $m = 1$, $k_x = 0.606$ and $\beta = 4$. The effect of the high $\beta$ is that $C_S > V_A$, where $C_S$ and $V_A$ are the sonic and the Alfvén velocities, respectively.

Results. The numerical solution has been carried out varying $\beta$ and the normalized current density $J_{0N}$. When $\beta > 1$ discrete Alfvén modes can overlap onto the continuum of slow modes. As a consequence the discrete Alfvén modes are expected to suffer a mode conversion into slow modes. Also some discrete Alfvén modes may oscillate at the same frequency of discrete slow modes.

For typical observational values; ion number density $= 1000 \text{ cm}^{-3}$, $B = 5 \times 10^{-6} G$ and tail disturbance length scale $= 10^6 km$, aperiod of the Alfvén wave is obtained of about 1.7 day.

Conclusions. This analysis on the MHD spectrum of cometary rays opens interesting possibilities for modeling and observationally testing the unusual character of the discrete Alfvén modes in high $\beta$ plasmas. With regards to the observational efforts, the indication of surviving alvenic and sonic modes in cometary rays is auspicious. However the future experiments should pay attention to the fact that both discrete modes have the same frequency. Therefore a way to distinguish one mode from the other is crucial for testing our results. This distinction seems to be viable in practice, since the sonic modes can show up as brightness variations, while alvenic modes can appear as morphological variations. Based on such a knowledge the task of disentangling mass motions from wave phenomena may become easier.

Acknowledgements. This work was partially supported by CAPES - Coordenação de Aperfeiçoamento do Pessoal de Ensino Superior, CNPq - Brazilian National Research Council, and FAPESP - Fundação de Amparo a Pesquisa do Estado de São Paulo for financial support.

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LOW-FREQUENCY RESONANT ION-BEAM TURBULENCE IN COMETARY PLASMAS

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A theoretical framework is established, based upon Fowler's theorem for unstable plasmas, for estimating the upper bound on magnetic field fluctuations produced in ion-beam systems, due to the excitation of low-frequency resonant modes. In the region upstream of the cometary bow shock such modes can be excited due to pickup of cometary ions, at Doppler-shifted frequencies equal to the cometary ion gyrofrequencies. The estimates of the present theory show that $\delta B/B_0$ is fairly small, in agreement with observational evidence faraway from the cometary nucleus.

INTRODUCTION AND THEORETICAL FRAMEWORK

In previous papers /1,2/ we used a version of Fowler's theorem for unstable plasmas /3/ to estimate the magnetic field turbulence levels for low-frequency nonresonant unstable modes in a multibeam plasma. Here we apply the same formalism to resonant modes excited due to pickup of cometary ions in the region upstream of the cometary bow shock. For such modes the Doppler-shifted frequencies equal the cometary ion gyrofrequencies.

We start with a homogeneous, charge neutral beam-plasma system immersed in a uniform magnetic field. Both beam and plasma constituents can in principle have parallel drift velocities, as perpendicular equilibrium drifts are eliminated by going to a deHoffmann-Teller frame. We assume charge and current neutrality for both beams, consisting of ions with a given set of characteristics $(N_s, e_s, m_s, U_s)$ plus the electrons necessary to provide the return current. Then in the context of cometary environments, by resonant ion-ion modes one means that the Doppler-shifted wave frequency $\omega - kU_{ci}$ resonates with the gyrofrequency $\Omega_{ci}$ of the pickup ions of cometary origin. Let us start from equation (6) of Lakhina and Verheest /4/, written as

$$((\omega - kU)^2 + k^2 (W - V_A^2))(\omega - kU_{ci} \pm \Omega_{ci}) = \rho(\omega - kU_{ci})^3.$$  

This dispersion law is valid for frequencies small compared with the proton gyrofrequencies $(\omega \ll \Omega_p = \Omega_{sw})$, and the line of reasoning followed is specifically for cometary environments, where the cometary ions of the water group have 16 to 17 times the proton mass /5,6/. In (1), $\overline{U}$ is the mean bulk velocity of the plasma as a whole, $V_A$ the global Alfvén velocity,

$$W = \frac{N_{sw}m_{sw}(U_{sw} - \overline{U})^2 + N_{ci}m_{ci}(U_{ci} - \overline{U})^2}{N_{sw}m_{sw} + N_{ci}m_{ci}} = \rho(1 - \rho)U_{ci}^2$$

the normalized kinetic energy in the relative drift motions /1/; and $\rho = N_{ci}m_{ci}/(N_{sw}m_{sw} + N_{ci}m_{ci})$ the dimensionless mass density of the energetic cometary ion beam, viewed in the solar wind frame (where $U_{sw} = 0$). We can now put

$$\omega = \omega - kU_{ci} \pm \Omega_{ci},$$
use the abbreviations
\[ A = kU'_{ci} \mp \Omega_{ci}, \quad B = k^2(V_A^2 - W) \equiv k^2 c_A^2, \] (4)
and get a compact form for the dispersion law:
\[ F(w) = w\left((w + A)^2 - B\right) = \rho(w + \Omega_{ci})^3. \] (5)

For resonant modes \( w \approx 0 \) and the dominant terms in \( F(w) \) are either \((A^2 - B)w \) or \( 2Aw^2 \), and on the r.h.s. of (5), \( \rho \Omega_{ci}^2 \). This supposes \( \rho \) small enough, so that the roots of the full dispersion law (5) are close to the roots of \( F(w)/\rho \). From the choices for the dominant terms one sees that unstable modes need at least \( A^2 - B \sim 0 \), in other words when \( W < V_A^2 \). Then
\[ w^2 = \pm \frac{\rho \Omega_{ci}^2}{2A} \] (6)
leads to unstable solutions provided \( A = \pm |A| \), depending on whether we deal with the right-hand (RH) or left-hand (LH) polarized mode. Now from \( A^2 \sim B \) and (4) we find that
\[ A \equiv kU'_{ci} \mp \Omega_{ci} = \pm c_A, \] (7)
with \( s = \pm 1 \) but not yet related to the RH/LH polarized modes and \( c_A \) defined as the positive root \( \sqrt{V_A^2 - W} \). Hence the wavenumber is given by
\[ k = \pm \frac{\Omega_{ci}}{U'_{ci} - sc_A}. \] (8)

In the solar wind frame the cometary ions form a sunward beam upstream of the nucleus, and it is their velocity component parallel to the interplanetary magnetic field (IMF) which is denoted by \( U_{ci} \). Hence a positive (negative) value for \( U_{ci} \) means that the IMF has a parallel (antiparallel) component, in other words pointing towards (away from) the Sun. Keeping the sign of \( U_{ci} \) in mind, the different quantities can be expressed as follows:
\[ \bar{U} = \rho U_{ci}, \quad U'_{ci} = (1 - \rho)U_{ci}, \quad c_A = (1 - \rho)(1 - \rho M^2)V_{A,su}^2, \] (9)
where \( M \) is the Alfvénic Mach number with the solar wind Alfvén velocity as the reference, \( M = |U_{ci}|/V_{A,su}, V_{A,su} \) being taken as a positive quantity.

Starting from \( s = +1 \), (7) shows that \( k = \pm |k| \), since for instability \( A = \pm |A| \). In (8) this means that \( U'_{ci} - c_A \) has to be positive, in other words only the choice \( U'_{ci} > c_A > 0 \) is possible and the IMF has a parallel component. In the opposite case where \( s = -1 \), (7) imposes \( k = \mp |k| \) and from (8) there follows that \( U'_{ci} + c_A \) should be negative. This means that \( U'_{ci} < c_A < 0 \), leading to an antiparallel component for the IMF. As one can check, \( |U'_{ci}| > c_A \) in both cases, implying \( M > 1 \). From (6) and (7) we get for the unstable waves that
\[ \text{Re } \omega = \pm \Omega_{ci} \frac{|\bar{U}| + c_A}{|U'_{ci}| - c_A}, \quad \text{Im } \omega = \gamma = \Omega_{ci} \sqrt{\frac{\rho(|U'_{ci}| - c_A)}{2c_A}}. \] (10)

Note that in both cases (\( s = +1 \) and \( s = -1 \)) the RH mode has \( \text{Re } \omega > 0 \) and \( k \) directed along the sunward direction, whereas the LH mode has \( \text{Re } \omega < 0 \) and \( k \) directed in the antisunward
direction. Hence both the RH and LH modes always propagate sunward, viewed in the solar wind frame, with phase velocities $|\overrightarrow{U}| + c_A$.

In order to compute an upper estimate for the ultimate levels of magnetic turbulence in the case of unstable modes, we will make use of Fowler’s equipartition ideas /3/. Performing the same approximations as when deriving the dispersion law (1), one gets

$$\left(V_A^2 + W + \left(\frac{\text{Re} \omega}{k - U} - \nu\omega\right)^2 - \rho \left(\frac{\text{Re} \omega}{k - U} - \nu\omega\right)^2 + (1 - \rho) \left(\frac{\text{Im} \omega}{k}ight)^2\right) + \rho \frac{|\omega - kU_{ci}|^2}{k^2} \frac{\Omega_{ci}^4 + \Omega_{ci}^2|\omega - kU_{ci}|^2}{(\Omega_{ci}^2 - (\omega - kU_{ci})^2)(\Omega_{ci}^2 - (\omega^2 - kU_{ci})^2)} \right) \leq \frac{\delta B^2}{B_0^2}.$$  

Here $F$ represents the normalized free energy associated with the unstable modes. The most unstable mode will grow fastest and use up the available free energy, so that $F$ will be equal to $\gamma^2/k^2$, as shown more clearly in the case of nonresonant modes /1/. Using (10) we can rework (11) and get

$$\frac{\delta B^2}{B_0^2} \leq \left\{ \frac{2k^2V_A^2}{\gamma^2} + 1 - \rho + \frac{\Omega_{ci}^4(2\Omega_{ci}^2 - \gamma^2)}{\gamma^4(4\Omega_{ci}^2 + \gamma^2)} \right\}^{-1}.$$  

**COMETARY APPLICATIONS**

In an 8 nT field one finds that $\Omega_{ci} = 0.046/s \ll \Omega_p \ll |\Omega_e|$, so that the restrictions inherent in our theory will be obeyed provided Re $\omega$ and $\gamma$ do not exceed $\Omega_{ci}$ very much. At an angle of about 45° between the solar wind flow and the IMF we have a Alfvénic Mach number of about 3.5, keeping in mind that $V_{A,sw} \simeq 80$ km/s and assuming a mean solar wind flow with $V_{SW} \simeq 400$ km/s. Table I then shows the relevant quantities computed for those parameters, namely $\rho$, Re $\omega$, $\gamma = \text{Im} \omega$, $\lambda = 2\pi/k$ and $\delta B^2/B_0^2$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Re $\omega$ (s$^{-1}$)</th>
<th>Im $\omega$ (s$^{-1}$)</th>
<th>$\lambda = 2\pi/k$ (10$^3$ km)</th>
<th>$\delta B^2/B_0^2$</th>
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<tbody>
<tr>
<td>0.1</td>
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The maximum levels of magnetic turbulence given in Table I are in good agreement with in situ observations near comet P/Halley /8-10/. We may point out that computer simulations of cometary-ion resonant instabilities as reported by Gary et al. /11,12/ predict saturation amplitudes which show good overall agreement with the estimates given in Table I. However, at strong injection of free energy, Gary et al. /11,12/ found that a single Fourier mode emerges from the noise to exhibit exponential growth. The peak value of the fluctuating magnetic field amplitude associated with this mode can be quite large ($\delta B \sim B_0$) but the energy decays within a few hundred proton gyroperiods to a much smaller value. Our formalism deals with the saturated amplitudes and we cannot predict what happens in the intermediate regime. Also, we consider the cometary ion density as given, whereas in the computer simulations /11,12/ cometary ions were continuously injected. These subtle differences must be kept in mind while making any comparison between our theory and the computer simulations!
CONCLUSIONS

Since the observed ULF turbulence at both comets P/Giacobini-Zinner and P/Halley is transverse and characterized by a spectral peak near 0.01 Hz and since water-group ions dominate the pickup ions, it is natural to investigate whether the water-group ions and waves are in gyroresonance and to compute the contribution of such resonant modes to the observed turbulence. However, in order to discriminate between resonant and nonresonant modes as being more important, one would need wavelength measurements, but these are not (yet) available. So the case for the resonant modes, as advanced by a majority of the researchers in the field and as investigated here, rests for the time being solely on frequency measurements, only half of the criterion.

Importantly, the linear polarization of the MHD turbulence observed closer to comets is explained naturally by nonresonant modes rather than by resonant modes which predict RH or LH polarization. However, farther away from the cometary nucleus the cometary ion density is lower and one would expect the resonant modes to dominate. It is thus clear that the last word has not been said yet on the identification of the modes responsible for MHD turbulence. Perhaps future cometary missions might shed more light on this aspect.

Acknowledgments. FV thanks the (Belgian) National Fund for Scientific Research for research and travel grants, and the Indian Institute of Geomagnetism for their kind hospitality during a stay in which the work was initiated.

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LOWER HYBRID RESONANCE ACCELERATION OF ELECTRONS AND IONS IN THE AURORA AND SOLAR FLARES

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Abstract
Nonlinear wave processes at the lower-hybrid frequency are extremely important in transferring energy between different particle populations and fields in both astrophysical and laboratory plasmas. In this paper we describe a new acceleration model for auroral and solar flare particles based on free energy in the form of ion anisotropies or plasma inhomogeneities generating lower-hybrid turbulence which undergoes collapse producing cavities within which particles are accelerated. We have identified a number of free energy sources in the aurora and solar flares for the generation of these wave modes, examples are anisotropic ion distribution functions such as loss cones produced by magnetic mirrors, ring distributions produced in shocks and gradients in temperature and density. Analytical studies reveal that a possible saturation mechanism for lower-hybrid wave growth is the modulational instability, which ultimately leads to wave collapse and cavity formation. Numerical simulations of ion conics and ion ring relaxation processes using a 2½-D fully electromagnetic, relativistic particle in cell code are described. The results indicate strong collapse at the lower-hybrid resonance resulting in simultaneous acceleration of both electrons and ions, together with electromagnetic wave generation at the electron cyclotron frequency. Arrest of wave collapse is due to particle acceleration. Comparison of theory and simulations with observations of measured electron and ion distributions in the aurora and inferred from X-ray and γ-ray emission from solar flares indicate that the acceleration model is in good agreement.

Introduction
The detailed model of the solar flare shock acceleration model is well described by McClements et al. This paper assumed that unstable ion ring distributions are formed due to shock formation. Reconnection can also produce such ion distribution functions, making the model much more general. In this paper we will show that not only electron acceleration takes place but also simultaneously ion acceleration to MeV energies. This result dispels the view that there are two different types of flares i.e. electron or proton flares, we show that the accelerated electrons and ions have a common mechanism. Particle simulations were carried out using a fully electromagnetic, relativistic particle-in-cell code. There are two space dimensions and three velocity dimensions with the possibility of the magnetic field lying in or out of the coordinate plane. The ratio of proton mass \( m_p \) to electrons mass \( m_e \) is taken to be either 100 or 400. This choice of mass ratio is sufficiently small that significant electron and ion acceleration can occur within a computationally convenient interval of simulated time. The initial ion distribution consisted of a monoenergetic perpendicular (to \( B_0 \)) ring and a Maxwellian core, the ratio of the
density of ring ions $n_r$ to core ions $n_c$ being 3.7. This is close to density ratios observed at the Earth's bow shock.\(^{(3)}\)

In the absence of in situ measurements, it is not possible to say with certainty that ring densities as high as this could occur in the solar case. However, our plasma parameters were chosen such that gyrating ions constituted only a small fraction (~4\%) of the energy potentially available in the downstream magnetic field.\(^{(1)}\)

The electron gyrofrequency $\omega_{pe}$ was taken to be twice the electron plasma frequency $\omega_{pe}$; this condition being satisfied in a flaring loop with, for example $n_e = 6 \times 10^9 \text{cm}^{-3}$ and $B_0 = 300 \text{G}$.

Ion ring distributions have been shown by Mikhailovskii\(^{(4)}\) to excite waves in the lower hybrid frequency range

$$\omega = \omega_{LH} \left( 1 + k^2 R^2 + \frac{m_i k^2}{m_e k^2} - \frac{\omega_{pe}^2}{k^2 c^2} \right) \quad (1)$$

with a growth rate

$$\gamma_0 \simeq \frac{n_r}{n_c} \omega_{LH} \quad (2)$$

where $\omega_{LH}$ is the lower hybrid frequency $\omega_{LH} = \omega_{pi} / \left( 1 + \omega_{pe}^2 / \omega_{ci}^2 \right)^{1/2}$, $R = \left( \frac{3 \gamma_i}{\omega_{LH} \omega_{pi}} + \frac{2 T_e}{m_e \omega_{pe} \omega_{ci}} \right)^{1/2}$, $\omega_{pi}$ is the ion plasma frequency and $T_{ei}$ is electron, ion temperature. The last term in equation (1) is the electromagnetic correction to the frequency. Resonant oscillations grow for $\omega = k . v_r$ where $v_r$ is the velocity of the ion ring.

The initial ion distribution is given by

$$f_i(v_{\parallel}, v_{\perp}) = \frac{n_c}{(2 \pi)^3/2 v_c} \exp \left( - \frac{v_{\parallel}^2 + v_{\perp}^2}{2 v_c^2} \right) + \frac{n_r}{2 \pi v_r} \delta(v_{\parallel}) \delta(v_r - v_{\perp}) \quad (3)$$

where $\delta$ is the Dirac delta function, $v_c$ the core ion thermal speed, $v_r$ the ring speed. As stated $n_r/n_c \simeq 3/7$ and $v_r \simeq 20 v_e$, with $m_e/m_i = 100$ this means $v_r = 2 v_e$ and with $m_e/m_i = 400, v_r = v_e$ where $v_e$ is the electron thermal speed. The ion ring speeds are easily computed from knowledge of the Alfvén speed in the solar flare\(^{(1)}\). The core ion temperature, which is assumed equal to the initial electron temperature, was set equal to 800eV.

The results of the simulation are shown in figures 1-3. Figure 1 depicts the time evolution of the wave spectral density as a function of the parallel phase velocity, the lower-hybrid waves appear at the low phase velocity and are seen to grow then fall to a saturated value level determined by particle absorption. Figure 2a represents the time evolution of the electron distribution as a function of $v_{\parallel}$ with the final electron distribution is represented in figure 2b. The flux of electrons with energy above 20keV, for a bimaxwellian plasma, is given by
\[ F(> 20\text{keV}) = 2\pi \int_0^\infty dv_{||} \int_0^\infty v_\perp f_e(v_{||}, v_\perp)dv_{\perp} \]

using a plasma density of \(10^{10}\text{cm}^{-3}\) we find from our simulations

\[ F(> 20\text{keV}) \sim 10^{18}\text{electrons cm}^{-2}\text{sec}^{-1} \]

ie. a typical "thick target flux". Figure 3 depicts the initial and final ion core distribution function showing significant energization in the perpendicular plane. Protons with energies in the MeV range are observed to be produced within distances of the order of 10's of meters and within a few ion-gyropariods, both electrons and ions are observed to be accelerated in localized regions of space indicating the presence of cavitons. The acceleration of electrons and ions takes place simultaneously indicating that it is not a two stage process but is due to a similar mechanism in this case due to the waves generated at the lower-hybrid resonance frequency. This is in agreement with the simultaneous observations of x-rays and \(\gamma\)-rays by Forrest and Chupp. Detailed studies of the electron distribution function reveal the presence of beams with \(+ve\) slopes in \(f_e\) ie. \(\partial f_e/\partial v > 0\). This cannot be due to a simple quasi-linear diffusion process which at best can only produce a plateau in \(f_e\). We find that at the end of the simulations approximately 10\% of the ring energy has been transferred to the background electrons and ions while less than 1\% of the energy is stored in all the waves generated.

Simulations show the transfer of perpendicular ion energy to energetic electrons via the process described above. The results have shown perpendicular shocks, such as might be associated with coronal mass ejection, or reconnection processes which produce ion ring distributions that can give rise to \(10^{18}\text{electrons cm}^{-2}\text{sec}^{-1}\) at hard x-ray emitting energies. Protons are accelerated to MeV energies in the same time scale. Therefore this mechanism for particle acceleration can account for both the nonthermal hard x-ray and \(\gamma\)-ray emission observed in solar flares. A similar mechanism is also responsible for auroral electron acceleration.

References

Fig. 1. Time evolution of the wave spectral density as a function of the parallel phase velocity.

Fig. 2. (a) Time evolution of the parallel component of the electron distribution function. (b) Final electron parallel distribution showing bump on tail.

Fig. 3. Initial and final perpendicular ion core distribution functions.
Solar Flare Mechanism Based on MHD Relaxation Model

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In order to reveal the solar flare mechanism, the dynamics of a solar coronal magnetic arcade are investigated by modeling it as an MHD relaxation process. Our work is composed of the following two parts: the equilibrium analysis and the dynamic simulations.

Equilibrium Analysis

According to Taylor’s conjecture [1], the minimum energy state for the magnetized plasma is described by

$$\text{rot} \mathbf{B} = \alpha \mathbf{B}. \quad (1)$$

In the two dimensional system which has a translational symmetry along \( z \)-axis, the magnetic field \( \mathbf{B} \) can be written by using the flux function \( \psi \),

$$\mathbf{B} = (\frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x}, B_z).$$

The equation (1) is modified to

$$\nabla^2 \psi = -B_z \frac{\partial B_z}{\partial \psi}, \quad (2)$$

$$B_z = \alpha \psi + c, \quad (3)$$

where the coefficients \( \alpha \) and \( c \) are constant. By numerically solving the equation (2) and (3) in the rectangular domain of the coronal region, we obtain the minimum energy state for the magnetic arcade. The boundary condition for \( \psi \) is given as following: \( \psi = \psi_0(1 - \cos 2\pi x)/2 \) at \( y = 0 \), \( \psi = 0 \) at \( y = L_y \) and at \( x = 0 \) and \( L_x \), where the bottom boundary \( (y = 0) \) corresponds to the photosphere. The solution is calculated as a function of the magnetic helicity \( K = \int Z \mathbf{A} \cdot \mathbf{B} dS \) and the magnetic flux across

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the domain $\Phi = \int_s B \cdot dS$. As a result, we find that the Taylor's minimum energy state for the magnetic arcade has three different types of solution, which have different topology each together; a simple arcade (type 1), an arcade plus an island above the arcade (type 2), and an arcade plus an island within the arcade (type 3), respectively. The typical solution is presented in Fig. 1 for each type. What type of solution appears depends on the parameters $K$ and $\Phi$. In the space of the helicity $K$ vs. the flux $\Phi$, the solution of type 1 occupies the low helicity region. The solution of type 1 changes to the types 2 and/or 3, as the magnetic helicity $K$ becomes larger than a critical amount. The critical helicity, above which the type 2 or 3 appears, depends on the magnetic flux across the arcade. When the helicity is sufficiently large, the solution has both features of type 2 and 3, having the islands above and also within the arcade. From the fact that the topology of the magnetic arcade is changed by the change of the helicity, the magnetic reconnection process is expected when the magnetic helicity is injected into the arcade by a certain mechanism.

**Dynamic Simulation**

Secondly, we directly carry out the dynamic simulations of the magnetic arcade, where the magnetic helicity is injected by the shearing motion on photosphere ($y = 0$) into the arcade. The basic equations are given by the following:

$$\rho \frac{dV}{dt} = J \times B,$$

$$\frac{\partial B}{\partial t} = -\text{rot} E,$$

$$E + V \times B = \eta J,$$

$$J = \text{rot} B,$$

where the pressure gradient force is neglected. The simulation box is same as the domain using in the equilibrium analysis, and the translational symmetry is also assumed. The boundary conditions are given by the impermeable and the perfectly conductive condition. Only for the photospheric boundary ($y = 0$), however, the plasma velocity $v_z$ is given by the analytic function of $z$ to model the shearing motion. Two different
shearing motions are considered, those being given by

\[ v_x = v_0 \sin 2\pi x / L_x \quad \text{for case 1}, \]
\[ v_x = v_0 \sin 4\pi x / L_x \quad \text{for case 2}. \]

The characteristic difference between these cases is that the flux injection rate \( \Phi = \int \psi v_x B_y dx \) is given by \( \pi v_0 \psi_0 / 2L_x \) for case 1, but is vanished for the case 2. On the other hand, the helicity injection rate is positive for both cases. In the case 2, hence, only the magnetic helicity is injected and the magnetic flux across the domain is kept to be zero. As a result of it, the resultant orbit for the case 1 in the \( K - \Phi \) space remains in the region for the type 1 of the minimum energy state considered in the equilibrium analysis. However, the orbit for the case 2 progresses in the region for the type 2 and 3 of it.

Figure 2 shows the resultant magnetic field lines of force from the simulation for the case 1. By investigation about the evolution of the total magnetic energy \( W = \int B^2 dS \) for this simulation, we find that the energy \( W \) from the simulation well agrees with the Taylor's minimum energy. On the other hand, the simulation result in the case 2 has a large excess energy compared to the Taylor's minimum energy. From these results, we can conclude that Taylor's minimum energy state can be obtained by 2D dynamics, if the helicity and the flux remains in the region where the minimum energy state is given by the type 1 of solution. However, if the parameters are in the region for types 2 and 3, 2D dynamics no longer generate Taylor's minimum energy state spontaneously.

The accumulation of the excess energy suggests the presence of three dimensional dynamics to release itself. Such an energy relaxation process is discussed as a possible mechanism of solar flare. We are now developing the three-dimensional simulation in order to confirm the relaxation model for solar flare. The details will be reported in the conference.

Reference

FIG. 1. The contour of the flux function of the Taylor's minimum energy state for three different types of solutions.

\[ t = 20 \]

FIG. 2. The three dimensional structure of the magnetic field lines of force, which is generated by the simulation with the case 1 of shearing motion.
Nonlinear (NL) MHD waves have been studied in connection with various plasma processes in magnetically structured solar atmosphere and solar wind. The propagation of MHD surface waves, supported by plasma-plasma, or plasma-magnetic field boundaries has been also a subject of intense study. During last decade some aspects of NL MHD surface waves have been considered. Roberts and Mangeney /1/ have proposed that the nonlinear behaviour of surface waves in a magnetic slab is governed by the Benjamin-Ono equation. The result by Roberts /2/ on dispersive MHD surface waves has been generalized by Merzlyakov and Ruderman /3/ for stratified plasmas. On the other hand, the self-modulation of NL MHD waves is examined by Hasegawa /4/, Mio et al. /5/ using perturbation method /6/ as a generalized scheme for deriving NL Schroedinger equation. It was shown that long-time behaviour of the complex amplitude of the MHD wave train is governed by modified, or derivative NL Schroedinger equation (NLSE). The problem of stability of NL MHD surface waves against a modulation was firstly examined by Merzlyakov /7/ and Sahyouni et al. /8/. It was shown that the behaviour of NL MHD surface waves in a magnetic slab depends on both parameters $\eta = \rho_0 / \rho$ and $K = k_0 x_0$, where $\eta$ is the ratio of plasma densities and $K$ is the normalized with respect to the slab half-thickness $x_0$ wavenumber. Merzlyakov's result yields a soliton-like propagation at great enough $K$'s. In Sahyouni et al. /8/ by using the ponderomotive forces approach a usual NLSE which governs the propagation of magnetosonic waves is derived. The analysis yields that only dark soliton solution are possible. This analysis was however performed under assumption that the folding of the slab interface is neglected. In the present study we use free boundary approach and consider modulation only along the MHD surface wave propagation.

Let us consider a magnetic slab of thickness $x_0$ surrounded by plasmas.
Fig. 1 Neutral curves \([\beta(K, \eta) = 0]\) for NL MHD surface waves regions. Hence, the dark soliton solution could be dominating either when the plasma and wave parameters \(\eta, x_0, v_A, k_0 = k_0(\omega)\) are such that we are away from the neutral curves, or as the soliton lengths are large enough in order the effects of the derivative NL term to be diminished.

The second neutral curve reduces the region of stability of the NL MHD surface waves. Merzlyakov's assumption for modulation in the form of a localized perturbation (with an evanescent amplitude) /7/ has a stabilizing rather than a destabilizing effect. In the usual modulation treatment (in \(z\) direction only) the region of stability is as follows:

\[
0.1x_0^{-1} < k_{01}^{\text{root}} < k_0^{02} < x_0^{-1}.
\]

Here \(k_{01,2}^{\text{root}}\) are the roots of the equation \(\beta(\eta, K, v, v_A, k, V) = 0\).

Which form of the evolution equation for the NL MHD surface waves is operative depends strongly on the plasma geometry and parameters, namely \(x_0, p_e, p, v_A\). In the limit \(K \rightarrow 0\) the MHD surface waves are dispersive and their evolution is governed by the Benjamin-Ono equation /1/. For large \(K\)'s the dispersion goes to zero (Fig. 2) and when the amplitude is small (but finite) a bright soliton solution is possible. In the intermediate case \(0.1 < k_{01}^{\text{root}} x_0 < K < k_{02}^{\text{root}} x_0 < 1\) the hybrid character of the MHD surface waves is most pronounced and the wave evolution is more sensitive to NL term conditions.

Finally, we mention that in our present analysis a one-dimensional
The static magnetic field in the slab is oriented along the z axis. The x coordinate is normal to the interfaces \( x = \pm x_0 \). Following the derivative expansion method /6/ and Merzlyakov’s /7/ notation we get the following NLSE

\[
\frac{\partial f_{11}}{\partial \tau} + 0.5 \frac{\partial^2 \omega_0}{\partial k_0^2} \frac{\partial^2 f_{11}}{\partial \zeta^2} + \beta |f_{11}|^2 f_{11} = 0.
\]

Here \( f_{11} \) is the complex amplitude of a sausage MHD surface-wave train along the slab, \( \tau \) is the slow time variable, \( \omega_0 = \omega_0(k_0) \) is the linear dispersion relation, \( \zeta = x_1 - \nu \tau \) is the slow spatial variable (\( \nu \) being equal to the wave group velocity \( v_g = \partial \omega_0 / \partial k_0 \)) and \( \beta \) is

\[
- k_0^4 (1 + \eta \tanh K)^{-1} \left\{ 0.5 \eta \tanh K [1 + 9 \cosh^2 K + 4.5 \sinh 2K + 4 \cosh^2 K \sinh 2K \\
+ 2 \sinh^2 2K] + 2.5 \eta \sinh^2 K (1 - \tanh K) + 8 \eta \sinh^2 K \tanh K + 2 \eta (1 - \nu^2)^{-1} \right\}
\]

\[
\times \left[ \tanh K (-3 + k_0 \nu / \omega_0 + \nu \nu / k_0 \nu) - 2 \cosh 2K (1 - k_0 \nu / \omega_0)^2 \right]
\]

\[
(1 - k_0 \nu / \omega_0) (1 - \nu^2)^{-1} (k_0 \nu / \omega_0)^2 - 1)^{-1} \left[ 8 \eta \tanh K \sinh^2 K (1 + k_0 \nu / \omega_0) \right.
\]

\[
- 2 k_0^2 (k_0 \nu / \omega_0)^2 + 2 (1 - k_0 \nu / \omega_0) \left[ 2 (k_0 \nu / \omega_0) (1 + \eta \tanh K) - 2 - \eta \tanh K \right] \right] / \omega_0
\]

Here \( \nu_g = \nu / \nu_A \) is the normalized wave group velocity, \( \nu_A \) being the Alfvén velocity.

This coefficient differs, of course, from that of Merzlyakov /7/. Examining it we obtain two roots instead of one as in Merzlyakov case. Surprisingly, one of the root coincides precisely with that of Merzlyakov’s (see Fig. 1). It is seen that in the region of \( K > 1 \) (above Merzlyakov curve) there exist compressive (bright) solitons whose parameters (velocity, amplitude, etc.) are given by

\[
f_{11} = A \text{sech} \left[ A (\beta / \alpha)^{1/2} (\zeta - \nu \tau) \right] \exp \left[ \beta^2 / 2 + 1(\zeta - \nu \tau / 2) / \alpha \right],
\]

where \( A \) is the soliton amplitude, \( \nu \) its velocity, \( \alpha = \partial^2 \omega_0 / \partial k_0^2 \) is the wave group velocity dispersion. Between the neutral curves, defined by \( \beta = 0 \) (Fig. 1), we observe a wide region of stable waves. In this region, where

\( 0.1 < K < 1 \) and \( \eta > 1 \) only dark solitons are possible which resemble the magnetosonic case in the plasma slab (see /8/). Near the neutral curves the NL term is however small and there a fourth order approximation should be adopted, i.e. a derivative NLSE should govern the wave evolution in those
Fig. 2 Normalized wave frequency \( \Omega = \omega_0/\chi_0 \), wave group velocity \( V_g = v/v_A \) and its derivative \( \partial V_g/\partial K \) as functions of \( K \) at \( \eta = 6.75 \) modulation has been assumed. Further studies have to be performed in order to account for the two- and/or three-dimensional feature of the NL MHD surface-wave problem. The latter includes two-dimensional modulation (in arbitrary form) and a propagation at arbitrary angles to the ambient static magnetic field. Such an analysis is in progress.

Acknowledgements: This work was supported by the Ministry of Education and Science under grant F-50/91.

References

Nonstationary astrophysical phenomena as Supernova burst and solar flares are accompanied by generation of waves and shocks. The problems of the generation are connected with collisionless interaction of the high-speed plasma flows. In general form this problem may be expressed as the collisionless expansion of the plasma cloud in the magnetized background. As a result of such expansion the energy of the plasma cloud is transformed into the directed heat energy of medium fast particles flows that allow the character of the formed disturbance. The energy change mechanism of the cloud-background interaction connected with the action of the curl electric field $E_p$ has been investigated in \([1,2]\). The physical model of such interaction (called magnetic laminar mechanism-MLM \([11]\)) is based on the displacement of the magnetic field $\vec{B}_0$ by the cloud, and a moving compressed magnetic field formed on its boundary, and generating $E_p[\vec{B}V_0]$. To supply the effective energy change of cloud-background system it is necessary to fulfill the requirement for magnitude of MLM-parameter $\delta=M^2/R_H^2R_H^*\geq 1$. That is the requirement for the ions magnetization at the scale $\tilde{R}=\sqrt{3N/4\pi n_*}$ of the gasdynamic deceleration of the cloud by the background. There $N$ is number of particles on the cloud $n_*$ is the background density $R_H,R_H^*$ -Larmor radius of the cloud and background respectively (calculated according to the initial velocity of the cloud expansion $V_0$).

This paper will discuss the character of background disturbance generated by the expanding cloud with large $M_\perp \geq 5$, 

**GENERATION OF LARGE AMPLITUDE PLASMA WAVES IN SPACE PHENOMENA**

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Fig 1: The plasma density in the r-z plane at t=2 μs, $M_A=10$.

The initial disturbances of magnetic field formed at small $M_A < 3$ are decayed into a wave packet at the distance $R \approx 2R$. In the calculations the disturbances structure is determined by the positive dispersion due to which the short wavelength one run forward. The oscillator disturbances on the profile of magnetic field for large $M_A > 5$ are not observed. The vector of transverse magnetic field was polarized linearly. The alteration of circular polarization of transverse magnetic field vector to the linear one with the increase of $M_A$ means a change possibility of the wave generation conditions and it is likely may be connected with the total decreasing of Hall currents with the growth of $M_A$.

In Figure 2 we show the distribution of cloud ions and background ions in the phase space at three times for $M_A \gg 1$. 

Fig 2: Ion phase space at t=2μs, 2.4μs, 3μs for parameters $M_A=1, 9=90^\circ$. 

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One can see the dynamics of the plasma cloud deceleration and the generation of large amplitude wave at the distance $R \sim \tilde{R}$. This wave breaks up with the time.


PARAMETRIC INSTABILITIES OF LANGMUIR WAVES IN DUSTY PLASMAS

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Recently, there has been a growing interest in dust plasma physics. A dusty plasma may contain electrons, ions, and a large number of charged dust grains of micrometer or submicrometer size. Such a plasma is usually found in many low-temperature laboratory devices, as well as in space and astrophysical environments.

Collective effects in dusty plasmas are of great importance. The presence of charged dust grains can affect low-frequency wave motions in an electron-ion plasma. For example, recently it has been shown [1-3] that the presence of stationary charged dust grains changes the quasi-neutrality condition, thereby modifying considerably the existing wave spectra. On the other hand, a novel dust acoustic wave [4] arises in a uniform plasma with mobile charged dust fluid and Boltzmann distributed electron and ion fluids.

In this paper, we consider parametric instabilities [3] of a large amplitude Langmuir wave with new types of low-frequency electrostatic oscillations in an unmagnetized dusty plasma. The latter consists of singly charged positive ions, electrons, and negatively charged dust grains. We assume that the dust grains all have uniform size and that they neither break up nor coalesce. When the grain size is much smaller than the inter-particle distance and the wavelength of any perturbations, then the dust grains can be treated as negatively charged point masses.

The nonlinear interaction of finite amplitude Langmuir waves with dusty plasmas is governed by

\[ \partial_t n_j + \nabla \cdot (n_j v_j) = 0, \]  

\[ (\partial_t + v_j \cdot \nabla)v_j = -(Z_j q_j/m_j)\nabla \phi - (\gamma_j T_j/m_j n_j)\nabla n_j, \]  

and

\[ \nabla^2 \phi = -4\pi \sum_j Z_j q_j n_j, \]

where \( n_j, v_j, \) and \( T_j \) are, respectively, the total particle number density, the fluid velocity, and the constant temperature of the particle species \( j \) (equals \( e \) for electrons, \( i \) for ions, and \( d \) for dust grains). Furthermore, \( q_d(q_i) = -e(e), q_d = -e, e \) is the magnitude of the electron charge, \( Z_d \) refers to the number of electrons that reside on a dust grain, \( m_j \) is the mass, and \( Z_{e,i} = 1 \). The electrostatic potential is denoted by \( \phi \). The ratio of specific heats \( \gamma_j \) takes the value 1(3) for isothermal (adiabatic) compression.

with mobile charged dust fluid and Boltzmann distributed electron and ion fluids.
In order to investigate the nonlinear interaction of high-frequency Langmuir waves with low-frequency oscillations in dusty plasmas, we decompose all the physical quantities as:

\[ n_i = n_{i0} + \sum_j n_{i1}^j + \sum_j n_{i2}^j, \quad v_i = v_{i0}^j + v_{i1}^j, \quad \phi = \phi_0^j + \phi_1^j, \]

where \( n_{i0} \) is the unperturbed particle number density \( (n_{e0} + Z_{i} m_{i0} = n_{i0}) \), and the superscript \( h(1) \) denotes the high- (low-) frequency plasma motion. Thus, the equation for the Langmuir sidebands, which is derived from (1) to (3), reads

\[
(\omega_{\pm}^2 - \omega_{pe}^2 - 3k_{\pm}^2 v_{Te}^2)E_{\pm} \approx (4\pi e^2/m_e)n_{ei1}^j E_{o\pm}, \tag{4}
\]

where \( E = -\nabla \phi^h \) is the electric field vector, \( \omega_{\pm} = \omega \pm \omega_0, k_{\pm} = k \pm k_0 \), and the subscript \( \pm(0+) \) denotes the sidebands (pump). The plasma frequency and the thermal velocity of the electron fluid are denoted by \( \omega_{pe} = (4\pi n_{e0} e^2/m_e)^{1/2} \) and \( v_{te} = (T_e/m_e)^{1/2} \), respectively. The plasma frequency of the electron fluid is much larger than those of the ion and dust fluids. Note that sidebands are created due to the coupling of the pump \((\omega_0, k_0)\) with low-frequency \((\omega, k)\) plasma motions.

Assuming that the phase velocity of the dusty plasma slow motion is much smaller than \( v_{te} \), we find that the electron number density perturbation \( n_{ei1}^j \) is given by

\[
n_{ei1}^j = (k^2/(4\pi e))\chi_e(\phi_1^j - \phi_{pe})/T_e, \tag{5}
\]

where \( \chi_e = 1/k^2 \lambda_{De}^2 \) and \( \phi_{pe} \approx e |\nabla \phi^h|^2/2m_e\omega_{pe}^2 \) is the low-frequency electron susceptibility and the ponderomotive potential, respectively. The electron Debye length is denoted by \( \lambda_{De} = (T_e/(4\pi n_{e0} e^2))^{1/2} \). The ion number density perturbation is given by

\[
n_{ii1}^j \approx -(k^2/(4\pi e))\chi_i\phi_1^j, \tag{6}
\]

where \( \chi_i = -\omega_{pi}^2/(\omega^2 - \gamma_i k^2 v_{Te}^2) \) is the low-frequency ion susceptibility, \( \omega_{pi} = (4\pi n_{i0} e^2/m_i)^{1/2} \) and \( v_{ri} = (T_i/m_i)^{1/2} \), are, respectively the plasma frequency and thermal velocity of the warm ion fluid. On the other hand, the number density perturbation of the cold dust fluid is

\[
n_{di1}^j \approx (k^2/(4\pi Z_{d} e))\chi_d\phi_1^j, \tag{7}
\]

where \( \chi_d = -\omega_{pd}^2/\omega^2 \) is the low-frequency dust susceptibility and \( \omega_{pd} = (4\pi n_{d0} Z_d^2 e^2/m_d)^{1/2} \) is the dust plasma frequency. The ponderomotive force acting on the dust fluid is negligibly small as \( Z_{d} m_e \ll m_d \).

Inserting (6) and (7) into (3), we obtain

\[
\phi_1^j = -4\pi e n_{ei1}^j / k^2 (1 + \chi_i + \chi_d). \tag{8}
\]

Eliminating \( \phi_1^j \) from (5) and (8), we have

\[
n_{ei1}^j = -n_{e0}(1 + \chi_i + \chi_d)e\phi_{pe}/T_e, \tag{9}
\]

where \( \epsilon = 1 + \chi_e + \chi_i + \chi_d \).

The dispersion relation for parametric instabilities in dusty plasmas is obtained by eliminating \( n_{ei1}^j \) from (4) and (9). The result is

\[
\epsilon = -k^2 \chi_e(1 + \chi_i + \chi_d) \sum_{\pm}(k_{\pm} \cdot v_o)^2/k_{\pm}^2 \omega_{\pm}^2 \epsilon_{\pm}, \tag{10}
\]
where \( c_\pm = 1 - (\omega^2_{pe}/\omega^2_\pm)(1 + 3k^2_\pm \lambda^2_{De}) \) and \( v_o = eE_0/m_e\omega_{pe} \).

We consider two types of plasma slow response. Firstly, we assume that the dust grains are static so that \( \chi_d = 0 \). For \( k^2 \lambda^2_{De} \ll 1 \), (10) takes the form

\[
(\omega^2 - \omega^2_\pm) = k^2\omega^2_{pe} \sum_\pm (k\pm \cdot v_o)^2/k^2_\pm D_\pm,
\]

(11)

where \( \omega^2_\pm = k^2c^2_\pm \equiv k^2[\gamma v_i^2 + (n_{io}/n_{ee})c^2_s], c^2_s = T_e/m_i, D_\pm \approx \pm 2\omega_{pe}(\omega - k \cdot v_g \mp \delta), v_g \approx 3k^2v^2_{te}/\omega_{pe}, \) and \( \delta \approx 3k^2v^2_{te}/2\omega_{pe} \).

For the three-wave resonant interaction, the upper-sideband is off-resonant (viz. \( D_+ \neq 0 \)), and the maximum growth rate found from (11) is

\[
\gamma \approx \omega_{pi} k \left| \hat{k}_- \cdot v_o \right| /2(\omega_{pe}\omega_{ee})^{1/2}.
\]

(12)

It follows that the presence of static charged dust grains reduces the growth rate because the dust acoustic frequency \( \omega_{as} \) is larger than \( k(3c^2_t + c^2_s)^{1/2} \), which is the frequency of the ion sound wave when the dust grains are absent.

For the modulational instability, the upper- and lower-sidebands are resonant. For quasi-static modulations (\( \omega \ll \omega_{as}, \gamma_i = 1 \)), (11) becomes

\[
(\omega - k \cdot v_g)^2 = \delta^2 \left\{ 1 - \omega^2_{pe}(\hat{k}_- \cdot v_o)^2/\omega_{pe}\delta c_s^2[(n_{io}/n_{ee}) + (T_i/T_e)] \right\}.
\]

(13)

It emerges that the growth rate of the modulational instability is also reduced in dusty plasma.

Next, we consider the dynamics of the dust grains and concentrate in the frequency regime \( \omega \ll k\nu_i \). Accordingly, we have \( \chi_s = 1/k^2\lambda^2_{De} \) and \( \chi_i = 1/k^2\lambda^2_{Di} \), where \( \lambda_{Di} = (T_i/4\pi\nu_i e^2)^{1/2} \) is the ion Debye length. Here the dispersion relation for \( k^2\lambda^2_{Di} \ll 1 \) is of the form

\[
(\omega^2 - \omega^2_{as}) = -[\omega^2 - \omega^2_D]/\lambda^2_{De} \sum_\pm (k\pm \cdot v_o)^2/\alpha k^2_\pm D_\pm,
\]

(14)

where \( \omega_D = k\lambda_{Di}\omega_{pd} \equiv k c_D, \omega_{rs} = \omega_D/\sqrt{\alpha} \equiv k c_{rs}, \) and \( \alpha = 1 + T_i n_{io}/T_e n_{io} \equiv 1 + \tau \).

The growth rate of the three-wave decay interaction for this case is

\[
\gamma \approx \omega_{pe} k \sqrt{\tau}\lambda_{Di} \left| \hat{k}_- \cdot v_o \right| /2\lambda_{De}(1 + \tau)^{1/2} \omega_{pe}\omega_{rs}\alpha^{1/2}.
\]

(15)

The modulational instabilities involving extremely low-frequency dust-acoustic perturbations are likewise investigated by analyzing (14). For example, for \( \omega \ll \omega_D \), we have

\[
(\omega - k \cdot v_g)^2 = \delta^2[1 - (\hat{k}_- \cdot v_o)^2/\lambda^2_{De}\omega_{pe}\delta].
\]

(16)

Equation (16) admits an oscillatory instability.

We now present the relevant equations governing the dynamics of the modulated Langmuir wave packets. For the Langmuir wave envelope, we have

\[
i(\partial_t + v_g \cdot \nabla)E + (3v^2_{te}/2\omega_{pe})\nabla^2E - \omega_{pe}(n_{io}/2n_{ee})E = 0.
\]

(17)

On the other hand, low-frequency density responses are given by

\[
(\delta_t^2 - \nabla^2)\n_{ee}^l = (1/2)(n_{io}e^2/m_o\omega_{pe}^2)\nabla^2 |E|^2,
\]

(18)
for the case in which the dust grains are static, and

\[(\partial_t^2 - \nabla^2) n_{ei} = -(1/4\pi T_e \alpha) (\partial_t^2 - c_D^2 \nabla^2) |E|^2, \tag{19}\]

for the case in which the dust grains are mobile and the electron and ion fluids have the Boltzmann distribution. Equations (17) to (19) can be used for the study of envelope soliton formation, wave collapse, etc.

To summarize, we have investigated parametric instabilities of a finite amplitude Langmuir wave in a uniform unmagnetized dusty plasma. It is found that the growth rates of the three-wave decay as well as the modulational instabilities are reduced as compared with those cases without the dust grains. Also presented are the dynamical equations that govern the spatio-temporal evolution of the modulated Langmuir wave packets in dusty plasma. Our results should be useful for the understanding of the nonlinear wave phenomena in dusty plasmas, such as those in cometary comae and tails, in interstellar clouds, as well as in noctilucent clouds on the night sky of the Earth's north pole during the summer season.

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MAGNETIC RECONNECTION IN THE PLASMA WITHIN CLUSTERS OF GALAXIES

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Abstract: The process of magnetic reconnection (MR) can be responsible for the liberation of magnetic energy in the plasma of the medium within clusters of galaxies in a rate \( \sim 10^{44} \) erg s\(^{-1} \) in the more central regions (i.e., \( < 10^{23} \) cm) of clusters. Such a process can work as a mechanism of ionization and heating of the gas intracluster, and certain observations of filaments with optical emission lines can perhaps be explained if MR is considered as an exciting source, as suggested by some recent observational studies. In this work we use hydrodynamical time-dependent calculations: 1) To follow the evolution of cooling condensations until the phase of optical line emission; and 2) To study the effects of MR as a mechanism of excitation of filaments in cooling flows in clusters of galaxies. We treat the cases in which the magnetic field is: a) Null; b) Present and passive (i.e., acting just as magnetic pressure); and c) Present and contributing energetically through the MR process. Our results clearly indicate that the agreement between theoretical and observational results concerning emission line intensities and line ratios is much better when the effect of MR is included. The approach proposed here has several implications related to important plasma physics processes of wide interest and which deserve further studies in those plasmas as, for example, processes of particle acceleration, generation of plasma wave modes, and transport.

1 Optical Filaments and Magnetic Reconnection (MR)

X-ray observations (in the range 1 – 10 kev) of clusters of galaxies show that in many cases the gas in the centre of the cluster presents a cooling time less than the age of the system. This leads to the formation of a pressure-driven mass flow (called “cooling flow”) towards the centre of the cluster (e.g., /1/). Moreover, in some cooling flows optical filaments are observed around the central dominant galaxy associated with these flows. Such filaments are expected to be a phase of the evolution of thermal instabilities taking place within the cooling flow. The flux ratio of the emission lines allows us to divide the filaments into two classes, namely: Class I, when \( <[\text{NII}]/\text{H\alpha}> = 2.0 \), and Class II when \( <[\text{NII}]/\text{H\alpha}> = 0.9 /2/ \).

What are the mechanisms which ionize and heat the gas of the filament? Several models consider different kind of such mechanisms; it is beyond the scope of this work to discuss these mechanisms here. The present work belongs to a series where time dependent models are developed for these optical filaments. In previous works of the series the effects of shock

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as well as those of photoionization (by soft X-rays produced within the cooling flow or by an OB stellar population formed within the filaments) have been included /3/. In the present work we consider the process of magnetic reconnection as a source of ionization and excitation of the gas of the filament.

The magnetic reconnection (MR) is a well known process of conversion of magnetic energy into other forms of energy (e.g., /4/). What characterizes the process of MR is the topological change it causes in the magnetic field. This aspect is very important in the treatment of transport processes in magnetized plasmas, particularly for collisionless plasmas as those observed in the intracluster medium of galaxies.

The possibility of MR as source of excitation for such filaments was generically suggested by Soker and Sarazin 1990 /5/. There have been some observations /6, 2/ which seem to support that suggestion. Here we treat the process of MR in those filaments in greater detail through a quantitative approach and the results are compared with observational ones.

2 Evolutive Models for the Optical Filaments

The evolution of the density perturbations which will give rise to the optical filaments is obtained by the resolution of the hydrodynamic equations of mass, momentum and energy conservation.

We assume here that: a) The perturbations are self-gravitating; b) The thermal conductivity is reduced by a factor of $10^{-4}$ in comparison with the classical (Spitzer) value, in order not to inhibit the growth of the thermal instabilities; c) The cooling function used is a non-equilibrium isochoric one, in order to take into account the fact that the recombination time of important ions is longer than the cooling time for temperatures below $10^6$ K; d) The initial density ($\rho$) perturbations are of the form $\delta \rho/\rho = A \sin x/x$, where $x = 2 \pi r/L$, with $A$ standing for a characteristic amplitude and $L$ a typical dimension (see below); e) The geometry is plane-parallel; and f) The perturbations are isobaric and non-linear ($A = 1$).

We analyse two regimes for the cooling flow, namely: 1. For the outer regions: $r \approx 20$ kpc; $\rho_0 = 5 \times 10^{-26}$ g cm$^{-3}$; and $T_0 = 2.5 \times 10^7$ K; and 2. For the inner regions: $r \approx 3$ kpc; $\rho_0 = 5 \times 10^{-25}$ g cm$^{-3}$; and $T_0 = 10^7$ K. These values were obtained from a time dependent modelling applied to the cooling flows of the clusters A496 and A1795 /7/.

We study six models, namely: A. For extended filaments far from the centre of the cooling flow: $L = 10$ kpc and regime 1; B. For inner filaments: $L = 1.5$ kpc and regime 2; C. Model B including magnetic pressure (i.e., including a passive participation of the magnetic field), where the initial value adopted for the plasma beta, $\beta_{pl}$, is 100, as suggested by radio and X-ray observations of the central regions in cooling flows (e.g., references in /5/); D. Model C with the parameter $\lambda \equiv l_{MR}/l_B = 1$. The parameters $l_{MR}$ and $l_B$ stand, respectively, for the length scale of the region where the MR takes place (i.e., the scale over which the magnetic field changes its direction) and the width of the region where the magnetic pressure becomes important (i.e., $\beta_{pl} \leq 1$); both length scales are usually $<< L$; E. Model D with $\lambda = 0.5$; F. Model D with $\lambda = 0.2$.

3 Discussion and Results

The properties of our models are presented in Table 1: $t_{col}$ is the time for the collapse, i.e., the time interval between the beginning of the collapse until the moment in which the temperature reaches $10^5$ K; $t_{op}$ is the duration of the phase of emission of optical lines; and $L_{H\alpha,max}$ is the maximum luminosity in H$\alpha$. We consider $t_{op}$ as the time interval between the
instant in which $T$ is less than $10^4$ K and that in which $L_{H\alpha}$ is reduced to $L_{H\alpha,max}/3$. The luminosities have been normalized to $\dot{M} = 100 M_\odot \text{yr}^{-1}$, where the rate of mass deposition through cooling, $\dot{M}$, is estimated from the ratio between the filament mass and its time of collapse (where one solar mass $M_\odot \approx 2 \times 10^{33}$ g).

For model A, as soon as the temperature decreases below $3 \times 10^4$ K a shock is created. For $t = 10^4$ years the shock velocity is $122 \text{ km s}^{-1}$, and it is dissipated in $t = 9 \times 10^4$ years. A similar scenario takes place with model B, with the only difference that in this case the timescales involved are about 2.5 times longer than those in model A. This fact added to that the time for collapse in model B is much shorter than that of model A implies that model B spends a much greater fraction of its life in the phase of optical emission than model A.

As there is no ionization equilibrium for $T < 10^6$ K, the state of ionization of the gas is obtained by solving the ionization equations during each temporal step by using the program SUMA (see /8/). In model A the line of [OI], and possibly of [SII] are more intense than those observed, and $L_{H\alpha}$ is too faint (Table 2). On the other hand, model B presents values of $L_{H\alpha}$ which are adequate for Class I filaments. The line ratios, however, are still in disagreement with observations (Figure 1). Therefore, apparently the models including only shock are not consistent with observations.

It remains to be investigated if magnetic fields can offer a solution for such difficulties. We will restrict this investigation only to models from regime 2, because they are those presenting a higher optical luminosity. We consider then model C. As one can see from Table 1 and from Figure 1 the effect of the magnetic pressure is nearly negligible. The term including such an additional pressure just contributes to slow down the collapse and to reduce the intensity of the emission, although always by very small amounts. The line ratios also remain practically unchanged.

On the other hand, while the kinetic energy of the gas is lost through radiative cooling, which occurs almost completely out of the optical range, the magnetic field represents an energy reservoir which remains available for processing the optical emission. This can occur thanks to the MR.

In fact, as the gas compression evolves the magnetic energy density increases proportionally to $\rho^2$, while the gas is cooling radiatively. As a consequence, soon $\beta_{pl}$ will become $<< 1$. In this regime there are a number of mechanisms of fast connection which proceed at a rate $\approx 0.1 v_A/l_{MR}$ (see /5/; where $v_A$ is the Alfvén velocity in the considered region). As previously mentioned, here we consider that the MR takes place only in the region where $\beta_M \leq 1$; such a region has its half-width given by $l_B$ ($\sim l_{MR} << L$).

From models D to F we have an increasingly greater efficiency of the process of MR. Although these models are not much different from model B in what concerns the H$\alpha$ luminosity and the time for collapse, the line ratios in these models become much nearer to observed ones. What happens is that in the case of models D, E and F the ratios [OI]/H$\alpha$ and [NII]/H$\alpha$ increase, in comparison with the model including only shock. This occurs because the H$\alpha$ line is produced by recombination but the forbidden lines are produced by collisional excitation. When one has a heating source, it contributes more effectively to the rate of collisional excitation rather than to the ionization rate.

One of the main conclusions from this work, therefore, is that the MR can be an important mechanism of heating and ionization of optical filaments in cooling flows (see Figure 1).

The future development of this work involves to extend the parameter space studied, i.e., the regime of the cooling flow, the thickness of the perturbation, the value of $\beta_{pl}$, and the efficiency of the MR. Besides that, a further improvement of this work is concerned with the inclusion of other processes associated with the MR, as the production of suprathermal
particles and the interference in the transport processes in the plasma.

References


Figure 1: Evolution of [NII]/Hα versus [OIII]/Hβ for the models with L = 1.1 L⊙. The degree upon the curves indicates (decreasing from the upper extreme) the instance of time t = 2.3, 3.4, 7.5, and 48 × 10^6 years, respectively. The dashed curve refers to the model without magnetic pressure and the solid curves to the models with initial value of B_0 = 100. Models with magnetic parameters are marked with the values of A below the right extreme of the curve. Observational points (Heckman et al. 1989) of several luminous nuclei are represented as follows: A3720, diamonds; Perseus, squares; P89/455-131, plus signs; Sc230, plus sign surrounded by circle; Virgo, asterisks; A1909, triangles; A2390, squares; A2397, circles. Notice the clear separation between Class I Hα active (higher values of [NII]/Hα) and those of Class II.

Table 1: Proportion of the models.

<table>
<thead>
<tr>
<th>Model</th>
<th>L_{bol} (10^6 L⊙)</th>
<th>L_{Hα} (10^6 L⊙)</th>
<th>L_{NII}/L_{Hα}</th>
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<tr>
<td>A</td>
<td>1.15</td>
<td>2.05</td>
<td>2.4</td>
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<tr>
<td>B</td>
<td>1.68</td>
<td>0.64</td>
<td>18.7</td>
</tr>
<tr>
<td>C</td>
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<td>0.04</td>
<td>18.3</td>
</tr>
<tr>
<td>D</td>
<td>1.20</td>
<td>0.04</td>
<td>12.5</td>
</tr>
<tr>
<td>E</td>
<td>1.43</td>
<td>0.42</td>
<td>15.9</td>
</tr>
<tr>
<td>F</td>
<td>1.83</td>
<td>0.85</td>
<td>10.0</td>
</tr>
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</table>

Table 2: Model A: Line ratios in relation to Hα.

<table>
<thead>
<tr>
<th></th>
<th>L_{bol} (10^6 L⊙)</th>
<th>[NII]/Hα</th>
<th>[OIII]/Hα</th>
<th>[OII]/Hα</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
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<td>0.322</td>
<td>0.54</td>
<td>0.91</td>
</tr>
<tr>
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<td>0.50</td>
<td>0.95</td>
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<td>0.332</td>
<td>0.57</td>
<td>1.93</td>
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</table>
QUANTITATIVE ANALYSIS OF COSMIC NOISE COMMING FROM PLASMA JETS OF QUASAR

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1. INTRODUCTION

This is an article of taking cosmic plasmas of quasar by means of cosmic noise and a measure of information entropy. We take microwave noise from quasars of the band width: 0.01 ≤ f ≤ 100 GHz (wave length: 3mm ≤ λ ≤ 30m). A quasar emits strong microwave from its plasma jets by an interaction with magnetic flux. The microwave noise is a superposition of noise from multiple jets (microwave sources). Since a quasar is distributed at the edge of the universe and the farthest galaxy (and very active), we observe microwave noise from it by a parabolic antenna as if it is from a point source. Observations of microwave from a quasar by VLBI show complexity in structure of its microwave sources.

Let x(t) be the intensity of microwave with t the time and P(f) the power spectral density with f the frequency of microwave, respectively. In x(t), information on the energy or something of the quasar's gravity or the jets, relating to physical events is contained. The frequency f in the spectral density are very probable to have a concern with the energy, reflecting a structure of from where (or which level of the gravity) in the quasar the plasma jets spout out.

2. MEASURE OF STRUCTURE

An information entropy is defined as a measure of structure of plasma jets. It needs an object of event. Let p(s) being the joint probability distribution of a time series x(t): t = 0, 1, ..., m of an event, then the entropy for the time series is

H = -∫p(s) log p(s) dv.

If p(s) follows Gaussian distribution, the entropy rate
h=\lim_{m \to \infty} \frac{1}{m+1} \sum_{k=m+1}^{m} \log P(f) df \cdot \log 2^{1-h_k}.

(1)

Eq. (1) is a transformation of the spectral density \( P(f) \) into the entropy rate \( h \). The author obtained \( h \) for a continuous spectral density function the \( n \)-th domain of which is \( P_n \) with \( a_n \) being the spectral index \( f_{n-1} < f_k < f_n \). We take important terms in \( h \) for our problem and call them as \( \Delta h \). It is

\[
\Delta h = \sum_{n} (a_{n+1} - a_n) \log f_n.
\]

(2)

Eq. (2) is the entropy rate of noise event in a needed frequency band. The measure \( \Delta h \) is a summation of the product of the difference between adjacent indices and the logarithm of the frequency the index changes. If \( a_{n+1} > a_n \), then the measure is positive and if \( a_n > a_{n+1} \), the measure is negative (provided that \( \log f_n > 0 \)). Usually an event is considered to create a structure for the negative entropy \( (\Delta h < 0) \). Higher the frequency \( f_n \) and sharper an angle the adjacent lines make, the absolute value of \( \Delta h \) is larger. A radiospectral index reflects a physical process of event; a different index means a different physical or locational (because of gravitation energy) process. The frequency \( f_n \) corresponds to the energy of quasar (velocity and density of plasma jets, magnetic flux density). As a whole, the measure \( \Delta h \) intuitively fits a measure of complexity in structure of the microwave sources.

The microwave frequency \( f_n \) at a quasar is different from the observation frequency \( f_n' \) at an antenna by the red shift \( z \) of the quasar. By substituting \( f_n = (1+z)f_n' \) into eq. (2), we have

\[
\Delta h = 1.66 \sum_n \Delta a \{ \log_{10} f_n' + \log_{10} (1+z) \} \text{ bit.}
\]

\[
\Delta a = a_{n+1} - a_n
\]

We use the unit 'bit' (amount of information) for \( \Delta h \) the measure of structure.
3. MEASURE OF QUasar PLASMA JETS

In many radio-observatories on the earth, they have observed microwave flux densities of quasars at their respective bandwidths according to the ability of antennas. Observation data were collected and dotted for the flux density and frequency, for example, shown in Fig.1. The lines in the figure were drawn by the author so as to fit on the dots, with slope index $a$. The dimensions of the flux density and the power spectral density are $\text{Wm}^{-2}\text{Hz}^{-1}$ and $\text{WHz}^{-1}$, respectively; however, it does not prevent for our discussion to apply eq.(3) to each diagram of Fig.1. We obtained the measure $\Delta h$ for each plasma jets of eight quasars and two radio garaxies in the frequency band $0.01 \leq f \leq 100 \text{GHz}$: for quasars, 3C273 ($z=0.16, a=0.59, f'=0.6 \text{GHz}; \Delta h=-8.7 \text{bit}$), 3C48 ($z=0.37, a=0.78, f'=0.2 \text{GHz}; \Delta h=11 \text{bit}$), 3C279 ($z=0.54, a=0.57, f'=0.6, 10 \text{GHz}; \Delta h=-1.2 \text{bit}$), 3C345 ($z=0.595, a=0.40, f'=1.3 \text{GHz}; \Delta h=-7.7 \text{bit}$), 3C216 ($z=0.67, a=0.79, f'=3.5 \text{GHz}; \Delta h=-7.4 \text{bit}$), 3C380 ($z=0.69, a=0.67; \Delta h=0$), 3C454.3 ($z=0.86, a=0.77, f'=0.2, 0.5 \text{GHz}; \Delta h=-8.5 \text{bit}$) and 3C448 ($z=1.40, a=0.83, f'=2.5 \text{GHz}; \Delta h=-12.7 \text{bit}$); for radio garaxies, 3C274:M87 ($z=0.0043, a=0.82; \Delta h=0$) and 3C84:Seyfert ($z=0.017, a=1.4, -0.37, f'=0.5, 10 \text{GHz}; \Delta h=-19.5 \text{bit}$). In Fig.2 we show the measure $\Delta h$ of the quasars and radio garaxies, with the abscissa being the red shift $z$ in logarithmic scale. Their optical magnitudes are shown as well/3/. The $\Delta h$ is a meaningful measure in a sense: 3C446 (-12.7bit) is a very active body; 3C454.3 (-8.5bit) is an active body; 3C380 (0bit) is a little active body; a structure of 3C279 (-1.2bit) has not yet been found; a pair of jets of 3C273 (-8.7bit) have not been yet found but one jet/4/. Now we cannot give $\Delta h$ a systematic meaning as the measure of structure by only such few examples. It depends on our future research. The red shift $z$ reflects each age of the galaxy and if we find a systematic distribution in the red shift $z$ and the measure $\Delta h$, we will discover an observational cosmogony.

We are thankful to Professor M. Morimoto for his comments and encouragement, to S.Kameno (under doctorate) and Dr.R. Kato for their advices, and to Professor H. Tabara for the data edited by him.
Fig. 1 Collected data of flux density and frequency (by H. Tabara)

Fig. 2 Distribution of the measure $\Delta h$ and red shift $z$.

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A Theory on the Evolution Process of the Fine Structure in Space

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I. Introduction

High speed and energetic streaming ions with the energies from 0.1MeV to 100MeV are frequently observed in the solar atmosphere and interplanetary space. When we assume that the ion energy depends only on the kinetic energy, such plasma ions have to attain relativistic speeds. By considering relativistic effects where the ion velocity is about 0.1c, we can describe the relativistic motion of such ions in the study of nonlinear interaction. In this situation, ion temperature and relativistic effects are important for energetic ion acoustic waves propagating in space. It has been suggested that relativistic double layers occur in space plasmas. However, little attention has been given to relativistic ion acoustic double layers. Interesting features such as the formation of the precursor of the ion acoustic waves and the long wavelength ion wave modes are discussed as astrophysical phenomena. Although the relativistic ion acoustic solitary waves in space are described by the nonlinear evolution equation, the effect of the ion temperature is an indispensable factor. It is recognized that, in the recent astrophysical observations, the fine structure is important to understand the properties of space plasmas. Double layers and spiky solitary waves form the fine structure. We show a new point of view with respect to both waves. The object of this paper is to show the formation of relativistic double layers, spiky solitary waves and explosive modes in a plasma.

II. A mixed MK-dV equation

We consider small but finite amplitude ion acoustic waves propagating in a relativistic plasma. The plasma is quasineutral. It is assumed that the velocity distribution of each species is Maxwellian everywhere and the relativistic plasma is composed of a mixed fluid with hot and isothermal electrons and hot ion species. We assume that the ion flow velocity is relativistic. The basic equations are described by the fully relativistic ion fluid equations:

\[
\frac{\partial}{\partial t} (\gamma n) + \frac{\partial}{\partial x} (\gamma n v) = 0 ,
\]

\[
\left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial x} \right) \gamma v + \frac{\alpha}{\gamma} \frac{\partial p}{\partial x} + \frac{\partial \phi}{\partial x} = 0 ,
\]

\[
\left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial x} \right) p + 3 \gamma^2 p \left( \frac{\partial}{\partial x} + \frac{1}{c^2} v \frac{\partial}{\partial t} \right) v = 0 .
\]
where the Lorentz factor is $\gamma = (1-(v/c)^2)^{-1/2}$. The system of equations is closed with the help of the Poisson's equation

$$\frac{\partial^2 \phi}{\partial z^2} - n_e + n = 0 \tag{1.d}$$

with $n_e = \exp(\phi) \tag{1.e}$.

We assume $1 > v/c$. The densities, the velocities, the pressure, the potential, the time and the distance are normalized. The parameter $\sigma$ is the ratio of the positive ion temperature to the electron temperature. In order to solve eqs. (1.a)-(1.e), we expand the quantities $n$, $v$, $p$, $\phi$ and $n_e$ as power series in terms of the small parameter $\varepsilon$:

$$n = 1 + \sum_{I=1}^\infty \varepsilon^I n_I, \quad v = v_0 + \sum_{I=1}^\infty \varepsilon^I v_I, \quad p = 1 + \sum_{I=1}^\infty \varepsilon^I p_I, \quad \phi = 0 + \sum_{I=1}^\infty \varepsilon^I \phi_I \tag{2}$$

We introduce the stretching coordinates $\xi = \varepsilon^{1/2}(x-ct)$, $\tau = \varepsilon^{3/2}t$, where $s$ refers to the phase velocity. We substitute (2), $\xi$ and $\tau$ into (1.a)-(1.e). To the lowest order of $\varepsilon$, we represent $n_I, v_I, p_I$ by $\phi_I$ and obtain the phase velocity. To the second order of $\varepsilon$, we have the K-dV equation. In order to take into account the higher-order non-linearity, we use the modified coordinates $\xi = \varepsilon(x-ct)$, $\tau = \varepsilon^{3/2}t$. Substitution (2) and (3) into (1.a)-(1.e) yields a mixed MK-dV equation

$$\frac{\partial \phi}{\partial \tau} + \alpha \phi \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \beta \phi^2 \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \beta \phi^2 \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \frac{\partial^2 \phi}{\partial \xi^2} = 0 \tag{4}$$

where $\alpha$ is positive and $\beta$ is negative. Here $\phi = \phi_I$.

### III. Stationary solutions of eq. (17)

We introduce a variable $\eta = \xi - u \tau$ in a stationary frame, where $u$ is a constant velocity. Inserting $\eta$ into (4), we obtain

$$(d \phi / d \eta)^2 + \Psi = 0 \quad \text{and} \quad \Psi = (\beta / 6) \phi^4 + (2 \alpha / 3) \phi^3 - 2 \alpha \phi^2 \tag{5}$$

under the boundary conditions $\phi, \phi^3 / 3 \eta \rightarrow 0$ at $|\eta| \rightarrow \infty$, when $n=1,2$.

### III.-A. We consider that $\Psi$ has two double roots. In order to obtain solutions, we transform eq. (5) to $\Psi = (\beta / 6) \phi^2 (\phi^2 - \phi^2)^2 \tag{6}$

where $\phi_\alpha = -2 \alpha / \beta$, $u = - \alpha^3 / 3 \beta$. Here, $\phi_\alpha$ denotes the maximum potential. Integrating eq. (5) with eq. (6), we have a solution

$$\phi = \frac{1}{2} \phi_\alpha \left\{ 1 - \tanh \left( \frac{1}{2} \left[ - \frac{\beta}{6} \right]^{1/2} \phi_\alpha (\eta - \eta_0) \right) \right\} \tag{7}$$

**Fig.1** Dependency of the potential drop $\Delta \phi$ of the double layer on the ratio $\sigma$, for $v_0/c=0.2$.

**Fig.2** The double layer thickness as a function of the ratio $\sigma$, for $v_0/c=0.2$. 
for $0 < \phi < \phi_\infty$. We regard eq. (7) as a relativistic compressive double layer. The dependency of the potential drop $\Delta \phi$ on the ion temperature ratio $\sigma$ is shown in Fig. 1 and the double layer thickness $L$ is shown in Fig. 2. For the condition $\phi > \phi_\infty$ or $\phi < 0$, we obtain an explosive mode.

III-B. We consider the case where $\Delta \phi$ has one double root. Then $\Psi$ of eq. (5) requires

$$\Psi = (\beta / 2) \phi^2 (\phi - \phi_\infty) (\phi - \phi_\infty),$$

with $\phi_\infty = (2 / \beta) \left( -\alpha \pm (\alpha^2 + 3 \beta u)^{1/2} \right)$. Here when we put $u = -\alpha^2 / 4 \beta > 0$, $\phi_\infty$ and $\phi_\infty$ take a form of $\phi_\infty = -\alpha / \beta > 0$, $\phi_\infty = 3 \alpha / \beta > 0$, provided that $\phi_\infty < \phi_\infty$. Equation (5) with eq. (8) has a solution

$$\phi (\alpha, \beta, \eta; \phi_\infty) = \frac{3 \alpha}{\beta} \text{sech}^2 \left[ \alpha (\alpha, \beta; \phi_\infty) - \alpha \left( \frac{1/2}{2 \beta} \right)^{1/2} (\eta - \eta_0) \right],$$

for $|\phi - (\phi^2 + (4 \alpha / \beta) \phi + (12 \alpha^2 / \beta^2))^{1/2}| \leq 2.3^{1/2} \alpha / \beta$, and

$$\phi (\alpha, \beta, \eta; \phi_\infty) = \frac{3 \alpha}{\beta} \text{cosech}^2 \left[ \alpha (\alpha, \beta; \phi_\infty) + \alpha \left( \frac{1/2}{2 \beta} \right)^{1/2} (\eta - \eta_0) \right],$$

for $|\phi - (\phi^2 + (4 \alpha / \beta) \phi + (12 \alpha^2 / \beta^2))^{1/2}| > 2.3^{1/2} \alpha / \beta$.

$$\mathcal{A}(\alpha, \beta; \phi_\infty) = \text{arctanh} \left\{ \frac{\beta}{2 \cdot 3^{1/2} \alpha} \left[ \phi_\infty - \left( \phi_\infty^2 + \frac{4 \alpha}{\beta} \phi_\infty + 12 \alpha^2 / \beta^2 \right)^{1/2} \right] \right\},$$

and

$$\mathcal{B}(\alpha, \beta; \phi_\infty) = \text{arccosh} \left\{ \frac{\beta}{2 \cdot 3^{1/2} \alpha} \left[ \phi_\infty - \left( \phi_\infty^2 + \frac{4 \alpha}{\beta} \phi_\infty + 12 \alpha^2 / \beta^2 \right)^{1/2} \right] \right\},$$

respectively. Here $\phi_\infty$ denotes the peak amplitude of the potential. Equations (9) and (10) are a spiky solitary wave solution and an explosive solution, respectively. When $\phi_\infty = 0.75$, we illustrate the spiky solitary wave in Fig. 3. We show the dependency of the amplitude of the solitary wave on the ion temperature effect in Fig. 4.

**Fig. 3** The profile of the relativistic spiky solitary wave, in the case where $v_0 / c = 0.2$ and $\sigma = 0.03$.  
**Fig. 4** Dependency of the amplitude of the spiky solitary wave on the ratio $\sigma$, for $v_0 / c = 0.2$.  

![Fig. 3](image_url)  
![Fig. 4](image_url)
The dispersion relation in this system is obtained as

$$\omega = k \left\{ v_0 + \left( \frac{(\gamma_1 + 3\gamma_2\gamma_0)/(\gamma_0\gamma_1)}{\gamma_1 + 3\gamma_2\gamma_0} \right)^{1/2} \right. $$

$$- \frac{1}{2} \left[ 1 + v_0 \left( \frac{\gamma_0\gamma_1}{\gamma_1 + 3\gamma_2\gamma_0} \right)^{1/2} \right] \left( \frac{3\gamma_0\gamma_2 + (2/3)\gamma_2}{\gamma_0\gamma_1} \right) - \frac{1}{2} \left[ \frac{\gamma_0\gamma_1 (s - v_0) + (1/3)\gamma_2 + \sigma \gamma_0\gamma_2 (1 + (v_0/c)^2)}{\gamma_1 - (2/3)\gamma_2 s} \right]^{-1} k^2 \right\} $$

V. Discussion

The author has derived the mixed MK-dV equation and has shown the new nonlinear wave modes, that is, the relativistic double layer, the spiky solitary wave and the explosive solutions. We have shown the peculiar feature that the amplitude of the double layer grows as the relativistic and the ion temperature effects increase, and its thickness narrows with both effects increasing. The potential drop of the double layer grows as the ion temperature increases. These are shown for the first time for the relativistic double layers. It is clarified that the amplitude of the spiky solitary wave grows as the ion temperature increases. Both the modes also exist complementarily; if spiky solitary waves exist, explosive modes disappear and vice versa. This is proved by the boundary condition. As is seen in space and computer simulations, the solitary waves are not bell-shaped but spiky-shaped. The author think that the formation of the double layer is essentially associated with strong nonlinearity. Hence, the solitary waves which contribute to the generation and the disappearance between solitary waves and double layers need to be spiky-shaped and need to possess the same order of nonlinearity as the order of nonlinearity of the double layer. Recently, the existence of the streaming ion flux is worthy of notice as the condition of the formation of spiky solitary waves and double layers. This is an idea associated with the thought that high speed streaming ions form solitary waves and double layers. The present investigation presents a point of view that not only spiky solitary waves and double layers but also explosive events are generated by the high speed streaming ions. This investigation finds the peculiar feature concerning the evolution process of the nonlinear wave structure in which the relativistic double layer, the spiky solitary wave and the explosive modes form the fine structure in space.

References
The region of space adjacent to the Earth's bow shock, as well as other shock waves in the solar wind, is filled with large-amplitude magnetohydrodynamic (MHD) waves, here generally referred to as Alfvén waves. The theory presented in this paper is addressed toward an understanding of these waves. These waves show a number of properties which are generally supposed to be the result of nonlinear wave evolutions. These properties become more pronounced as one goes deeper into the "foreshock" (the term for the region upstream of the bow shock), corresponding to more advanced nonlinear development. The characteristics of interest are as follows. First, the initially sinusoidal waveforms become steepened. Second, there appears a shorter wavelength wavepacket near the front of the initial wavepacket. These short wavelength wavepackets are referred to as "shocklets," and presumably are generated from the initial wavepacket by nonlinear processes. Third, there is an evolution in the polarization of the Alfvén waves, from circular at shallow depths in the foreshock to elliptical/linear at greater depths, where nonlinear effects have become pronounced.

For some time we have been interested in interpreting these waves in terms of the Derivative Nonlinear Schrödinger Equation (DNLS), a nonlinear wave equation for Alfvén and Fast Magnetosonic waves\cite{1},\cite{2}. For the simplest case of wave propagation parallel to the large-scale magnetic field (the case considered in the aforementioned papers), the equation may be written as follows.

\[
\frac{\partial \phi}{\partial t} + \frac{i}{2} \left[ \frac{1}{8 \pi \rho_0 V_A(1 - \beta)} \right] \frac{\partial}{\partial x} \left\{ \phi(|\phi|^2 - \phi_0^2) \right\} + \frac{V_A^2}{2 \Omega_i} \frac{\partial^2 \phi}{\partial x^2} = 0
\]

Equation (1) describes one-dimensional waves propagating in the z direction. The wave field is \( \phi = b_y + i b_z \), where \( b_y \) and \( b_z \) are the components of the wave field in directions perpendicular to the mean field. The plasma through which the wave propagates is characterized by the plasma density \( \rho_0 \), Alfvén speed \( V_A \), ion-cyclotron frequency \( \Omega_i \), and plasma \( \beta \). In equation (2) the second term describes wave nonlinearity, and the third term describes dispersion due to right- or left-hand polarized waves.

As discussed in\cite{1} and\cite{2}, equation (1) yields solutions which reproduce some of the properties of the observed upstream waves. For the case of left-hand polarized waves and \( \beta < 1 \), or right-hand polarized waves with \( \beta > 1 \), an amplitude-modulated wavepacket will steepen and produce an associated wavepacket with a much smaller wavelength. These requirements regarding \( \beta \) and the sense of polarization are the same as for the occurrence of a modulational instability of a plane MHD wave.
The aforementioned conditions for the occurrence of wavepacket steepening, as well as equation (1) from which they are derived, are based on fluid theory which will be a poor description of the plasma when $\beta \approx 1$. A treatment based on kinetic theory /3/ has shown that substantial changes will occur in the structure of (1) in that the coefficient in the nonlinear term is not such a strong function of $\beta$, and that an additional term is introduced in the equation. It has been pointed out /4/ that this has serious ramifications for interpreting the nonlinear evolution of foreshock waves in terms of (1), since right-hand polarized waves will not readily undergo steepening. The observed waves are predominantly right-hand polarized.

We are presently considering the possibility that other physical processes, not included in (1) for the simple case of parallel propagation, actually cause the observed properties summarized at the beginning of the paper.

This paper is concerned with the effect of slightly oblique wave propagation. The waves are observed to be propagating at slight angles with respect to the large-scale magnetic field. Propagation angles of $5^\circ$–$15^\circ$ are typical. A nonlinear wave equation can be simply derived in the case $|b_y| \ll |B_0 \sin \Theta| \ll |B_0|$, where $B_0$ is the large-scale field and $\Theta$ is the angle of propagation with respect to the field /5/. A rigorous but more complex derivation is also available in the literature /6/.

The novel features introduced by oblique propagation may be illustrated by the equation for the $y$ component of the magnetic field /5/.

$$\frac{\partial b_y}{\partial t} + \frac{1}{2} \left[ \frac{1}{8\pi \rho_0 V_A(1-\beta)} \right] \frac{\partial}{\partial x} \left\{ b_y (b_y^2 + b_z^2) \right\} + \frac{B_{0y}}{4\pi \rho_0 V_A(1-\beta)} b_y \frac{\partial b_y}{\partial x} + \frac{B_{0y}^2}{8\pi \rho_0 V_A(1-\beta)} \frac{\partial b_y}{\partial x} = 0$$

In equation (2) we have not written terms describing the effect of dispersion on the equation of motion for $b_y$. The convention employed in equation (2) is that $b_y$ is the component of the wave field which lies in the plane determined by the large-scale magnetic field $\vec{B}_0$ and the direction of wave propagation ($\hat{z}$).

There are two important features about equation (2). First, the underlined terms represent Korteweg-deVries like nonlinearities which vanish in the limit of parallel propagation ($B_{0y} \rightarrow 0$). In view of the well-known proclivity of the Korteweg-deVries equation for wave steepening, this result immediately suggests that a slight degree of oblique propagation might be important for the steepening of nonlinear MHD waves. The second important fact is that the equation for $b_z$ (the component of the wave field perpendicular to the plane containing $\vec{B}_0$ and $\hat{z}$) differs from (2) in that it does not contain Korteweg-deVries like nonlinearities. Since the differential equations for $b_y$ and $b_z$ differ in the case of oblique propagation, polarization evolution is an inevitable consequence of nonlinear evolution.
While equation (2) appears to be much more complicated than the DNLS, it has been shown in /3/, /6/ that obliquely propagating nonlinear waves also obey the DNLS, provided that the field is redefined as

\[ \phi = b_y + ib_x + B_0 \]  

(3)

in other words, the real part of the field now has a constant part. The results presented in this paper concern analytic and numerical solutions to (1), given (3) as a field variable.

We have explored stationary solutions to (1), i.e., solutions of the form \( \phi(x,t) = \Phi(x - V_0 t) \). Such solutions were considered at length in /7/; but our work has consisted mainly of a detailed inspection of the solutions. We solve the equation using the pseudopotential method /5/.

For extremely small angles of obliquity, simple analytic expressions for the wave field are possible. An example of our results is shown in Fig. 1. The left panel represents \( b_y \) as a function of the comoving coordinate \( \xi = x - V_0 t \), and the right panel \( b_x \). The dashed line shows a circularly polarized wave (a stationary solution in the case of parallel propagation) but with a constant term added to \( y \) component. Such a wave train might be considered a zeroth-order model for an obliquely propagating Alfvén wave. The solid line displays a true stationary solution for the case of very small obliquity. A comparison of the two waveforms yields two conclusions of immediate interest. First, the noncircular polarization is obvious. The stationary waveforms in \( b_y \) and \( b_x \) differ. It will be noticed that the stationary solution for \( b_y \) is more rounded near maximum than the circularly polarized wave. The solution for \( b_x \) shows a continual decrease at the position where the circularly polarized wave is at maximum. Second, the waveform in \( b_x \) shows a more rapid increase to maximum than the circularly polarized wave, a tendency which may be considered incipient steepening of the wavepacket.

In the case of more pronounced oblique propagation, analytic solutions for the stationary waves exist, but they are not of such simple functional form as those shown in Fig. 1. However, the properties noted in Fig. 1 persist and become more pronounced /5/.

We do not expect waves in real space plasmas to be formed as stationary wave trains, so the question arises as to the relation between these stationary solutions and more arbitrary, obliquely propagating wavepackets. To explore this, we have carried out numerical solutions to the DNLS, using circularly polarized, oblique wavepackets of the sort shown in Fig. 1 as initial conditions. We expect that if the initial wave train does not differ significantly from a stationary solution wave, subsequent evolution will be relatively minor, whereas if the initial wave train differs markedly from any stationary solution, substantial evolution will occur. Criteria for assessing similarity of an initial condition to a stationary wave train are presented in /5/.

An example of a case in which substantial evolution occurred is shown in Fig. 2. The four rows show the wave intensity, components of the wave field, and spatial power spectrum at \( t = 0 \) (initial condition, top row) and three subsequent times. The calculation shows strong evolution of the initial wave train, as expected from the remarks above. The evolved wave train shows some of the properties of the stationary solutions, as may be seen by comparing the \( b_y \)
and $b_z$ waveforms at $t = 40$ and 80 in Fig. 2 with those in Fig. 1. This may be interpreted as a tendency of the initial wave train to evolve towards a stationary solution wave train.

The most striking feature of the calculation in Fig. 2 is the development of short wavelength wave packets late in the evolution. This development is most clearly illustrated in the plots of $b_y^2 + b_z^2$ at $t = 80$ and 100. These wave packets closely resemble the shocklets observed in spacecraft data.

We conclude that oblique propagation of nonlinear Alfvén and Fast Magnetosonic waves is an important effect which can account for several observed characteristics of MHD waves in space plasmas.

MATHEMATICAL THEORIES OF RESONANT ABSORPTION OF ALFVÉN WAVES IN SPACE PLASMAS: TIME DEPENDENT SOLUTION

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The Alfvén wave equation governing the dynamics of an ideal MHD system in the presence of inhomogeneous magnetic fields has been extensively studied both by normal mode analysis and as an initial value problem. The time-dependent solutions, however, have been either found asymptotically or for very specific initial conditions. Recently, Uberoi and Sedlacek /1/ have given an integro-differential formulation for this equation and constructed a time-dependent solution for very general smooth initial conditions. The aim of this note is to discuss this solution, firstly, to understand the phase mixing as current sheets crossover phenomena and secondly, with relevance to earlier studies especially those which were made to understand some of the magnetospheric phenomena.

Considering an incompressible medium with density $\rho$ and the magnetic field $B_0(x) = [0, B_{01}(x), B_{02}(x)]$ the linearized ideal MHD equations give the Alfvén wave equation:

$$\nabla \cdot \left\{ \rho \left( \frac{\partial^2}{\partial t^2} - \frac{(B_0 \cdot k)^2}{4\pi \rho} \right) \nabla U_{xk} \right\} = 0 \quad (1)$$

where $U_{xk}(x,k,t)$ is the Fourier transform in $y$ and $z$ of the perturbed velocity component $v_x(x,y,z,t)$. The wave number $k=(o,k_y,k_z)$. The integro-differential formulation of eqn.(1) as given in /1/ is:

$$\frac{\partial}{\partial x^2} \left( \frac{\partial^2}{\partial t^2} - k^2 \right) \left( \frac{\partial^2}{\partial t^2} + \omega_A^2(x) \right) e_k(x,t) = \frac{\partial \omega_A^2(x)}{\partial x} \int e_k(s,t) \, ds \quad (2)$$

Eqn. (2) is to be solved with boundary conditions and initial conditions: $e_k(x,t) \to 0$ as $|x| \to \infty$ and $e_k(x,0) = \phi(x,k)$ and $\dot{e}_k(x,0) = \psi(x,k)$ where $|\phi(x,k)|$ and $|\psi(x,k)| \to 0$ as $|x| \to \infty$. Here $e_k(x,t) = \partial v_x / \partial x$, where $v_x = (\partial v / \partial t)$ and $\omega_A(x) = (B_0(x)k)^2/4\pi\rho_0$.

The left side of eqn. (2) shows the coupling of surface waves to the local Alfvén oscillations arising due to the inhomogeneous Alfvén frequency $\omega_A(x)$ as seen on the right hand side of the eqn. The solution of (2) is

$$e_k(x,t) = \frac{\partial v_x}{\partial x} = \phi(x,k) \cos t \omega_A(x) + \psi(x,k) \frac{\sin t \omega_A(x)}{\omega_A(x)}$$

$$+ \int_0^t \frac{\sin(t-\tau)}{\omega_A(x)} K(x,\tau,k) d\tau,$$

where $K(x,\tau,k) = \sum (l+T)^{-1} Z^{-1} T^{-1} Z^{+ ...} e_{k0}(x,\tau)$. $e_{k0}(x,t)$ is the first two terms of the solution in (3). $Z$ is an integral operator with kernel $Q(s,t)$ /1/. $T$ is the operator $[\partial^2 / \partial t^2 + \omega_A^2(x)]$.

Considering $\phi(x,k) = Ae^{-kx}$ and $\psi(x,k) = Be^{-kx}$ we get dominant asymptotic terms for $\xi_x$ by integrating (3) and taking the limit $t \to \infty$. For $\omega_A(x)$ as a linear function of $x$:

$$\xi_x = Ae^{-kx} \frac{\sin \omega(x) t}{t} + Be^{-kx} \frac{\cos \omega(x) t}{t} + O\left(\frac{1}{t^2}\right).$$

This value corresponds to the asymptotic solution obtained by Barston /2/. Also we note that $\xi_z = \partial \xi_x / \partial x$ does not show decay with time whereas $\xi_x$ decays as $1/t$. Hence, eventually for large $t$ the wave becomes a longitudinal wave. This result was arrived at by numerical simulation of the Alfvén wave equation /3/.


To understand the phase-mixing we take $B_0(x) \cdot k = B_0(x)k_z$, calculation of the current density $J$ shows that

$$
4\pi J_y = \frac{(4\pi \rho_0)^{1/2}}{k_z} \frac{\omega_A(x)t}{t_A(x)} (\frac{\phi(x,k)\cos \omega_A(x) t}{\omega_A(x)}) + \text{terms showing no increase with } t.
$$

From (4) we see that $J_y$ has terms which show that the amplitude increases linearly with time. This is the manifestation of the cross-over phenomena of current sheets. Each current sheet oscillates with its local Alfvén frequency $\omega_A(x)$ but after time $t > \tau$ the current sheets begin to cross-over and get phase mixed. The phase mixing time $\tau \equiv \tau/(d\omega_A/dx)$ as obtained from eqn. (4) is same as calculated by initial value problem /4/. The eqn. (4) shows that $J_y$ increases at every point in space and not just near the neutral sheet /5/. The implication of this result on the surface wave induced magnetic reconnection is discussed by Uberoi /6/.

Now we consider an initial perturbation of the type $\Phi(x,k) = A e^{-k_x x} \exp[i(k_y y + k_z z)]$ with $u_0 = k_x/k_y$ as an inclination of the surface 'wavelet'. For simplicity $\psi(x,k) = 0$. Taking a linear function for the magnetic field, $\omega_A(x) = (v_A k_z + v_A k_y)/a$. Integrating (3) and taking $A = (k_x^2 + k_y^2)^{1/2}$, we have

$$
\xi_x = \left[ \frac{1 + u_0^2}{1 + (u_0 - v_A t/a)^2} \right]^{1/2} \exp i [k_y y + k_z (z - v_A t) + i k_x (u_0 - v_A t/a)]
$$

+ other terms.


The first part of eqn.(5) gives the amplification of the amplitude for $t < \frac{a_0}{v_A}$, the other terms do not add to the amplification. This transient amplification was first noted by Lau /7/ by solving the eqn.(1) approximately and using numerical analysis technique. The application of this to understand the excitation of resonant Alfvén waves by sudden impulse in magnetoplasma was discussed by Uberoi /8/. The solution (3), however, points that such a transient amplification can be experienced by a general class of perturbations which are like a surface 'wavelet' when released on to the shear hydromagnetic flow.

In order to explain certain observations of magnetic pulsations a time-dependent solution of eqn.(1) was discussed in the presence of a wide-band source /9/. The solution given by eqn.(2) in this paper can be shown to correspond to the solution in /9/, thus suggesting that the presence of an external wide band source is not necessary for driving the local field line oscillations in the magnetosphere but as discussed in our earlier paper /10/ with the approximate solution of the Alfvén wave equation, any surface perturbation caused by K-H instability, for example, can be responsible for driving these oscillations.

RUNAWAY ELECTRONS IN THE PRESENCE OF KINETIC ALFVÉN WAVES

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ABSTRACT

It is shown by the quasilinear Fokker–Planck approach that the shear kinetic Alfven wave (KAW) cannot by itself produce runaway electrons, although it carries an electric field aligned with the ambient magnetic field. However, it can enhance the runaway production rate if it propagates in the presence of a background DC electric field.

1—INTRODUCTION

It is well known that when an uniform electric field is acting on an uniform plasma, a certain fraction of the electrons will run away, that is, they will gain an energy such that the electric force acting on them exceeds the collisional drag force (Kulsrud et al. 1973 and references therein).

In this work we consider the runaway production by a DC electric field when the KAW is present. This is a relevant and open problem in space plasma physics, mainly in solar, magnetospheric and extragalactic plasmas, where there is observational evidence of energetic particles (electrons/ions) being generated when the KWA or/and a DC electric field is/are present in that plasmas (Jafelice and Opher 1999; Hasegawa 1987; Borovsky 1986; Otani and Silberstein 1991; Melrose 1980). Also, there are still questions on how the KAW affects the runaway production rate and the nonthermal emission (Hollweg 1981).

The KWA was introduced by Hasegawa and Chen 1975 in the context of plasma heating: it is the shear Alfven wave with a finite perpendicular wavelength, and its name was chosen because of its kinetic properties. Along the direction of the background magnetic field its scale length is that of MHD waves, whereas in the direction perpendicular to the background magnetic field, its scale length is that of the ion Larmor radius. It can be excited by an MHD surface wave through linear resonant mode conversion, or by a drift wave instability (Hasegawa and Mima 1978). The kinetic nature makes it possible that the KWA can be damped via Cherenkov interaction (CHD), that is the Landau damping (LD) and Transit Time Magnetic Pumping (TTMP). Here, only Landau damping will be considered since the KWA has no TTMP due to its vanishing parallel wave magnetic field. The Landau damping of the KWA can affect the runaway production rate via the wave particle interaction (de Assis and Tsui 1991).

To determine this runaway production rate, which is the main goal here, it is necessary to use the Fokker–Planck quasilinear approach.

2—FOKKER–PLANCK QUASILINEAR EQUATIONS

For simplicity, we consider a homogeneous, uniformly magnetized, singly ionized plasma situated in a constant uniform background DC electric field. Also, as a first approach, we restrict ourselves to the one–dimensional case. Therefore, we can write:

\[
\frac{\partial F}{\partial t} = \frac{\partial F}{\partial t}_{KAW} + \frac{\partial F}{\partial t}_{\text{col}} + \frac{\partial F}{\partial t}_{\text{E-field}} + S
\]  

and,
\[ \frac{\partial \mathbf{U}_k}{\partial t} = 2\text{Im} \omega_k^{\text{KAW}} - \nu_{\text{col}}/2 \mathbf{U}_k + (\mathbf{v} \cdot \mathbf{F})_k. \] (2)

The quasilinear operator can be written as (Tsui and de Assis 1989),
\[ \frac{\partial F}{\partial t} \bigg|_{\text{KAW}} = \frac{d}{dv} D(v_\|) \frac{dF}{dv}. \] (3)

with
\[ D(v_\|) = \pi e^2/m_e \int |E_{\|,k}^*|^2 \delta(\omega_k - k_n v_\||) d^3k. \] (4)

The collision operator is
\[ \frac{\partial F}{\partial t} \bigg|_{\text{col}} = \frac{d}{dv} \nu(v_\|)[F(v_\|) + v_\| \frac{dF}{dv}], \] (5)

where
\[ \nu(v_\|) = \begin{cases} \nu_{ee} + 3 \nu_{ei}/4, & v_\| << v_{th} \\ \nu_0(2 + Z)/v_\|, & v_\| >> v_{th} \end{cases} \] (6)

with \( \nu_0 = \nu_{ee} v_{th}^3 \). \( \nu_{ee} \) is the rate at which the electrons collide with particles of species \( \alpha \) (\( \alpha = e, i \)): \( \nu_{ee} = 4 \sqrt{2} \ln \lambda (e_0/2)^2 n_\alpha/(3 \sqrt{m_e T_{e0}}) \); \( \ln \lambda \) is the Coulomb logarithm and \( Z \) is the ion charge state [Trubinikov 1965]. This collision operator is appropriate to describe collisions between particles with velocities much smaller than the electron thermal velocity (first case), and much larger than the thermal velocity (second case). This term is linear and models the situation where the dissipated energy goes into a thermal reservoir [Kulsrud et al. 1973]. It is suitable to model the relaxation of the perturbed distribution function towards a Maxwellian.

The DC electric field term is
\[ \frac{\partial F}{\partial t} \bigg|_{E\text{-field}} = \frac{dE}{dv}. \] (7)

Finally, \( S \) is the necessary particle source term, since a stationary distribution function involving a loss of particles at \( + \infty \) implies an equivalent source at \( v_\| = 0 \), and \( \mathbf{P} \) is the Poynting vector. Assuming the steady - state situation \( \partial F/\partial t = 0 \), Eq. (1) can be rewritten as
\[ \frac{d}{dv}[\nu(v_\|)(F(v_\|) + v_\| \frac{dF}{dv})] - \frac{d}{dv}[D(v_\|) + v_\| \frac{dF}{dv}] + eE \frac{dF}{dv} + A \delta(v_\|) = 0. \] (8)

On introducing the Heaviside function in the source term of (8), all terms are total derivatives with respect to \( v_\| \), and can immediately be integrated to yield,
\[
\frac{dF}{dv_{th}}\left[\nu(v_{th})v_{th}^2 + D(v_{th})\right] + F(v_{th})\left[\nu(v_{th})v_{th} + \frac{eE}{m_e}\right] = C - AH(v_{th}),
\]
(9)

where a convenient regrouping has been carried out.

In order to determine the constant of integration \(C\), we consider the limit \(v_{th} \to \infty\), where \(H(v_{th})\) is equal to unity, and \(\frac{dF}{dv_{th}}\) as well as \(\nu(v_{th})\) are equal to zero. Hence, (9) becomes

\[
F(\infty) = C - A
\]
(10)

The first term is the flux of the runaway electrons, which in the steady-state must be equal to the source term constant \(A\). Therefore, consistency requires that the integration constant \(C\) is taken to be equal to zero. As a consequence, the general solution of Eq. (9) is

\[
F(v_{th}) = F(0)\exp\left[-\int_0^{v_{th}} \frac{\nu(u)u + \frac{eE}{m_e}}{\nu(u)v_{th}^2 + D(u)}\,du\right]
- A\int_0^{v_{th}} \frac{\exp\left[-\int_0^{v_0} \frac{\nu(w)w + \frac{eE}{m_e}}{\nu(w)v_{th}^2 + D(w)}\,dw\right]}{\nu(u)v_{th}^2 + D(u)}\,du.
\]
(11)

Note that the result (11) remains valid if \(\nu(v_{th})\) and \(D(v_{th})\) are taken to be generalized functions,

\[
\nu(v_{th}) = \nu_{ei}[H(v_{th} - v_{th}) + \frac{v_{th}^2}{v_{th}^2}H(v_{th} - v_{th})],
\]
(12)

\[
D(v_{th}) = D_0[H(v_{th} - v_{th}) - H(v_{th} - v_{th})].
\]
(13)

In order to gain information about the runaway production rate \(A\), we let \(v_{th} \to \infty\), with the result

\[
A = F(0)\exp\left[-\int_0^{\infty} \frac{\nu(u)u + \frac{eE}{m_e}}{\nu(u)v_{th}^2 + D(u)}\,du\right],
\]
(14)

where use has been made of Eq. (14) (with \(C = 0\)) in order to eliminate \(F(\infty)\) in favor of \(A\).
In Figure 1, we plot Eq. (14) for low phase velocity, taking $v_{1n}$ and $v_{2n}$. We see that the KAW cannot by itself generate runaway electrons, but can only enhance the production rate. Also, this enhancement is not so strong as in the case of lower hybrid waves (An et al. 1982), the discrepancy stemming from different velocity dependence of the diffusion coefficient of these two modes. The high phase velocity case was also investigated, but is not shown here for it does not show any new features over the low phase velocity one.

![Flux vs. Diffusion Coefficient](image)

**FIGURE 1:** Normalized runaway flux $A/nv$ versus normalized diffusion coefficient $D_0 = D_{KAW}/n\nu \nu_{he}$ for three different electric fields $E$ in units of the critical field $E_c$.

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POLARIZATION CHANGES THROUGH STIMULATED RAMAN SCATTERING

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Change of polarization of electromagnetic radiation due to its propagation in a magnetized plasma (Faraday rotation) and due to electron scattering is well known. In this paper, the change in the polarization due to the coherent scattering of the electromagnetic wave off the electron plasma wave (stimulated Raman scattering (SRS)) is investigated /1/, /2/. It is found that some of the observed polarization properties of the pulsar and quasar radio radiation, not accountable by Faraday rotation and electron scattering, can be explained by the SRS.

Consider a large amplitude elliptically polarized electromagnetic wave

\[ \vec{E}_0 = \varepsilon_{10} [\cos(k_0 z - \omega_0 t) \hat{x} + \alpha_0 \cos(k_0 z - \omega_0 t + \delta_0) \hat{y}], \]  

where \( \alpha_0 = \varepsilon_{20}/\varepsilon_{10} \), which can be thought of as the superposition of two linearly polarized waves, \( \vec{E}_1 = \varepsilon_{10} \cos(k_0 z - \omega_0 t) \hat{x} \) and \( \vec{E}_2 = \varepsilon_{20} \cos(k_0 z - \omega_0 t + \delta_0) \hat{y} \), propagating in a plasma of density \( n_0 \) and temperature \( T \). Let \( \vec{E}_1 \) induces density perturbation \( \delta n_{1z} = \delta n_1 \cos(kz - \omega t) \) and \( \vec{E}_2 \) induces \( \delta n_{2z} = \delta n_2 \cos(kz - \omega t + \delta_e) \).

The scattered wave

\[ \vec{E}_s = \varepsilon_{1s} [\cos(k_s z - \omega_s t) \hat{x} + \alpha_s \cos(k_s z - \omega_s t + \delta_s) \hat{y}], \]  

where \( \alpha_s = \varepsilon_{2s}/\varepsilon_{1s} \), satisfies the wave equation

\[ \left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E}_s = \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t}. \]  

The current density is given by \( \vec{J} = -e n_e \vec{u}_e \), where \( \vec{u}_e \) is the velocity of oscillation of electrons in the radiation field. We obtain \( \vec{u}_e \) from the equation:

\[ \frac{\partial \vec{u}_e}{\partial t} = -\frac{e}{m} [\vec{E}_0 + \vec{E}_s], \]

where \( e \) and \( m \) are the charge and the mass of an electron.
From equation (3), we get

\[ D_s \varepsilon_{1s} = -\frac{4\pi \varepsilon_0^2 \varepsilon_{1a}}{m} \delta n_1, \]  

\[ D_s \varepsilon_{2s} = -\frac{4\pi \varepsilon_0^2 \varepsilon_{2a}}{m} \frac{\cos(kz - \omega t + \delta_\epsilon)}{\cos(kz - \omega t + \delta_\sigma - \delta_\delta)}, \]  

where \( D_s = k_s^2 c^2 - \omega_s^2 + \omega_p^2 \), \( \omega_s = \omega_0 - \omega \), \( k_s = \vec{k}_0 - \vec{k} \).

The density fluctuations \( \delta n_1 \) and \( \delta n_2 \) can be determined using the standard procedure /3/:

\[ \delta n_1 = -\frac{\varepsilon_{1s} \varepsilon_{1a}}{8\pi m \omega_0^2 k_s^2} \frac{\chi}{(1 + \chi)}, \]  

\[ \delta n_2 = -\frac{\varepsilon_{2s} \varepsilon_{2a}}{8\pi m \omega_0^2 k_s^2} \frac{\chi}{(1 + \chi)} \frac{\sin(kz - \omega t + \delta_\sigma - \delta_\delta)}{\sin(kz - \omega t + \delta_\epsilon)}, \]  

where \( \chi \) is the electron susceptibility.

We find the dispersion relation from equations (4) and (5):

\[ 1 + \frac{1}{\chi} = \frac{1}{8 \varepsilon_0^2 k_s^2} \left[ \frac{1}{D_-} + \frac{1}{D_+} \right] \]  

with \( \delta_\sigma - \delta_\delta = \delta_\epsilon, \alpha_o^2 = 1, p = \delta n_2/\delta n_1, \alpha_\sigma = p/\alpha_o, \nu_o^2 = \varepsilon_0^2 \varepsilon_3^2 / m^2 \omega_0^2 \) and \( D_\pm = (k_s \pm k)^2 c^2 - (\omega_0 \pm \omega)^2 + \omega_p^2 \).

**Stokes Parameters:** For the incident wave (\( \alpha_o = \pm 1 \)), the Stokes parameters are /4/:

\( I_o = 2\varepsilon_0^2, \quad Q_o = 0, \quad U_o = 2\varepsilon_0^2 \cos(\delta_\sigma), \quad V_o = 2\varepsilon_0^2 \sin(\delta_\sigma) \).

The angle \( \beta \) describing the sense of rotation of the electric field is given by \( \sin(2\beta_o) = V_o/I_o = \sin(\delta_\sigma) \) or \( \beta_o = \delta_\sigma/2 \). The magnitudes of the principle axes of the ellipse are \( a_o = \sqrt{I_o} / \cos(\beta_o) \) and \( b_o = \sqrt{I_o} / \sin(\beta_o) \). The orientation of the axes of the ellipse relative to \( x \) and \( y \) axes, is given by \( \tan(2\chi_o) = U_o/Q_o \), so that \( \chi_o = \pi/4 \) for \( 0 \leq \delta_\sigma < \pi/2 \).

The corresponding parameters for the scattered wave are:

\( I_s = \varepsilon_{1s}^2 (1 + p^2), \quad Q_s = \varepsilon_{1s}^2 (1 - p^2), \quad U_s = 2\varepsilon_{1s}^2 p \cos(\delta_\epsilon), \quad V_s = 2\varepsilon_{1s}^2 p \sin(\delta_\epsilon), \quad \sin(2\beta_s) = V_s/I_s = 2p \sin(\delta_\epsilon)/(1 + p^2), \quad a_s = \sqrt{I_s} / \cos(\beta_s), \quad b_s = \sqrt{I_s} / \sin(\beta_s), \quad \) and \( \tan(2\chi_s) = 2p \cos(\delta_\epsilon)/(1 - p^2) \).

From the condition, \( \delta_\sigma - \delta_\delta = \delta_\epsilon \) we find:

1. for \( \delta_\sigma - \delta_\epsilon = 0; \delta_\sigma = 0 \), the incident elliptically polarized wave scatters into a linearly polarized wave; (2) for \( \delta_\sigma = 0; \delta_\epsilon = 0 \), the linearly polarized wave scatters into an elliptically polarized...
wave; (3) for $\delta_e = 0$; $\delta_0 = \delta_s$, there is no change in polarization; (4) for $\delta_0 - \delta_e = \delta_s$, in general for $\delta_0 \neq 0$, $\delta_s \neq 0$ and $\delta_e \neq 0$ elliptically polarized wave scatters into another elliptically polarized wave with or without change in the sense of rotation.

When $\omega \gg k v_e$, $v_e$ is the thermal velocity of electrons, the electrostatic modes are relatively well defined since they are not heavily damped on electrons. In this limit, the expression for growth rate, obtained from (8), is given by

$$\gamma = \frac{-1}{2} (\Gamma_p + \Gamma_-) \pm \frac{1}{2} \sqrt{\left( \Gamma_p - \Gamma_- \right)^2 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \omega_p \omega_p \cos^2(\theta)},$$

where $\theta$ is the angle between $\vec{k}_0$ and $\vec{k}$, $\Gamma_p$ and $\Gamma_-$ are the Landau and collisional damping rates, and $\omega_p$ is the plasma frequency. The typical plasma parameters of a quasar at a distance of 0.01 pc from the central engine are: $n_e = 10^{10} \text{cm}^{-3}$, $T = 10^4 K$. The luminosity of radiation in the radio band $\Delta \nu < 10^{10} \text{Hz}$ is $10^{42} \text{erg sec}^{-1}$. For these parameters, the value of the growth rate of the SRS is $\gamma = 7.5 \times 10^5 \text{sec}^{-1}$. Which shows that changes in the polarization can occur extremely rapidly with a characteristic time as small as a few micro seconds. Reversal in the sense of polarization over a few milli to micro seconds has been observed in some pulsars/5/.

We conclude that SRS may provide a possible explanation for such a fast change in polarization.

Topic 10: Basic Collisionless Plasma Physics
TRANSLATION TO CHAOS IN PERIODICALLY DRIVEN THERMIONIC DIODES AT LOW PRESSURE

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1. Introduction. The static I(U) characteristic of thermionic diodes at mbar pressures shows a large hysteresis, which describes the transition from the 'anode-glow-mode' (AGM), with essentially negative plasma potential, to the 'temperature-limited-mode' (TLM), with positive plasma potential. Many features of these modes are also found in magnetic-box discharges with filament cathodes at pressures of $10^{-2} - 10^{-1}$ Pa. Although these two pressure regimes are basically different concerning the transport properties (diffusion vs. free streaming), the elementary processes that establish the AGM in the low pressure regime are very similar to the high pressure regime. Ions are produced in that part of the anode sheath where the potential exceeds the ionisation energy. The production rate is enhanced by multiple reflections of electrons between the magnetic fields of the permanent magnet array at the anode and the repulsive potential of the cathode plasma. Although the mean free path for charge exchange or elastic collisions substantially exceeds the anode-cathode distance, some few ions are stopped and trapped within the potential well of the virtual cathode. This accumulation of ions forms a cathodic plasma, which is essentially at cathode potential. Plasma formation in the anode sheath is suppressed as long as the ion production time is larger than the ion transit time through the sheath. These model ideas are supported by 1d-Particle-in-cell simulations using a modified PDP1-code. The AGM is attractive for studies of nonlinear dynamics because of its feedback processes and oscillations, which occur close to the hysteresis point.

When the static point of operation is chosen in the AGM, and the system is periodically forced to attain the TLM transiently, two different routes to chaos have been observed. Cheung et al. reported the appearance of intermittency for a diode with superimposed sine-modulation, when the selfspiking was suppressed by a suitable choice of the system's length. In the opposite case the same authors found mode-locking, Arnol'd tongues, and a quasi-periodic way to chaos. Timm and Piel, by applying sine-waves, observed a truncated period-doubling sequence for the case, in which selfspiking was not excited. The system attained a 'chaotic' state, which could be characterized by short laminar sequences of periods 4, 2, and 1. Apparently the system was hopping between different stages of a period doubling sequence.

2. Period doubling. For sine-modulation it is not distinguishable, whether the modulation amplitude or an effective width of the sine-wave, for which the hysteresis threshold is exceeded, acts as control parameter. We have therefore changed the modulation to pulse-trains of independently controllable width and height (Fig. 1(a)). Our experiments are performed in a stainless steel vessel of 325mm dia. and 330mm height, which acts as anode. The cathode is a single straight tungsten filament (0.2mm dia., 160mm length, 4.0-4.5A) in the center of the vessel. The outer wall is covered by rows of permanent magnets (10x20x15, 0.12T) that form a set of line cusps of alternating polarity with a separation of 10cm. The discharge is operated in Argon at $p = 10^{-2} - 10^{-1}$ Pa. Figure 1(b) shows the typical hysteresis of the I(U) characteristic. The part A-B represents the AGM, whereas E-D is the TLM. In the interval B'-B selfspiking is observed. In comparing the voltage of the hysteresis point B with the ionisation energy $\Phi_{argon} = 15.6V$, one must take the
voltage drop along the cathode filament \((U_{fil} = 15V)\) into consideration, which raises the starting potential of primary electrons.

A period doubling sequence is observed, when the discharge is periodically pulsed from a stable state in the AGM \((U = 27V)\) to an unstable state \((U = 41V)\), and when the pulse repetition frequency is chosen between 2.5kHz and 3kHz. An optimum choice is 2.82 kHz and the pulse width is varied from 60 — 90μs. Figure 2(a) shows the time series of current pulses, Fig. 2(b) gives the corresponding bifurcation diagram of the stroboscopically sampled current pulses, taken at the end of the driving pulse. Period 1, 2, 4, and chaos are clearly distinguishable. A nearly identical bifurcation diagram is found, when the pulse frequency is varied for constant pulse width. The choice of the pulse height in the regime 40-60V has only a minor influence. It may therefore be concluded that the pulse-width, i.e., the rate of ions produced in the anode sheath is the control parameter for the dynamics, whereas the pulse height is unimportant, because the associated electron energy corresponds to the maximum of the ionisation cross-section.

The return map for the chaotic state \((D)\) is displayed in Fig. 2(c). Evidently, the chaotic dynamics is given by a non-invertible quadratic map. The correlation dimension at this point is \(D_2 = (0.68 \pm 0.2)\), which has to be compared with the dimensionality of the standard logistic mapping at the accumulation point \(D_2 = 0.5\). We attribute the slightly increased dimensionality to the influence of the noise in the system. The logistic mapping with a noisy control parameter: 
\[ x_{n+1} = (\lambda + \Delta \lambda \cdot \text{rand}(n))x_n(1 - x_n) \]
yields a very similar bifurcation diagram for \(\Delta \lambda = 0.03\), that only consists of periods 1, 2, 4, and chaos without windows.

3. Mode locking. Operating the discharge in the regime \(B' - B\) of the characteristic, unstable selfspiking occurs, which becomes nearly periodic close to \(B\). The formation of a current spike is initiated by a Buneman-instability of the cathodic plasma, formation of an electron hole, and reduction of negative space charge by ions streaming to the cathode. Two time scales are involved, the removal of ions from the cathodic plasma by a double layer type field, and the refilling and expansion of the cathodic plasma by trapping of new ions. Similar processes have recently been suggested for oscillations in thermionic converters and Q-machines.

A small sinusoidal signal applied to the cathode is found considerably amplified on the plasma potential halfway between anode and cathode. This amplification is indicative of the inherent plasma instability. The selfspiking can be mode-locked even by weak signals (i.e. a few per-cent of the dc-voltage). Figure 3 gives a measured phase diagram for the mode-locked states. A typical pattern of Arnol’d tongues is found. The arrangement of mode-locked states gives the expected Farey sequence. Within the Arnol’d tongues, period doubling transitions to chaos are observed. Outside the mode-locked states quasi-periodic states are observed. For strong modulation various transition to chaos can be observed. Especially for the period-doubling route within the mode-lock-2 state, the reconstructed phase space attractor has a simple unimodal shape similar to Fig. 2(c). This finding hints at close relationships between the period-doubling sequence reported above and the period-doubling observed in the mode-lock-2 state. The Arnol’d tongues observed in Ref.5 are in two respects rather different from our case. They are extremely narrow for low modulation voltage, and overlap of the tongues is only observed for modulation voltages in excess of 30V. In our case the modulation index is typically an order of magnitude smaller.

For the case of suppressed selfspiking, it is justified to represent the internal oscillator by a complex frequency, whose imaginary part changes sign at point \(B'\). In addition it
is reasonable to assume only a small change of the real part when crossing $B'$. Hence, we can compare the frequency of selfspiking with that value of the modulation frequency that yields the period-doubling route. This frequency is typically twice the natural spiking frequency. Therefore, the period-doubling route may be interpreted as mode-locking of a damped oscillation, which is destabilized by the modulation.

Conclusions. We have demonstrated how period doubling and mode-locking phenomena in thermionic diodes are connected with selfspiking instabilities of the cathodic plasma in the AGM. The internal control parameter of the nonlinear dynamics is the number of ions per driver period produced in the anode sheath, which can be varied by altering either the pulse-width or repetition frequency. Similar shapes of phase space attractors with a simple quadratic extremum were found at the accumulation point of the period-doubling sequence and at chaos-onset inside the period-2 Arnol’d tongue.

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Fig. 1 (a) Thermionic diode with pulse modulation (left), (b) static hysteresis curve. A-B: anode-glow-mode, B'-B: selfspiking regime, E-D: temperature-limited-mode (right)
Fig. 2(a) Time series of current pulses for increasing pulse width (left), (b) Corresponding bifurcation diagram (top right), (c) Chaotic return map $I_{n+1} = f(I_n)$ (bottom right).

Fig. 3 Mode locking regimes in the $m$ vs. $f_0/f$ plane.
EQUILIBRIUM DOUBLE LAYERS IN EXTENDED PIERCE DIODES

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1. Introduction

The extended Pierce diode is similar to the standard (or classical) Pierce diode, but has passive circuit elements (R, L, C) in place of the short circuit between the electrodes (Fig.1)/1//2/. This device is important as an approximation to real bounded plasma systems. It consists of two parallel plane electrodes (an emitter located at x=0 and a collector located at x=L) and a collisionless cold electron beam traveling between them. The electrons are neutralized by a background of comoving massive ions. This situation is analysed in this paper and new equilibrium double layer (DL) plasma structures are obtained.

2. Basic Equations

We introduce the adimensional quantities:

\[ \omega = \frac{\omega_p}{\omega} \] (Pierce parameter), \[ \rho' = \rho/\rho_0, \quad v' = v/v_0, \quad x' = x/L, \]

\[ E' = qL E/mv_0^2, \quad \xi = q\rho/mv_0^2, \quad \lambda = \omega x', \quad \xi = E'/\omega, \quad E'/\omega = \epsilon, \quad r = \rho' - 1, \]

where \[ \omega_p = \left( n_0 e^2/\varepsilon_0 m \right)^{1/2} \equiv (q\rho_0/\varepsilon_0 m)^{1/2} \] is the electron plasma frequency of the beam, \( m = \) electron mass, \( \rho_0 = -e_n \), \( \rho = \) the electron charge density, \( n_0 = \) both the ion background density and electron emission density, \( \omega_t = v_0/\lambda = \) the transit frequency. The electrons leave the emitter with velocity \( v_0 \), and are absorbed when hitting either electrode.

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The equilibrium equations are

\[ p'v' = 1, \quad v'(dv'/dx') = E', \quad dE'/dx' = \omega^2 (p' - 1) \]  

(2.1)

with the boundary conditions

\[ p'(0) = 1, \quad v'(dv'/dx') = 1, \quad \int_0^1 E'(x')dx' = -\bar{\varphi}_t, \]  

(2.2)

where \( \bar{\varphi}_t \) is the (transit) potential drop across the system. To solve these equations, we define \( t' \) (orbit variable /3/) such that

\[
\begin{align*}
    t' &= \int_0^{x'} \frac{dx'}{\sqrt{\frac{v'(x')}{v'(x')}}} = \int_0^1 \frac{dv'}{E'(v')} = \int_0^{E'(x')} \frac{dE'}{\omega^2 (1-v')} ,
\end{align*}
\]

(2.3)

This time represents the time it took a fluid element (or particle) to arrive at position \( x' \) from its time of injection at emitter, that is, \( t' \) is a local transit time. It is not an independent variable in the usual sense, but rather a parameter of which \( x', v', E' \) and \( \bar{\varphi} \) depend on. From the equations (2.1)-(2.3) we find

\[
\begin{align*}
    x &= \tau + (1 - \cos \tau), \quad (2.4) \\
    v' &= 1 + \epsilon \sin \tau \equiv V, \quad (2.5) \\
    \bar{\varphi} &= \epsilon \cos \tau, \quad (2.6) \\
    \bar{\varphi}(\tau) &= \epsilon \sin \tau (1 + 0.5 \epsilon \sin \tau), \quad (2.7) \\
    r &= (dE'/dx')/(\omega^2) \equiv -1 + (1/v') \equiv -1 + (1 + \epsilon \sin \tau)^{-1}, \quad (2.8)
\end{align*}
\]

where \( \epsilon \) is the "extended Pierce parameter", \( \tau = \omega t' \), \( r \) is the charge density, and

\[
\bar{\varphi}(\tau) = \int_0^\tau \frac{E'dx'}{d\tau} \, d\tau \quad (2.9)
\]

is the local(equilibrium) potential drop. It is interesting to note that the equations (2.5)-(2.7) describe a curve in three-dimensional phase space \((V, \bar{\varphi}, \bar{\varphi})\), and \( \tau \) is some parameter along the curve. A set of neighboring curves, forming a congruence, may be obtained by the variation of the parameter \( \epsilon \). For example, in the 2D phase space \((\bar{\varphi}, \bar{\varphi})\) we have the diagrams indicated in Fig.2. These phase space plots can be related to the presence of DLs, but we shall not be concerned with this method here (see /4/).
3. Double Layers

The equilibrium of classical and extended Pierce diodes is described by the local behavior of functions $V, \xi, \xi$, and $r$ given by the equations (2.5)-(2.8). These quantities, as functions of $x$, are diagrammed in Fig. 3 for $\epsilon = 0.5$. The curves enable us to conclude the following:

(a) The appearance in a collisionless cold electron beam of a Pierce diode of potential drops corresponding to those of DLs can be correlated with the energy conservation law. An electron of the beam, coupled to the electric field of the plasma, has an energy $(mv^2/2) + q\phi = \text{constant}$, and forms an electrostatic structure which we call "spatial plasmon" or "spatial quantum". The electrons lose and gain their kinetic energy in interaction with the essentially changing electric field, but the energy of spatial plasmon remains constant.

(b) As in the case of a collisional plasma, the generation of DLs can be correlated with certain quantum processes (spatial plasmon $\leftrightarrow$ electron + photon). Although the DLs discussed in this work refer to a state of equilibrium, these remain essentially dynamical in nature. The electrons in DLs are continuously changed. The electrons passing the potential minimum form a beam accelerated by the potential difference $\phi_{DL}$ on the high potential side. Hence we have the characteristics of "strong" DLs.

4. An Example: New Methods of UHF Generation

For the case of $v_o \approx 10^{10}$ cm/sec, $\Delta \approx 0.1$ cm, $\omega_t$ corresponds to cm-range which may be of interest in developing of new methods of UHF generation. If the diode works as a classical one the electrons will hit the collector with $V=1$, that is the device is a passive element of circuit, and $X=\omega t = 2\pi$, so that $\phi_{collect} = 0$. If the diode works as an extended Pierce diode with positive (or negative) resistance, the electrons will hit the collector with $V < 1$ (or $V > 1$).
Fig. 3. $V, S, \delta, r$, and $\tau$ as functions of $X$ for $\epsilon = 0.5$.

An interesting, and to my knowledge unreported, result is noticeable: A Pierce diode can work as a classical (or an extended) one even if there is (or there is not) an external circuit; the condition is to realize $\phi = 0$ (or $\phi \neq 0$) at $X = \omega$.

Acknowledgements. I gratefully acknowledge the advice, assistance and support of Professors A.M. Pointu, M. Fitaire, M. Sanduloviciu, and G. Popa.

References
NEW RESULTS ON WEAKLY NONLINEAR DRIVEN OSCILLATIONS IN THE PIERCE DIODE

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Introduction and Model. The simplest and most fundamental model of a bounded plasma system is the "classical" Pierce diode,\(^1\sim^5\) in which a collisionless cold electron beam is injected into the plasma region with constant density \(n_0\) and velocity \(v_0\) at the emitter surface \((x = 0)\) and the electrons are absorbed on impact on either the emitter or the collector (located at \(x = d\)), the two electrodes being externally shorted. The ions form an immobile neutralizing background of uniform density \(n_0\).

In the present paper, however, we consider an "extended" Pierce–diode model including nontrivial (RLC) external–circuit elements and an ac source in the external circuit (Fig. 1). In particular, we study small–amplitude, steady–state, driven oscillations about the uniform dc ("equilibrium") state (in which the electrons have constant density \(n_0\) and velocity \(v_0\), and the electric field vanishes everywhere inside the diode gap). The linear stability behavior of this system, including the influence of external–circuit elements, was discussed in Refs. 2 and 3, and weakly nonlinear steady–state oscillations of the undriven system were studied in Refs. 4 and 5.

An analytical theory of weakly nonlinear, steady–state driven oscillations was first presented by us in Ref. 6, where the basic principles and assumptions were outlined and first results were presented. The driving ac generator produces a small–amplitude periodic signal with fundamental frequency \(\omega\). This theory is applicable to situations where (i) linear theory yields no unstable eigenmodes and (ii) we are far away from "resonance", e.g. we must avoid the case \(\omega \approx \text{Re} \omega_{\text{lin}}\), where \(\omega_{\text{lin}}\) is the dominant linear eigenfrequency. As in the undriven case\(^5\), all perturbation quantities are expanded in time Fourier series with fundamental frequency \(\omega\); the \(n\)–th harmonic is assumed to be small of \(n\)–th order, and the time–averaged perturbation is assumed to be of second order. All quantities of higher than third order are neglected.

In Ref. 6, some dependencies of the nonlinear current amplitudes on the driving frequency in the short–circuit case were given. In this paper, we present new results for a different short–circuit case and for a diode with a purely inductive external circuit. These analytic results are compared with particle simulations performed with the code PDPI\(^7\).

Basic Equations. The basic equations governing the extended Pierce–diode model of
with \( v(x,t) \) the electron velocity, \( E(x,t) \) the electrostatic field, \( n(x,t) \) the electron number density, \( \tilde{U}_g(t) \) the periodic, small-amplitude driving ac voltage, \( j_p(x,t) = -en_0 v \) the plasma current density, and \( J(t) = I(t)/A \) the total, or external—circuit, current density. \( I(t) \) is the total external—circuit current and \( A \) the electrode area; \( R, L \) and \( C \) are the external—circuit resistance, inductance and capacitance, respectively, and \( Q(t) \) is the electric charge on the right—hand plate of the capacitor \( C \). The latter is short—circuited dc—wise by the (large) inductance \( L_0 \). The external dc battery has to compensate the dc—potential drop across the resistance \( R \), so that \( U_0 = -IR \), with \( I \) the dc current. In addition, the following particle boundary conditions must be satisfied at the emitter surface:

\[
\begin{align*}
\hat{n}(0,t) &= n_0 \quad & (6) \\
\hat{v}(0,t) &= v_0. \quad & (7)
\end{align*}
\]

Nonlinear Perturbation Theory and Fourier Analysis. We now write any physical quantity involved in the form

\[
u(x,t) = \bar{\nu} + \tilde{\nu}(x,t)
\]

with \( \bar{\nu} \) the constant equilibrium value \( \bar{n} = n_0, \bar{v} = v_0, \bar{E} = 0, \bar{J} = -en_0v_0, Q = 0 \) and \( \tilde{\nu} \) the (small) perturbation. Inserting (8) into the basic equations (1)—(7) and expressing \( \bar{E} \) and \( \bar{n} \) in terms of \( \bar{\nu} \) results in the integro—differential system

\[
\begin{align*}
\frac{\partial \tilde{\nu}}{\partial t} + \frac{\partial}{\partial x} (\tilde{\nu} v) &= 0 \quad & (1) \\
\frac{\partial E}{\partial x} &= \frac{e}{\varepsilon_0} (n_0 - n) \quad & (3) \\
V(l,t) - V(0,t) &= -\int_0^l dx E(x,t) = \frac{dI}{dt} + R I + \frac{\tilde{Q}}{C} + U_B + \tilde{U}_g(t), \quad & (5)
\end{align*}
\]

with \( \tilde{U}_g(t) \) the periodic, small—amplitude driving ac voltage, \( j_p(x,t) = -en_0 v \) the plasma current density, and \( J(t) = I(t)/A \) the total, or external—circuit, current density. \( I(t) \) is the total external—circuit current and \( A \) the electrode area; \( R, L \) and \( C \) are the external—circuit resistance, inductance and capacitance, respectively, and \( Q(t) \) is the electric charge on the right—hand plate of the capacitor \( C \). The latter is short—circuited dc—wise by the (large) inductance \( L_0 \). The external dc battery has to compensate the dc—potential drop across the resistance \( R \), so that \( U_0 = -IR \), with \( I \) the dc current. In addition, the following particle boundary conditions must be satisfied at the emitter surface:

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with \( \bar{\nu} \) the constant equilibrium value \( \bar{n} = n_0, \bar{v} = v_0, \bar{E} = 0, \bar{J} = -en_0v_0, Q = 0 \) and \( \tilde{\nu} \) the (small) perturbation. Inserting (8) into the basic equations (1)—(7) and expressing \( \bar{E} \) and \( \bar{n} \) in terms of \( \bar{\nu} \) results in the integro—differential system

\[
\begin{align*}
\frac{\partial \tilde{\nu}}{\partial t} + \frac{\partial}{\partial x} (\tilde{\nu} v) &= 0 \quad & (1) \\
\frac{\partial E}{\partial x} &= \frac{e}{\varepsilon_0} (n_0 - n) \quad & (3) \\
V(l,t) - V(0,t) &= -\int_0^l dx E(x,t) = \frac{dI}{dt} + R I + \frac{\tilde{Q}}{C} + U_B + \tilde{U}_g(t), \quad & (5)
\end{align*}
\]

with the boundary conditions for \( v \) and \( n \) now expressed in the form

\[
\tilde{\nu}(0,t) = 0 \quad & (11) \\
\tilde{n}(0,t) = \frac{m e^2}{\varepsilon_0} \left[ \frac{\partial^2 \tilde{\nu}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial x^2} + \left( \frac{\partial v}{\partial x} \right)^2 \right] \bigg|_{x=0} = 0 \quad & (12)
\]

Since we are only interested in periodic steady—state oscillations, we expand all per—
turbation quantities in Fourier series of the form
\[ \ddot{u}(x,t) = \sum_{n=-\infty}^{\infty} \ddot{u}_n(x)e^{-in\omega t}. \] (13)

Since all perturbations and the driving frequency \( \omega \) are assumed to be real, each spatial profile function must satisfy the relation \( \ddot{u}_n(x) = [\ddot{u}_n(x)]^* \). Inserting (13) into (9)–(12) and collecting terms belonging to the same harmonic orders, we obtain relations between the velocity profile functions \( \ddot{u}_n(x) \), external-circuit current amplitudes \( j_n \), and driving-voltage amplitudes \( \hat{U}_{gn} \). These relations are given in Eqs. (35)–(38) of Ref. 5. As already mentioned above, all quantities with \( |n| > 3 \) are neglected here ("3-harmonic approximation").

**Numerical Results.** (a) Short-circuit case: In this section we present results for the special short-circuit case characterized by the normalized length \( \alpha = 2.95\pi \) (where \( \alpha = \omega_p d/v_0 \), with \( \omega_p \) the electron plasma frequency). The current history for a sinusoidal driving voltage with amplitudes \( \hat{U}_{g11} = 0.01 v_o^m e/c \) and \( \hat{U}_{g12} = \hat{U}_{g13} = 0 \) and a driving frequency \( \omega = 2.5 v_o/d \) is shown in Figs. 2a (theory) and 2b (simulation). From the Table, where the theoretical current amplitudes are quantitatively compared with those obtained from the simulations, we see that the agreement is very good. This becomes plausible because the dominant linear eigenfrequency is \( \omega_{lin}/v_0 = 1.95082 - 0.23902i \), which means that we are quite far away from resonance and are dealing with a strongly damped eigenmode, as required by the theory.

(b) Inductive case: Figures 3a (theory) and 3b (simulation) show the current history for a Pierce diode having normalized length \( \alpha/\pi = 0.55 \) and a purely inductive external circuit with normalized inductance \( L_{nd} = \omega_0^2 (\epsilon_0 A/d) L = 0.1/\pi \), for which linear theory yields a damped dominant eigenmode with \( \omega_{lin}/v_0 = 0.74478 - 0.058958i \). The driving voltage amplitudes are \( \hat{U}_{g11} = 0.1 v_o^m e/c \) and \( \hat{U}_{g12} = \hat{U}_{g13} = 0 \), and the driving frequency is chosen to be \( \omega = 5 v_o/d \), i.e., far away from resonance. According to the Table, the agreement between theory and simulation is good again.

In the cases shown here the resulting oscillations are clearly regular, but simulations show that for larger amplitudes and/or weaker damping the system seems to undergo subharmonic bifurcations and finally becomes chaotic. It would thus be desirable to have a predominantly analytic theory of such dynamic phenomena as well, and pertinent efforts are under way.

**Acknowledgments.** This work was supported by the Austrian Research Association (Project 09/0018) and the Austrian Science Foundation (Project P8405–PHY).
| Figure | $a/\pi$ | $I_{nd}$ | $R_{nd}$ | $G_{nd}$ | $J_{-1/2}^{i=\pm}$ | $J_{-3/2}^{h=\pm}$ | $|J_{-3/2}^{\pm}| - |J_{-1/2}^{\pm}|$ |
|--------|--------|--------|--------|--------|----------------|----------------|------------------|
| 2a, 2b | 2.95   | 0      | 0      | 0      | ±0.103         | ±0.105         | 2%               |
| 3a, 3b | 0.5    | 0.1/\pi| 0      | 0      | ±0.38          | -0.391         | 4%               |

I Introduction

Recently in astrophysics and geophysics one can see a great interest in plasma collisionless effects in the interaction processes between expanded plasma clouds and magnetized background. Possible processes [1,2] deceleration of such clouds by the background plasma (with concentration $n_M$) or by the magnetic field $B_0$ are being under consideration as well as the generation [3] of shock waves at $M\approx 1$ and last global effects similar to the influence [4] of SN explosions with high energy $E_0$ and mass $M$ upon the structure and evolution of diffuse galactic medium.

The same processes must play the determinable role in the dynamics of some geoeffective phenomena connected to the injection of space plasma clouds of the different scale. Such clouds expanding with large velocity $U\sim 10 - 1000$ m/s in rarefied interplanetary (coronal mass ejections) or near-Earth plasma (active experiments of the AMPTE, CRPES or "Starfish" type) may not experience pronounced collisions on scale $D$ of their deceleration by the background ($D\sim R_0\sim M/\sqrt{\rho n}$) or field ($D\sim R_0\sim \sqrt{3E_0/B^2}$). Under such conditions (non-adequately described by the known MHD-models) with allowance for the problems of natural measurements of such seldom-repeated explosion phenomena (as a rule, having the global and 3 D-character) the application of laboratory simulation methods allows us to obtain detailed and even unique data about the character of early-not investigated processes of the cloud-background interaction.

II "KI-1" Simulation Facility

In the table one can see the basic parameters of the "KI-1" facility (in the Institute of Laser Physics), created at the end of the 70-s and designed for the simulation of a wide range of non-stationary processes of the explosion character in space plasma [5]. The experimental opportunities of the facility supply the generation with the controlled delay of several clouds of spherical laser plasma with different energy ($E_0\sim 0.1 - 300$ J) by the two-side radiation method [6] of the spherical carbon-hydrogen targets. This allows us to investigate the processes of the interaction:
- between the clouds;
- between the clouds and neutral gas in the magnetic field;
- between the clouds and surrounding magnetized background.
(including inhomogeneous in density: \( \nabla n_1 \mid \vec{E}_o \) or \( \nabla n_2 \mid \vec{E}_o \));

- between the clouds and magnetospherical plasma configuration, created by the background flow in a nonuniform field \( \vec{B}_o \) of the magnetic dipole.

<table>
<thead>
<tr>
<th>&quot;KI-1&quot; chamber scales</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>background pressure</td>
<td>( P )</td>
</tr>
<tr>
<td>axial magnetic field</td>
<td>( B_o )</td>
</tr>
<tr>
<td>magnetic moment of the dipole (( \Theta \ 20 ) cm)</td>
<td>( M )</td>
</tr>
<tr>
<td>background plasma density (( H_1 ).......)</td>
<td>( n_1 )</td>
</tr>
<tr>
<td>output energy of the pulsed CO(_2)-lasers (( 0.1-3 ) mcs duration)</td>
<td>( Q_r )</td>
</tr>
<tr>
<td>laser-target diameter</td>
<td>( \Theta_t )</td>
</tr>
<tr>
<td>expansion velocity of the laser-produced plasma cloud (C(^+), H(^+))</td>
<td>( V_o )</td>
</tr>
</tbody>
</table>

The facility is supplied with different systems of plasma diagnostics to measure space distributions of \( n, T, \vec{B}, \ldots \) with high time-resolution (\(< 50 \) ns), namely: double electric and 3D-magnetic probes, calorimeter and ion collectors, mass analysers, interferometers in a visible, infrared and microwave ranges as well as frame/streak camera and various spectral methods.

### III Collisionless interaction processes.

The main advantage of the simulation experiments carried out at the "KI-1" facility consists in the fact that at different regimes of the cloud-background interaction we have managed for the first time to realize the laboratory conditions sufficiently close to space ones in such important similarity criterion as Larmor parameter \( \ell \). It describes the degree of ion magnetization \([5]\) and defines as \( \ell = R_\ell / R \), where \( R_\ell = v_0 m c / e B \) is a directed ion Larmor radius in cloud.

a) In particular, in regime of super-Alfvenic cloud expansion (\( M_v < 3 \cdot 69 \)) it has been established\([7]\) that its deceleration by the background at \( D-R_\ell \) may be effectively fulfilled owing to the Magnetic Laminar Mechanism (MLM) connected to the action on ions of the transverse electric field \( E_\perp \mid (B_0 \times \vec{v}) \), caused by the effect of magnetic field exclusion from the high-\( \beta \) cloud interior. The necessary condition for collisionless MLM-interaction is namely \( \ell < \ell_i \).

b) In the opposite, "vacuum" case (\( M_v \ll 1 \)) we have proved\([3]\) the important role of anomalous magnetic field diffusion (with \( \ell < \ell_i \)) in the processes of cloud-field interaction and established the similarity criterion \( \ell = R_\ell / R_\ell \), determining them. Only at \( \ell \ll 1 \) the cloud may be effectively decelerated by field up to \( \Delta \Theta_0 / 2 \) at radius \( R \sim R_\ell \), but it does not stop there and
propagates further in the form of separate jets (flutes),
being curled in the direction of ion Larmor rotation [91].

In the transition regime \( \left( M_p \approx \sqrt{m_e/m_i} \right) \) the effect of the
superseding influence of the background plasma upon the develop-
ment of the flute and lower-hybrid instabilities of the
cloud boundary [10] has been revealed for the first time.

(c) With the interaction of the clouds of sufficiently large
energy \( E_0 \) with real inhomogeneous space medium the so-called
global effects may play the crucial role, when the scales
of the cloud deceleration region reaches the scales of medium
inhomogeneity: \( L_\kappa \sim |V_n/N_x| \) or \( L_B \sim \|B/\|B\| \).
The preliminary data of experiments [10] in the uniform field
have shown, that in this case also (at the initial
stage of the asymmetrical cloud deceleration, \( t \leq D_0/c \)) one can
use in the "sectorial" approximation the described above con-
formities, obtained under homogeneous conditions.

IV Simulation results for some cosmophysical phenomena.

To analyze the statement and results of the "KI–1" simu-
lation experiments we apply [5, 11] the similarity criteria method,
based upon the theorem of the dimension analysis. Besides given
above determinable criteria of processes ( \( M_p \) and \( E_0 \)), we are
carrying out the comparison with the cosmophysical conditions
upon the set of such criteria as: \( Re, \ Kn = \omega_{ce}/D, m/m_*, \ z/z_*, \)
\( z_0/m_*, V/c_*, \ldots \) and at last, in the simplest inhomogeneous
cases \( z_0/D \sim D/L \). The relations between laboratory and space tem-
poral and spatial scales one can establish according the condi-
tion of the maintenance the next dimensionless variables:
\( \zeta = V/D \) and \( \sigma = R/D \).

a) Collisionless deceleration of Supernova Remnants (SNR)
was simulated for the first time [9] at the "KI–1" facility namely
owing to realization the necessary \( E_0 \leq 1 \) condition for MLM inter-
action, in contrast to experiments [11] at "HELIOS" and "THAROS"
facilities with negative results. The comparison of our simu-
lation deceleration low of cloud front with corresponding data
of the historical SNR allow us to speculate about initial, non-
"Sedov" phase of their deceleration, characterized by \( M_p \) value.

b) Evolution of the Ba-clouds in magnetosphere at \( M_p \leq 1 \)
condition (typical for modern active experiments) was simulated in
our experiments without background plasma [9, 12], where it was
predicted for the AMPTE or CRRES class releases (with \( E_0 \leq 1 \)) the
development of flute-type instability with mode-number \( K = 8\pi/\kappa \).
This dependence was confirmed later on in computer [13] and re-
cent laboratory [14] experiments with symmetrical plasma clouds.

c) The development of global perturbations of the magnetos-
phere it is very possible while realizing the some projects
of the Earth's protection against collisions with asteroids by
super-high energy explosions of the "Starfish" class. Indeed
the modern level of energy \( E_0 \leq 10^{17} \) in such kind of phenomena in
goplasm are comparable with the proper energy of geomagnetic
field \( E_m = m_e/3R^3 \). To investigate the plasma processes connec-
ted with this perturbations we begun the new class of simulati-
on experiments [15] devoted the research of global dynamics of
plasma clouds, injected at the distance \( R \) from the center of
magnetic dipole, imbedded in vacuum or background plasma flow.
This experiment is carried out for the first time under conditions rather strong magnetization of cloud ions ($E_B \ll d$) and at the different values of the similarity criterion of the problem $\mathcal{E} = \gamma B \cdot S / E_m \sim 0.01 + 10$. For example on Fig. it is shown the frame foto result of "vacuum" dipole experiment with $E_B = 10^7$ at $r = 22$ cm and moment $t = 2,5 \mu s$ after laser-produced-plasma ejections in the equatorial plane (at point denote by asterisk). One can see the filling of the "first" and "second" artificial Van Allen belts by the cloud ions, with their precipitation on both poles (top and bottom on Fig.).

TOTAL ENERGY CONSERVATION AND MANLEY-ROWE RELATIONS FOR PLASMA-MASER INTERACTION

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ABSTRACT

The total energy conservation relation between particles kinetic energy and wave energy is satisfied for the plasma-maser process. The Manley-Rowe relation among plasma waves is violated and as a result an efficient energy up-conversion from the low-frequency mode to the high-frequency mode is possible even for a normal unreversed electrons population in plasma turbulence.

Since the prediction of the plasma-maser[1,2] which is an interesting mode-coupling process in plasma turbulence[3], it has been subjected to careful scrutiny regarding its validity and probable limitations as an energy up-conversion process in plasma turbulence. Let us first define the plasma-maser interaction. We consider the nonlinear mode-mode coupling between two kinds of plasma waves. The first type is the low-frequency waves with the frequency \( \omega \) and the wavevector \( k \). These waves satisfy the Landau resonance condition \( [\omega - kv = 0] \) with the resonant electrons. The second type of the plasma wave is the nonresonant mode with the frequency \( \Omega \) and the wavevector \( K \) which do not obey the Landau resonance \( [\Omega - Kv \neq 0] \) and nor the nonlinear Landau resonance \( [\Omega - \omega - (K - k)v \neq 0] \). This paper reports on the total energy conservation relation and the violation of the Manley-Rowe relation for the plasma-maser process.

According to the standard results[1-3], the nonlinear dielectric constant of the Langmuir wave in the presence of the ion-sound turbulence which causes plasma-maser is

\[
e_h(K,\Omega) = \left( \frac{e_m^2}{\omega^2} \right) \sum_{k,\omega} \left| E_2(k,\omega) \right|^2 \left[ \frac{1}{\Omega - kv} \right] \frac{\partial}{\partial v} \omega - kx \frac{\partial}{\partial v} f_0 e_0 dv,
\]

where \( \Omega, K \) and \( \omega, k \) are frequency, wavenumber for the Langmuir and ion sound wave, respectively. \( E_2(k,\omega), f_0 \) are the electric field of the ion sound wave, the electron distribution function, respectively, \(+i0\) shows small imaginary part, \(-e\) and \( m \) are charge and mass for electrons. The plasma-maser contribution comes from the condition \( \omega = kv \) which produces high frequency nonlinear forces for electrons [4]. It should be stressed that the plasma-maser contribution is always accompanied by the reverse absorption process due to the slow time change of medium due to the quasilinear interaction between ion-sound wave and resonant electrons.
For closed system, the both processes cancell out each other exactly and the up-conversion does not occur. Here, we consider the plasma-maser process for open system where the electron distribution function is fixed by external agent.

The growth rate of the Langmuir wave \([\gamma_h(K,\Omega)]\) due to the plasma-maser for open system is

\[
\gamma_h(K,\Omega) = -\frac{I_m e_h(K,\Omega)}{\frac{\partial \varepsilon_0(K,\Omega)}{\partial \Omega}} \tag{2}
\]

where \(\varepsilon_0(K,\Omega)\) is the linear dielectric constant of the Langmuir wave, thus we get \(\frac{\partial \varepsilon_0(K,\Omega)}{\partial \Omega} \approx 2/\Omega\), and \(I_m\) shows the imaginary part. Inserting Eq.(1) into Eq.(2), we obtain the total energy change of the Langmuir wave energy density \([W_h(K,\Omega) = |E_h(K,\Omega)|^2/4\pi]\) as

\[
\frac{\partial}{\partial t} \sum_K W_h = 2 \sum_K \gamma_h(K,\Omega) W_h = -\pi \frac{|\alpha|^4}{m^3} \sum_{K,k} \frac{\Omega}{K} |E_q(k,\omega)|^2 |E_h(K,\Omega)|^2 \times \left(1 + \frac{1}{\Omega - kv} \frac{\partial}{\partial \Omega} \frac{1}{[\Omega - \omega - (K-k)v]} \frac{\partial}{\partial \Omega} \delta(\omega - kv) \right) f_{oe} dv. \tag{3}
\]

In deriving Eq.(3), we put \(\mu[(\omega - kv - 10)]^{-1} = \pi \delta(\omega - kv)\).

According to the linear response theory of a turbulent plasma, the inverse of the plasma-maser effect, i.e., the nonlinear effect of resonant and nonresonant waves simultaneously on the electron distribution reduces to

\[
\frac{\partial f_{oe}}{\partial t} = i \frac{|\alpha|^4}{m^3} \sum_{K,k} |E_q(k,\omega)|^2 |E_h(K,\Omega)|^2 \times \left(1 + \frac{1}{\Omega - kv} \frac{\partial}{\partial \Omega} \frac{1}{[\Omega - \omega - (K-k)v]} \frac{\partial}{\partial \Omega} \delta(\omega - kv - 10) \right) f_{oe}. \tag{4}
\]

From Eq.(4), the rate of change of total electron kinetic energy density \(\partial/\partial t E\) may be expressed as

\[
\frac{\partial}{\partial t} E = \frac{\alpha}{2} \frac{mv^2}{\partial t} dv. \tag{5}
\]

Inserting Eq.(4) into Eq.(5), we get

\[
\frac{\partial}{\partial t} E = \pi \frac{|\alpha|^4}{m^3} \sum_{K,k} \frac{\Omega}{K} |E_q(k,\omega)|^2 |E_h(K,\Omega)|^2 \times \left(1 + \frac{1}{\Omega - kv} \frac{\partial}{\partial \Omega} \frac{1}{[\Omega - \omega - (K-k)v]} \frac{\partial}{\partial \Omega} \delta(\omega - kv) \right) f_{oe} dv.
\]
which simply states that the total energy density is conserved by the plasma-maser interaction between Langmuir waves and electrons.

Next, we consider the rate of decrease of ion-sound wave energy corresponding to the plasma-maser emission of Langmuir wave. The most dominant imaginary part contribution to the ion-sound waves comes from the polarization term as

\[ \Re \varepsilon_2(k, \omega) = - \left( \frac{e}{m} \right)^2 \omega^4 \sum_K \frac{|E_h(K, \Omega)|^2}{\varepsilon_0(k-K, \omega-\Omega)} \times \Re \varepsilon_{m}[A^2], \]

where

\[ \Lambda = \int \frac{1}{(\omega - kv + 10)(\Omega - Ky)[\omega - \Omega - (k-K)v]} d\omega \]

\[ \Omega^2 \int \frac{1}{\omega - kv + 10} \frac{\partial}{\partial \omega} \frac{f_{oe}}{d\omega} \]

The plasma-maser interaction is effective even without the electron population inversion. Here, we assume the Maxwell distribution function for \( f_{oe} \) and obtain

\[ \Re \varepsilon_2(k, \omega) = - \left( \frac{e}{m} \right)^2 \left( \frac{\omega}{\Omega} \right)^4 \frac{2m^2 \pi c_s}{k|k|T^2} \left( \frac{m}{2\pi T} \right)^{1/2} \sum_K \frac{|E_h(K, \Omega)|^2}{\varepsilon_0(k-K, \omega-\Omega)}, \]

where \( c_s = (2T/M)^{1/2} \) is the phase velocity for ion-sound wave, \( T \) and \( \omega_0 \) is the temperature and plasma frequency for electrons, \( M \) is the mass for the ion. Accordingly, the damping rate of the ion-sound wave due to the plasma-maser is

\[ \gamma_2(k, \omega) = - \pi \omega k \left( \frac{k_e}{k} \right)^2 \left( \frac{m}{M} \right)^{1/2} \sum_K \frac{|E_h(K, \Omega)|^2}{4\pi N T}, \]

here \( k_e \) and \( N \) are electron Debye wavenumber and electron number density, respectively. In deriving Eq.(10), we put \( \varepsilon_0(k-K, \omega-\Omega) \propto -(k/k_e)^2 \). Accordingly, the rate of the change of the ion sound wave energy density \( \Re \varepsilon_2(k, \omega) \) reduces to

\[ \frac{\partial}{\partial t} \sum_k \Re \varepsilon_2 = - \frac{2\pi}{NT} \frac{m^{1/2}}{M} \sum_{k,K} \left( \frac{k_e}{k} \right)^2 \omega k |E_h(k, \omega)| W_2 W_h. \]
plasma-maser process. However, as is shown below, it is violated and as a result an efficient energy up-conversion from the ion-sound to Langmuir wave is possible. The plasmon number for ion-sound \( N_\varphi (k, \omega) \) and for Langmuir wave \( N_h (K, \Omega) \) is defined as

\[
N_\varphi (k, \omega) = \frac{W_\varphi (k, \omega)}{\omega}
\]

\[
N_h (K, \Omega) = \frac{W_h (K, \Omega)}{\Omega}
\]  \( (12) \)

From Eq. (3), for the Maxwell electron distribution function, the rate of increase of the Langmuir wave density for mode \((K, \Omega)\) reduces to

\[
\frac{\partial}{\partial t} W_h (K, \Omega) = \frac{3\pi}{NT} (\frac{m}{M})^{1/2} K \Omega W_h (K, \Omega) \sum_k \frac{W_\varphi (k, \omega)}{|k|} (\frac{k}{k_e})^2 . \]  \( (13) \)

Inserting Eqs. (11) and (13) into Eq. (12), we obtain the rate of change of the plasmon number for ion-sound wave \((k_0, \omega_0)\) and Langmuir wave \((K_0, \Omega_0)\)

\[
\frac{\partial}{\partial t} N_\varphi (k_0, \omega_0) = - \frac{2\pi}{NT} (\frac{m}{M})^{1/2} \frac{k_e}{k_0^2} W_\varphi (k_0, \omega_0) \frac{k_0}{|k_0|} \sum_k W_h (K, \Omega) ,
\]

\[
\frac{\partial}{\partial t} N_h (K_0, \Omega_0) = \frac{3\pi}{NT} (\frac{m}{M})^{1/2} K_0 W_h (K_0, \Omega_0) \sum_k \frac{W_\varphi (k, \omega)}{|k|} (\frac{k}{k_e})^2 . \]  \( (14) \)

If we put \( W_\varphi (K, \Omega) = W_\varphi (K, \Omega) \delta (K-K_0) \delta (\Omega-\Omega_0) \), \( W_h (k, \omega) = W_h (k, \omega) \delta (k-k_0) \delta (\omega-\omega_0) \) in Eq. (14), then we find for \( k_0 K_0 \ll k_e \)

\[
\frac{\partial}{\partial t} N_\varphi \approx \frac{k_e^4}{k_0^3 K_0} >> 1 .
\]  \( (15) \)

Accordingly, we may conclude that the Manley-Rowe relation is violated for the plasma-maser interaction and as a result an efficient energy up-conversion from the low-frequency mode to the high-frequency mode is possible even for a normal unreversed electrons population. This is markedly different from the standard nonlinear Landau resonance where the Manley-Rowe relation is satisfied and the energy up-conversion is possible only for a reversed electrons population.

REFERENCES

PHASE-SPACE TOPOLOGY IN THE DYNAMICS OF CYCLOTRON-MASERS

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With the advent of radiation generating systems like free-electron lasers, cyclotron autoresonance masers, gyrotrons, ion-channel lasers and others, a good deal of effort has been directed toward the analysis of the nonlinear interaction of relativistic particles and large amplitude electromagnetic waves. In most of the cases, quantities of interest like energy gain or energy exchange are obtained by considering the electron dynamics in given external wave fields. This kind of approximation can be fairly justified if one considers either waves with large enough amplitudes or beams with very tenuous densities, so that particles do not appreciably react back on the electromagnetic fluctuations. On the other hand, when these conditions are not satisfied, a self-consistent analysis is necessary where particles and fields are to be simultaneously treated as dynamical entities [1], [2]. The purpose of the present paper is to analyze cyclotron masers systems. To do so we consider a model in which a beam of homogeneously magnetized electrons propagating along a z axis coexists with an electromagnetic wave polarized along the x axis with the fast wave-vector lying along z.

With that model we basically intend to show that the topology of the phase space in those cases where we disregard the dynamics of the fluctuating field may be very different from the one corresponding to the self-consistent situation where fields are also treated as dynamical entities. To begin with, let us write the Hamiltonian that governs the electron dynamics. If one writes the vector potential of the electromagnetic field as

\[ e \frac{1}{mc^2} A_x = -\frac{1}{2} \sqrt{\rho} e^{i\omega t} e^{i(z-t)} + c.c. \]  

(1)

where \( \rho \) and \( \sigma \) have a slow time dependence, \( e \) is the electron charge and where time and space have been normalized to \( \omega \) and \( \omega/c \approx k \) with \( \omega \) and \( k \) respectively as the wave frequency and wave-vector, one obtains

\[ \mathcal{H} = [1 + (P_x + \sqrt{\rho} \cos(z - t + \sigma))^2 + (z + P_y)^2 + P_z^2]^{1/2}, \]

(2)

with \( \mathcal{H}/mc^2 \rightarrow \mathcal{H} \) and \( P/mc \rightarrow P \), and where we assume \( eB_{z,\text{equilibrium}}/mc \approx \omega \). The simpler form

\[ \mathcal{H} = -I + \Gamma + \frac{\sqrt{\rho} \sqrt{2I}}{2\Gamma} \cos(\phi + \sigma) \]

(3)

is obtained if now one performs a power series expansion in \( \sqrt{\rho} \) considering only the leading order contribution and introduces new canonical variables defined by the relationships

\[ P_x = \sqrt{2I} \cos\phi, \]

\[ P_z = P'_z, \]
with $I \equiv P'_x, \phi \equiv x'$. Besides, we have used the cyclotron approximation $d_t(\phi + z(t) - t) \approx 0$, $d_t \equiv d/dt$ and

$$\Gamma \equiv \sqrt{1 + 2I + (P_x + I)^2}.$$
where \( \rho \) is the "momentum" corresponding to the wave field and \( \sigma' (= -\sigma) \) is the canonically conjugated co-ordinate. If one drops the primes once again, the following canonical transformation

\[
\phi - \sigma \rightarrow \phi \\
\rho \rightarrow \rho - I,
\]

reduces the degrees of freedom (\( \rho \) is now a constant of motion) and allows to write a final effective Hamiltonian as

\[
H = -I + \Pi + \frac{\sqrt{\lambda(\rho - I)}\sqrt{2I}}{2\Gamma} \cos \phi,
\]

which is the function we shall analyze next. The key point of this paper is to compare the overall characteristics of the system with and without wave dynamics included. When such a dynamics is considered, the Hamiltonian is given by eq.(8) and when it is not, what one is really doing is to assume \( \rho >> I \), approximating therefore the factor \( \rho - I \) of \( H \) simply by \( \rho \). So, by assuming small values of \( \rho, I \), and \( \lambda \) as to represent usual experimental conditions, from now one let us proceed with the comparative study focusing our attention on the calculation of fixed points for the dynamics in the following physically relevant situations; \( P_z = 0 \) and \( P_z >> 1 \).

\( P_z = 0 \)

In that case the fixed point equations indicate that \( \sin \phi_{fp} = 0 \) and that the two values of \( I_{fp} \) (one for each value of \( \cos \phi_{fp} \)) are to be calculated approximately from the relation

\[
\rho - 2I_{fp} = 0,
\]

which yields \( I_{fp} = \rho/2 \) in both cases. One can not satisfy eq.(9) under the assumption of absence of wave dynamics (in that case one should set \( I_{fp} \rightarrow 0 \) in eq.(9)) from which it is possible to conclude that there is a dramatic change of phase space topology between the two cases. These topological changes can be better appreciated from figs.1 and 2, where we plot the phase space corresponding to the cases without and with wave dynamics respectively. From fig.(1), it is particularly seen that some particles that have small initial energies may be accelerated to large final values of this quantity, for the curves of constant \( H \) with \( \phi_{initial} = \pi/2 (mod 2\pi) \) are open upwards. This process, known as the autoresonance one, is seen to be saturated for large values of \( I \), when the dynamics of the wave field is included.

\( P_z \rightarrow \infty \)

In this case, the fixed point is approximately determined by

\[
1 + \frac{\lambda}{2P_z} \frac{(\rho - 2I_{fp})S}{\sqrt{2I_{fp} \rho - I_{fp}}} = 0,
\]

where \( S = \cos \phi_{fp} = \pm 1 \). From this equation it is possible to notice the following variety of situations: i) \( S = \pm 1 \) with wave dynamics \( I_{fp} \approx \rho \); ii) \( S = \pm 1 \) without wave dynamics \( \Rightarrow \) no possible \( I_{fp} \); iii) \( S = -1 \) with wave dynamics \( I_{fp} \approx 0 \); iv) \( S = -1 \) without wave dynamics \( \Rightarrow I_{fp} \approx 0 \). In other words, in the cases of large values of \( P_z \), there exists a root for "large" values of \( I \) which is not present when wave dynamics is not taken into account. Once again, like in the previous cases of small values of the momentum, a dramatic change in the topology of the phase space takes place. Such a change can be observed with help of figs.3 and 4, where once again we plot the phase space without and with wave dynamics respectively.
References


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**Fig. 1** - Phase-space of the effective Hamiltonian for $P_1 = 0.3$, without wave dynamics; $\lambda = 0.2$ and $\rho = 0.2$

**Fig. 2** - The same as in fig.1 with wave dynamics included.

**Fig. 3** - Phase-space without wave dynamics for large values of $P_1$; $P_1 = 30$. Other quantities as in the previous figures.

**Fig. 4** - The same as in fig.3 with wave dynamics included.
RF PRODUCTION OF LONG, DENSE PLASMA COLUMNS

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Novel accelerator concepts such as wake-field accelerators, plasma lenses, and inverse free-electron lasers, often call for uniform plasma sources of high, controllable densities of order $10^{14}$ cm$^{-3}$ or above. The beat-wave concept, which needs the order of $10^{17}$ cm$^{-3}$, falls in a different category. To produce $10^{14}$ cm$^{-3}$ plasmas, we have been studying the use of helicon waves, which themselves are accelerators, in this case bringing primary electrons to their optimum energy by Landau damping. Evidence for this absorption mechanism has been given previously. Measurements on 4- and 2-cm diameter tubes showed that a right-hand helical antenna is more efficient than others, and that densities of order $10^{14}$ cm$^{-3}$ can be attained at 60 mTorr of A and a field of 1.2 kG, using 2 kW of rf power at 27.12 MHz.

Creation of a plasma of $10^{14}$ cm$^{-3}$ density is facilitated by experimental techniques not easily foreseen from theory. Ions are confined in a radial potential well but escape freely along the axial magnetic field $B$ at the acoustic speed $c_a$. Electrons are confined by an axial potential well and are scattered out radially. To supply the ion losses, neutral gas must be injected into the discharge at a fast enough rate. We have previously found that a 4-cm diam tube housing a 2-cm diam plasma acts as a reservoir of gas which can feed radially into the plasma. The constriction of the plasma can be accomplished in three ways. An antenna wound helically in a direction to launch a right-hand polarized wave toward the midplane tends to produce a peaked density profile. By reversing the endcoils and producing a cusp field at the antenna, we can produce a magnetic aperture limiter which is very effective in narrowing the column. We can also insert a carbon limiter into the tube to constrict the plasma directly. To optimize these effects, the apparatus was configured as shown in Fig. 1.

Argon gas was fed in at four places along the tube. The endcoils near the antenna were controlled independently to shape the magnetic field there. To avoid overheating, both the field and the rf power were pulsed for 100-150 msec, with less than 5% duty factor. At these densities, the sputtering of tungsten probe tips would change the collection area during a run. This was remedied by using carbon tips (0.3 mm pencil lead), 1.5 mm long, centered in an alumina tube of 1.6 mm o.d. The probe was biased at $-130$ V with floating batteries, and it was made to follow potential fluctuations at the rf frequency by an rf choke with a resonance frequency around 30 MHz, connected within 5 cm of the probe tip. The probe was thus effectively terminated in 200
KΩ at rf frequencies, and in 50 Ω at low frequencies. Though the ion saturation current was calibrated against the density measured by microwave interferometry, the densities reported here are uncertain by ±10%, mainly due to variations in probe area.

The effect of varying the magnetic field shape is shown in Fig. 2, which gives the density on axis vs. voltage on the end coils for two antennas—a right-hand helical (R.H.) and a plane-polarized Nagoya Type III. The standard conditions were $B = 600$ G, $p = 5.5$ mTorr of argon, and $P_{\text{rf}} = 1.9$ kW at 27.12 MHz. A voltage of +40 gives a nearly uniform field; and -40, a strongly cusped field. The density from a plane-polarized antenna increases about a factor of 5 with a cusp field. Fig. 3 shows the density profiles with a uniform field, with the end coils off, and with the end coils reversed. At least part of the density increase is due to the peaking of the profile with cusped fields. The density increase is smaller with the R.H. antenna (Fig. 2), because this gives a peaked profile even in a uniform field.

![Fig. 2](image1)

![Fig. 3](image2)

We next investigated the effect of material limiters, carbon disks with holes of 1.2 and 2.0 cm. The density profiles with the 1.2-cm limiter are shown in Fig. 4 for a uniform $B$-field, and in Fig. 5 for a cusped $B$-field (end coils reversed). Other conditions were the same as above. Fig. 4 shows that the density was sensitive to the position of the limiter; a large increase in density occurred when the limiter was located just under the rear loop of the antenna, at -5 cm from the midplane of the antenna. When a cusped field was added (Fig. 5), the profile was more sharply peaked, and the density further increased, showing that a magnetic limiter is more effective. In this case, the position of the limiter was not important, except when it is located well downstream, near the probe (+22 cm). Restricting the column at that point decreased the density.

![Fig. 4](image3)

![Fig. 5](image4)
The combined effect of magnetic and material limiters is shown in Fig. 6, which shows the density on axis under standard conditions for different positions of the 1.2-cm limiter as the endcoil voltage is varied. Since the limiter is effective at the rear end of the antenna (—6 cm), there is relatively little improvement with a cusp field. When the limiter is at the front end of the antenna, however, there is a great improvement with a cusped field, because the performance in a uniform field is so poor. These observations are not yet understood.

Even a solid carbon block has an effect on the density, depending on its position. Fig. 7 shows density profiles with the block at the rear end of the antenna (—6 cm), far back near the pump (—22 cm), and in between (—12 cm). There is an increase in density at the —6 cm position. Possible explanations include image currents in the limiter, reflection of helicon waves by the limiter, and the recirculation of neutral atoms formed by recombination of ions on the surface. By comparison, Fig. 8 shows the performance of the limiter with a 2-cm diam hole. These data were for a uniform B-field.

To obtain the highest density with the available rf power, we operated at 1 kG with a cusped field, with no limiter, using a helical antenna and a probe located near it. The importance of gas feed is shown in the pressure scan of Fig. 9. The density pulse had a peak of about 5 msec, followed by a plateau for the remainder of the 100-msec pulse. The peak density did not vary with pressure above a few mTorr, but the plateau density fell off at low pressures, indicating a
deficiency in neutral gas. Apparently, unless the flow rate is very high, densities of order $10^{14}$ cm$^{-3}$ are sustained only by the gas stored inside the tube, feeding radially into the plasma. Fig. 10 shows radial profiles at the peak and the plateau for the upstream probe, as well as for the downstream probe, which does not show a density spike at the beginning of the pulse. The field was increased to 1.2 kG. There was a large axial density gradient because of the high pressure of 20-30 mTorr. At normal operating pressure, the plasma is much more uniform along the axis.

![Fig. 9](image)

![Fig. 10](image)

Since there is no need for internal electrodes in this device, it should be possible to produce arbitrarily long plasma columns of density $10^{14}$ cm$^{-3}$ by adding antennas periodically. This paper has shown the importance of magnetic field shaping near the antenna and of arranging for radial gas feed. Other investigators [6]/[7] have found that, above a power threshold of 2.3 kW, the helicon discharge can burn out all the neutral atoms near the axis and constrict itself to a narrow, fully ionized column. We hope to add enough power to see this in the near future.

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PLASMA POTENTIAL FORMATION IN THE PRESENCE OF LOCALIZED RADIO-FREQUENCY ELECTRIC FIELDS

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Researches on plasma potential formation have recently attracted considerable attention in basic plasma, space plasma and fusion plasma physics. Electrostatic potential formations as a result of interaction between radio frequency (rf) fields and plasmas are of particular interest concerning effective plasma heating and confinement. Experiments on rf sustained electrostatic potential barrier like thermal barrier /1/ and rf plugging /2/ have been performed in open ended machines, resulting in reducing end plasma losses. In this connection it is basically significant to experimentally clarify the potential formation process in the presence of spatially localized rf fields. Here we demonstrate a clear potential formation along the magnetized plasma flow when externally-excited nonlinear electron plasma waves are generated.

A collisionless plasma, 5.0 cm in diameter, is produced in a single-ended Q machine, as shown in Fig.1. The plasma of density \( n_0 = (5-50) \times 10^6 \text{ cm}^{-3} \) (electron plasma frequency \( \omega_{pe}/2\pi = 20-64 \text{ MHz} \)) and electron temperature \( T_e = 0.2 \text{ eV} \) is confined radially by a uniform axial magnetic field \( B = 2.3 \text{ kG} \). An electron plasma wave is excited by applying an external rf potential \( V_{RF} \) of a frequency \( \omega/2\pi = 10-100 \text{ MHz} \) to a 5-cm-diam ring antenna, which is axisymmetrically set near the center of the plasma column (z=0 cm). The plasma with a speed \( V_0 = 2.7 \times 10^5 \text{ cm/sec} \) flows downstream almost without being disturbed by the antenna when \( V_{RF} = 0 \text{ V} \).

The electron wave is observed to symmetrically propagate from the exciter toward the upstream (z<0) and downstream (z>0) sides along the plasma column, satisfying the linear dispersion relation for \( V_{RF} \leq 50 \text{ mV} \) (\( \ll T_e/\epsilon \)). With an increase in \( V_{RF} \) over \( T_e/\epsilon \), well-known nonlinear phenomena such as
amplitude oscillation appear, and asymmetric propagations simultaneously occur. For much larger values of $V_{RF}$, the wave is measured to damp even more strongly near the exciter and the wavelength at $z > 0$ becomes shorter than at $z < 0$, as typically shown at the top of Fig. 2. When $\omega/2\pi$ and $n_0$ are changed, this nonlinear behavior of the wave is typically observed under the condition that $P = \omega/\omega_{pe} < 1$.

Measurements of the plasma potential $\phi$ profile for the case of $P < 1$ show that $\phi$ at $z > 0$ is varied to be deeply negative and $\phi$ around $z = 0$ becomes a positive maximum in spite of almost no change of $\phi$ at $z < 0$, as given in Fig. 2. A large-scale potential drop, $\Delta \phi = \phi(z < 0) - \phi(z > 0) \approx 20 T_{eo}/e$, is formed in the result. A drastic increase of the electron temperature $T_e$ is also observed on the downstream side $[\Delta T_e = T_e(z > 0) - T_e(z < 0) - 30 T_{eo}]$. A density drop $\Delta n \approx 0.2 n_0$ arises around $z = 0$ in addition to a plasma loss over the whole column. Here, the characteristic scale length of $\Delta \phi$ observed is several ten times the Debye length, which is nearly equal to the theoretical width of classical electric double layer.

In the case of $P > 1$, no particular changes of both $\phi$ and $T_e$ are recognized even for $V_{RF} > 2 T_{eo}/e$ on the upstream and downstream sides except a region near the exciter, where $\phi$ and $T_e$ are increased to a positive and a high value, respectively, as shown in Fig. 3. Instead, $n$ at $z < 0$ becomes larger than $n_0$ and the resultant density drop is formed, indicating a clear plugging effect for the plasma flow along the $B$-field. It is to be noted that the electron wave can not propagate over the exciter because of the

Fig. 2: Spatial profiles of the electrostatic potential, electron temperature and plasma density. $V_{RF} = 8$ V, and the subscript "o" refers to a value with $V_{RF} = 0$ V. $n_0 \approx 5 \times 10^7$ cm$^{-3}$, $\omega/2\pi = 45$ MHz ($P = 0.71$).
strong Landau damping for $P>1$ [seen at the top of Fig.3].

Typical temporal evolutions of the $\phi$ profile are presented for the case of $P<1$ in Fig.4. Applying $V_{RF}$ ($t>0$), $\phi$ at both $z>0$ and $z<0$ are positively raised with the profile symmetrical up to $\approx 60$ $\mu$s (a few tens of ion plasma period), while $\phi$ near the exciter almost keeps constant. This potential increase over the exciter suggests that electrons are axially repelled by the strong damping of the nonlinear electron wave. However, $\phi$ near the exciter increases to a positive value up to $\approx 150$ $\mu$s during the whole decrease of $\phi$. At the second stage ($t=150-\omega$ $\mu$s), $\phi$ at $z>0$ becomes deeply negative and $\phi$ at $z<0$ approaches the value before applying $V_{RF}$ with $\phi$ near the exciter positively constant. Not only electrons but also ions can be reflected near the exciter under such a potential structure. For $P>1$, $\phi$ and $T_e$ near the exciter monotonously increase as a function of time. Independently of $P$, the potential profile attains to a stationary state around 700 $\mu$s, which is of the order of the plasma relaxation time (plasma-length/$V_0$). According to time-resolved measurements of electron energy distributions, the high-temperature Maxwellian distribution yielding $\Delta T_e$ and also $\Delta n$ is formed within 100–200 $\mu$s well before the $\Delta \phi$ generation.

In our experiment, the large-amplitude localized rf electric fields are present in the magnetized collisionless plasma flowing toward the endplate. Since plasma parameter variations near the exciter are almost independent of the frequency applied (or $P$), near fields of the antenna are expected to predominantly determine the phenomenon in the vicinity of the exciter. The localized rf field with the axial gradient provides a ponderomotive force repelling electrons. Thus there should appear a potential increase in this region of the localized rf field. A part of the ions are reflected by this local increase in $\phi$. As a result, $n$ decreases in the downstream

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Spatial profiles of the plasma parameters. $\omega/2\pi=95$ MHz ($P=1.49$). Other parameters are same as those of Fig.2.}
\end{figure}
region. As shown in Fig. 3, however, a large $\Delta \phi (\gg T_{e0}/e)$ is not generated by only the plugging effect due to the ponderomotive force. On the other hand ($P<1$), the enhanced wave damping gives rise to a number of detrapped electrons /3/. All electrons passed through $z=0$ are largely accelerated and heated in the region of $z>0$, while the electron reflected at $z=0$ are not heated in the hot plate side because of the high electron thermal conductivity. The electron energy increase implies a decrease of the electron density in the region of $z>0$ because the electron flux is conserved. Since the ion density must decrease at $z>0$ to satisfy the charge neutrality, the potential becomes negative at $z>0$ to accelerate ions, yielding the $\Delta \phi$. The $\Delta \phi$ generation is essentially ascribed to the nonlinear electron acceleration (or heating) by waves in the downstream region.

In summary, a potential structure with local hill and large drop has been observed across a localized rf electric field in a collisionless plasma flow. The electrostatic potential formation is considered to result from the electron acceleration due to the highly nonlinear electron wave in addition to the ponderomotive force.

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INWARD AND OUTWARD PLASMA MOTION INDUCED BY AXISYMMETRIC ELECTROMAGNETIC FIELDS AROUND THE ION CYCLOTRON FREQUENCY

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ABSTRACT: Large-scale plasma motions across a magnetic field are observed in the vicinity of an antenna which inductively generates axisymmetric radio-frequency fields. The phenomenon is due to the ion cyclotron resonance under the antenna near-zone nonuniform electromagnetic fields.

To understand mechanisms of impurity production during rf plasma heating is a topic of considerable contemporary interest. An ion-Berstein-wave heating is observed to result in a peaked density profile and plasma-confinement improvement, although the mechanism is not elucidated. It is basically important to clarify rf induced plasma-transport processes in the vicinity of powered antennas. Here we present a significant finding of inward and outward plasma motion induced by axisymmetric rf fields around the ion cyclotron frequency.

Experiments are carried out in a single-ended Q machine, where a potassium plasma of n=10^{10} cm^{-3} and T_e=0.2 eV is produced on a hot tungsten plate in a uniform magnetic field B_0=1.5-3.5 kG. The plasma (4 cm in diameter) flows toward a negatively biased endplate at a distance of 142 cm from the hot plate. An antenna consisting of a 1.2-cm-long solenoid (6.9 cm in average diameter, 4x5 turns) is set at distance 54 cm from the hot plate. Faraday shields (4.6 cm in inner-diameter) are mounted on the antenna to allow magnetic coupling and excitation of cylindrically symmetric rf mode. The frequency ratio \omega/\omega_{ci} to the ion cyclotron frequency \omega_{ci}/2\pi is varied by changing the B-field (\omega/2\pi=0.1 MHz: applied frequency). Profiles of near-zone electromagnetic fields are calculated by using the exact dyadic Green's function for the cylindrical structure in case of a typical antenna rf current.
The oscillating axial magnetic field just under the antenna (z=0) is almost constant (h=230 G) within the plasma radius (r<2 cm), and the azimuthal electric field simply increases from the center (E_θ=0 V/cm) toward the antenna edge (1 V/cm at r=1.5 cm). They rapidly damp in the axial direction, almost disappearing beyond |z|=10 cm (z>0: on the downstream side toward the endplate).

Langmuir probe measurements in the vicinity of the antenna show that small density perturbations are induced for the case of ω/ω_ci<1 or >1 even when large-amplitude rf fields are applied to the plasma. However, a large-scale density modification is caused as ω/ω_ci approaches to unity. The maximum modulation on the order of 100 % is observed for 0.95<ω/ω_ci<1.05 and I_A>60 A. Typical density profiles at z=0 are presented as a function of time in Fig.1 for the case of ω/ω_ci=1.05 and I_A=60 A. The profile is periodically modified to be largely peaked and hollow in strict correspondence with the applied rf period 2π/ω (=10 μsec). An ion temperature T_i⊥ perpendicular to the B-field at z=0 is observed to oscillate strongly with the rf period when ω/ω_ci approaches to unity, as shown in Fig.2(a) for I_A=60 A. Here T_i⊥ is estimated from e-folding slope of the ion energy distribution.

Fig.1. A temporal evolution of plasma density profile at z=2 cm. A profile without rf is also shown.
measured by a grided energy analyzer. Radial profiles of \( T_{ii} \) for typical phases are given in Fig.2(b). High-energy ions concentrate near the center of the plasma cross section at a phase, while they come up to the antenna edge after half a period. Time-averaged \( T_{ii} \) radially increases a little bit toward the antenna edge, while \( T_{ii} \) for \( T_A = 0 \) A is almost constant (≈0.3 eV). The result implies that ions strike the antenna, leading to the impurity generation.

It is observed that the electrostatic potential becomes positive in the antenna region, the difference of which (0.5–1 V) is comparable to the ion flow energy. Since the estimated ponderomotive potential at \( \omega/\omega_{ci} = 1 \) is also comparable to the flow energy, the plasma potential increase may be attributed to the ion deceleration by the axial ponderomotive force.

Ion trajectories around the cyclotron resonance are computed by numerically solving the equation of motion in order to clarify a mechanism for the phenomenon observed. Figure 3 depicts the trajectory for the typical electromagnetic field profile at \( z=0 \) (\( \vec{B}/B_0 = 0.09 \)) as described previously, where the ion drifts from an initial position toward the antenna edge with its Larmor radius increasing. More importantly, the ion at the typical initial position never overshoots the axisymmetry center. These ion behaviors are insensitive to initial phase of the ion velocity for larger rf fields. In our experiments most of the ions flowing along the B-field are considered to circulate several times between the column center and the antenna edge.
and edge due to the cyclotron resonance during passing through the antenna region (~50 μsec), where the ion-flow speed is greatly reduced by the axial ponderomotive force. Since electrons can neutralize ions by crossing the B-field at the conducting end (~4 μsec), strong radial motions arise in the plasma near the antenna, as if a collective mechanism acts.

In summary, nonuniform axisymmetric-fields around the ion cyclotron frequency induce a large-scale plasma motion across the B-field with the rf period. Our results imply an important rf-induced plasma transport, being instructive for understanding impurity generation mechanisms or peaking phenomena in ion-cyclotron heating experiments.

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The Exact and Drift Hamiltonian

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Abstract The Hamiltonian of the exact trajectory and first order corrections to the standard drift Hamiltonian are given in the Boozer coordinates for vacuum magnetic field with magnetic surfaces. The exact Hamiltonian depends on both the field strength and the shape of the magnetic surfaces while the standard drift Hamiltonian depends only on the field strength. The first order correction, in gyroradius to system size, to the standard drift Hamiltonian depends in a generic way on the metric tensor of the magnetic coordinates, as does the exact Hamiltonian.

The speed and efficiency with which the drift equations can be integrated has mandated their use for studying the trajectories of charged particles in complicated magnetic fields. An extreme example of such a study is the effect of toroidal asymmetries on the confinement of fusion α's. Fusion α's have a slowing down time that is of order $(\epsilon^3 \Omega)^{-1}$ with $\Omega$ the gyrofrequency and $\epsilon$ the ratio of their gyroradius to the system size, $\epsilon \sim \rho/a$. Unfortunately, the standard derivation of the drift equations, which is due to Alfvén, is only guaranteed to be valid for a time $(\epsilon \Omega)^{-1}$. Despite the obvious importance, there has been little study of the accuracy with which the drift trajectories approximate the exact particle trajectories, which is the subject of this paper.

We call the Hamiltonian that most accurately describes the guiding center motion using only four canonical coordinates the guiding center Hamiltonian, $H_g$. The exact Hamiltonian $H_e$ for the particle trajectories, which has six canonical coordinates, can be written as the sum of the guiding center Hamiltonian $H_g$ and a perturbation $H'_e$, which is chosen to be as small as possible,

$$H_e(\mu, \theta, P, Q) = H_g(\mu, P, Q) + H'_e(\mu, \theta, P, Q).$$

The four canonical coordinates of the guiding center Hamiltonian are $(P, Q)$, the magnetic moment is $\mu = mv^2/2B$, and the canonically conjugate gyrophase is $\theta_g$. As a field becomes
spatially uniform $\varepsilon \to 0$, the evolution of the magnetic moment goes to zero faster than $\varepsilon$ to any power,\(^2,3\) which implies, using $d\mu/dt = -\partial H_e'/\partial \theta_g$, that the perturbation $H_e'$ scales as $\exp(-c_0/\varepsilon)$ with $c_0$ a positive constant.

The standard drift Hamiltonian $H_d$, which is equivalent to the drift equations of Alfvén, differs from the guiding center Hamiltonian $H_g$ by $H_g'$:

$$H_g'(\mu, P, Q) = H_d(\mu, P, Q) + H_g'(\mu, P, Q).$$

The perturbation $H_g'$ has a magnitude of order $\varepsilon H_d$.

The importance of the term $H_g'$ is made clearer if one uses the magnetic coordinates $(\psi, \theta, \varphi)$ that were introduced by Boozer\(^4,5\) to study particle drift motion. Boozer found that in these coordinates the standard drift Hamiltonian $H_d$ depends on the complexity of the magnetic field structure only through the magnetic field strength $B(\psi, \theta, \varphi)$\(^6\). We find that both the exact Hamiltonian $H_e$ and the lowest order in $\varepsilon$ expansion of $H_g'$ depend on the metric tensor $g^{ij}$ of the transformation from the ordinary spatial coordinates $x$ to the magnetic coordinates $(\psi, \theta, \varphi)$. The importance of this property of the drift equations is manifest for the quasi-helically symmetric stellarator in which the field strength has the form $B(\psi, \theta - N\varphi)$ with $N$ an integer but the metric tensor $g^{ij}$ depends in a complicated way on all three spatial coordinates.\(^7,8\) For quasi-helical symmetry, the standard drift Hamiltonian $H_d$ has an exact symmetry, and a conserved canonical momentum $P_h$, that does not exist for either the exact Hamiltonian $H_e$ or the guiding center Hamiltonian $H_g$. Indeed, we find that the lowest order in $\varepsilon$ expansion of $H_g'$ depends on the metric tensor $g^{ij}$ in a generic way and breaks the the invariance of $P_h$. Although quasi-helical symmetry is broken by terms of order $\varepsilon$ in the Hamiltonian, the primary effect is an oscillation in value of the canonical momentum $P_h$ rather than a total loss of this invariant.

To explain these results in more detail we assume a curl-free magnetic field that is written in the forms $B = \nabla \psi \times \nabla \theta + i(\psi)\nabla \varphi \times \nabla \psi$ and $B = \mu_0 G_0 \nabla \varphi$ given by Boozer\(^4\). One can show that $\psi$ is the toroidal flux, $-\chi$ is the poloidal flux, $\theta$ is the poloidal angle, $\varphi$ is the toroidal angle, and $G_0$ is the current, in ampere-turns, flowing in the external coils. The rotational transform is $i = d\chi/d\psi$. It is useful to define $e_\varphi = \nabla \theta - i\nabla \varphi$, so $B = \nabla \psi \times e_\varphi$. 
The metric tensor $g^{ij}$ can be given in terms of $g^x = \nabla \psi \cdot \nabla \psi$, $g^y = e_z \cdot e_z$, $g^z = \nabla \psi \cdot e_z$, the field strength $B^2$, and the external current $G_0$. It is a general result of the canonical transformation theory that if the position $x$ is given as a function of coordinates $\xi^i$, $x = x(\xi^1, \xi^2, \xi^3)$, the canonical coordinates $x, p$ of a Hamiltonian can be canonically transformed to $\xi^i, p_{\xi^i}$ by letting $p_{\xi^i} = p \cdot \partial x / \partial \xi^i$. The theory of coordinate transformations implies $\nabla \xi^i \cdot \partial x / \partial \xi^i = \delta^i_j$. The linear momentum $mv$ can be written in a covariant representation

$$mv = mv_\parallel B + e s e_z - p_s \nabla \psi,$$

with the particle charge $e$, and the canonical momentum $p = mv + eA$ with the vector potential $A = \psi \nabla \theta - \chi \nabla \varphi$. One can then show that the exact Hamiltonian, $H_\varepsilon = mv^2/2$, has canonical coordinates $(s, \Theta, \varphi)$ and canonical momenta $p_s$, $P_\Theta = e \psi = e(\psi + s)$, and $p_\varphi = (\mu_0 G_0 / B)mv - e \chi - els$. In these canonical coordinates, the exact Hamiltonian is

$$H_\varepsilon(p_s, s; P_\Theta, \Theta; p_\varphi, \varphi) = \frac{1}{2m} \left( \frac{B}{\mu_0 G_0} \right)^2 [p_\varphi + e \chi + est]^2 + \frac{1}{2m} \left[ e^2 g^2 s^2 - 2eg^c sp_s + g^\psi p^2_s \right].$$

If the magnetic field is uniform in space, one can easily show that $\psi$ and $\Theta$ are constants of the motion and are the coordinates of the center of the circle about which a particle located at $(\psi, \Theta)$ gy rates, the coordinate $\varphi$ advances due to the motion along the field lines, and the canonical coordinates $p_s$ and $s$ oscillate at the gyrofrequency.

To obtain Hamiltonians that are in the forms that were discussed earlier, we use a generating function to effect a transformation from the $(p_s, s)$ coordinate pair to the $(\mu, \theta)$ pair. This canonical transformation makes changes of order $\varepsilon^2$ to the $(P_\Theta, \Theta; p_\varphi, \varphi)$ canonical coordinates, which become the $(P, Q)$ coordinates. If one is retaining only the lowest order in $\varepsilon$ expansion of $H_\varepsilon'$, one need not make a distinction between $(P, Q)$ and $(P_\Theta, \Theta; p_\varphi, \varphi)$. The expressions that we obtain for $H_d$ and the lowest order expansion of $H_\varepsilon'$ are

$$H_d = \frac{1}{2m} \left( \frac{B(\Xi)}{\mu_0 G_0} \right)^2 [p_\varphi + e \chi(\psi)]^2 + \mu B(\Xi).$$
and \( H_g' = \left[ p_\phi + e \chi (\Psi) \right] A(\Xi) \) with \( \Xi = (\Psi, \Theta, \varphi) \), \( \Psi = P_\phi / e \) and

\[
\Delta = \frac{\mu B(\Xi)}{2e(\mu_0 G_0)^2} \left[ \frac{dt(\Psi)}{d\Psi} g^\psi(\Xi) + \left( t \frac{\partial}{\partial \Theta} + \frac{\partial}{\partial \varphi} \right) g^\varphi(\Xi) - \frac{g^c(\Xi)}{g^s(\Xi)} \left( t \frac{\partial}{\partial \Theta} + \frac{\partial}{\partial \varphi} \right) g^s(\Xi) \right].
\]

One of the biggest differences between the exact Hamiltonian and the standard drift Hamiltonian is that the exact Hamiltonian depends on the metric tensor elements while the standard drift Hamiltonian does not. The first order correction is of great interest because it depends on the metric tensor elements in a generic way. The generic dependence follows from the fact that the elements of the metric tensor must be periodic functions of \( (\Theta, \varphi) \). Whenever the symmetry of the exact Hamiltonian \( H_* \) is broken by the metric, the symmetry of the guiding center Hamiltonian \( H_g \) is also broken.

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 COLLISIONLESS DRIFT WAVES IN ANISOTROPIC CYLINDRICAL PLASMAS

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An integral formulation for the normal mode structure of electrostatic drift waves in cylindrical and slab geometry is developed. No drift ordering is assumed nor is the mode frequency $\omega$ assumed to be ordered compared with the ion gyrofrequency $\Omega_i$. Two extensions are presented, namely anisotropic temperatures and arbitrary density and temperature profiles. A local analysis of the k-mode is performed yielding a dispersion relation. It is evaluated by means of a four-pole approximation of the plasma dispersion function to get growth rates. Furthermore a Nyquist technique is applied resulting in a stability criterion for $\eta_{||}$. As an application, the concept of marginal stability combined with the requirement of a minimum critical threshold is used for profile determination. Finally for the isotropic case, the absolute and convective character of unstable modes is discussed.

I. General Integral Formulation for Collisionless Drift Waves

We consider the stability of an infinitely long axially symmetric collisionless cylindrical plasma imbedded in an uniform axial magnetic field $B_0$. The equilibrium distribution functions for the ions and electrons are chosen to be of the form:

$$f_{e0} = \left( \frac{M_s}{2\pi} \right)^{\frac{3}{2}} \frac{n_{e0}(R)}{T_{e0}(R)} \exp \left[ -\frac{M_s}{2} \left( \frac{v_e^2}{T_{e1}(R)} + \frac{v_i^2}{T_{i0}(R)} \right) \right]. \tag{1}$$

They are exact solutions of the Vlasov-equation since $v_\perp$, $T_\perp$ and $R$ are constants of motion, where $R$ represents the radial coordinate of the guiding center. It is a generalization of MARCHAND et al. /1/ due to the anisotropy and the $R$-dependence of the temperatures in (1).

We assume electrostatic perturbations of the form

$$\phi_1 = \tilde{\phi}(r, \theta) \exp \left[ i (m\theta + k_z z - \omega t) \right] \tag{2}$$

with the azimuthal mode number $m$, wave number $k_z$ and wave frequency $\omega$. Linearizing the perturbed Vlasov-equation, we get the response $f_{e1}$ of the system by integrating along the unperturbed helical particle trajectories. In this solution the rather complicated time-dependence of the integrand can be handled by writing (2) as a superposition of plane waves, an idea suggested by /1/. This means that any perturbed quantity $g_1$ is written in the form:

$$g_1 = \tilde{g}(r, \theta) \exp \left[ i (m\theta + k_z z - \omega t) \right] \tag{3}$$

where $f$, $k_\perp$ are the position and wave vectors in a plane perpendicular to the cylinder axis. $\alpha$ is the azimuth of $k_\perp$. The solution reads, dropping subscripts:

$$f = -\frac{q_f}{T_k} \frac{1}{\tau} \int_{-\infty}^{t} \int_{-\infty}^{t} dt' \int_{-\infty}^{2\pi} dk_{\perp} \int_{-\infty}^{2\pi} d\alpha' \tilde{\phi}(k_{\perp}) \cdot \exp \left[ i \left( m\alpha' + k_{\perp} \cdot \vec{R} + k_{\perp} \cdot \vec{b} \times \vec{\tilde{\phi}}(t') + [k_z (z(t') - z) - \omega (t' - t)] \right) \right] \cdot \left\{ \omega \frac{q_f}{T_k} - k_z \frac{q_f}{T_k} \left( 1 - \frac{T_{k\perp}}{T_{k\parallel}} \right) \right\} \frac{1}{B_0 k_{\parallel} \sin (\alpha' - \theta)} \frac{\partial f_0}{\partial R} \tag{4}$$
In (4), \( \Theta \) is the azimuth of the position vector of the guiding center \( \vec{R} \). Making use of (3) and integrating over the velocity, the FOURIER-transform \( \tilde{N}(k) \) of the perturbed density is obtained:

\[
\tilde{N}(k) = \int_{0}^{\infty} dRR \int_{0}^{\infty} dk_{1} \hat{k}_{1} \hat{\phi}(k) \sum J_{m+1}(k_{1} R) J_{m+1}(k_{1} \vec{R}) \frac{q_{n0}}{T_{i}} \cdot \\
\left\{ \Gamma_{m}(b_{1}, b_{2}) \left[ 1 + \zeta_{k} \tilde{Z}(\zeta_{k}) + \left( 1 - \frac{T_{i}}{T_{e}} \right) \left( 1 - \zeta_{k} \tilde{Z}(\zeta_{k}) \right) \right] - \\
\frac{T_{i}}{q_{n0}} m + j \frac{T_{i}}{R} \frac{\partial}{\partial R} n_{0} \frac{1}{T_{i}} Z(\zeta_{k}) \Gamma_{m}(b_{1}, b_{2}) \right\} \exp \left[ i (m \alpha) \right]
\]

(5)

with

\[
J_{n}(x) \text{ BESSEL-function of first kind and } n\text{-th order, } I_{n}(x) \text{ modified BESSEL-function of first kind and } n\text{-th order, } \Gamma_{n}(x, y) = I_{n}(\frac{x y}{2}) \exp \left\{ -x^{2} + y^{2} \right\}, Z(x) = 2i \exp \left\{ -x^{2} \right\} \int_{-\infty}^{x} dt \exp \left\{ -t^{2} \right\} \text{ plasma dispersion function, } v_{n0} = \sqrt{\frac{T_{i}}{M}} \text{ parallel and perpendicular thermal velocity, } \zeta_{k} = \frac{v_{n0} \vec{k}}{b_{1}}, b_{1} = \frac{v_{n0} \vec{k}_{1}}, b_{2} = \frac{v_{n0} \vec{k}_{2}}{b_{1}}.
\]

Inserting \( \tilde{N}(k) \) into the FOURIER-transformed POISSON's equation we obtain the integral formulation of the normal mode spectrum:

\[
(k_{1}^{2} + k_{2}^{2}) \Delta \phi(k_{1}) = \frac{1}{\epsilon_{0}} \sum_{\sigma} q_{n} \tilde{N}_{\sigma}(k_{1}) \exp \left( -i \omega \right)
\]

(6)

with \( \tilde{N}_{\sigma}(k_{1}) \) given by (5).

It generalizes previous expressions by

- \( R \) dependent temperature profiles
- anisotropic temperatures.

We also note that no drift ordering is assumed and that the mode frequency \( \omega \) is arbitrary and not assumed to be small against the ion gyrofrequency \( \Omega_{i} \). It is also possible to get the corresponding slab version \( /2/ \).

II. Local Analysis

Assuming an adiabatic electron response, quasineutrality, drift approximation and the following ordering of the frequencies

\[
\frac{\left| \omega \right|}{k_{2} v_{th}^{\perp}} \ll \frac{\left| \omega \right|}{k_{2} v_{th}^{\|}} \ll 1
\]

(7)

we get the local dispersion relation for the \( n \)-mode in slab geometry (\( \zeta_{k} \equiv \zeta_{n} \)):

\[
D(\omega, k_{1}) = 1 + \tau_{l} \left[ 1 + S_{0} \zeta(\zeta_{k}) - \left( 1 - \epsilon \right) (1 + \zeta(\zeta_{k})) S_{0} \right] - \\
- \zeta_{n} S_{0} \left[ -\eta_{k} \tilde{G}(b) \zeta(\zeta_{k}) + \eta_{1} \left( \frac{2 - 1}{2} \right) \zeta(\zeta_{k}) + \zeta_{n} + \zeta(\zeta_{k}) \right] = 0
\]

(8)

with

\[
S_{0} = S_{0}(b) := e^{-b} I_{0}(b), \epsilon := \frac{\Delta_{1}}{\Delta_{1}}, \tau_{l} := \frac{\Delta_{l}}{\Delta_{1}}, b := \frac{k_{2} v_{th}^{\perp}}{M \Omega_{i}}, \zeta_{k} = \frac{\omega}{k_{2} v_{th}^{\perp}}, \omega_{sc} := \frac{k_{2} T_{e}}{\epsilon d_{n0}}, \zeta_{n} := \frac{k_{2} v_{th}^{\parallel}}{M \Omega_{i}}, \eta_{k} := \frac{\Delta_{0}}{\Delta_{1}}, \eta_{n} := \frac{k_{2} v_{th}^{\perp}}{M \Omega_{i}}.
\]

For isotropic systems, \( \epsilon = 1 \), (8) reduces to the dispersion relation of ANTONSEN et al. /3/.
use of the NYQUIST technique \(3/\) it is possible to find a criterion for instability, namely (\(\mu := \frac{\partial \ln \gamma}{\partial \ln \nu} \),
\(\eta_\parallel \equiv \eta_\|)\):

\[
\eta_\parallel > \frac{1}{1 + 2(1 + \mu) bG} \cdot \left\{ 1 + \frac{k^2 r_n^2}{bc} \left\{ -\varepsilon^2 + \frac{4}{S_0^2} \left[ \frac{1}{\tau_1} + (1 - S_0) + \frac{1}{2} S_0 \right] \right\}^2 \right\} [1 + 2(1 + \mu) bG]
\]

\[
\eta_\parallel < \frac{1}{1 + 2(1 + \mu) bG} \cdot \left\{ 1 - \frac{k^2 r_n^2}{bc} \left\{ -\varepsilon^2 + \frac{4}{S_0^2} \left[ \frac{1}{\tau_1} + (1 - S_0) + \frac{1}{2} S_0 \right] \right\}^2 \right\} [1 + 2(1 + \mu) bG]
\]

A typical example of the threshold as a function of \(b\) for (9) is given in Fig. 1 for three values of the parameters \(k, r_n\). It shows that there exists a minimum critical threshold for \(b\) of order unity. We have also solved the dispersion relation (8) numerically by using a four-pole-approximation for the \(z\)-function \(4/\), suggested by HAMMETT and PERKINS. Some results are shown in Fig. 2

\[ z = \frac{R_s}{k, \nu \lambda_k}, \ y = \frac{m, \nu \lambda_k}{}\]

Fig. 1: The stability criterion (9)

Fig. 2: Numerical solution of the dispersion relation (8)

III. Marginal Stability Concept for Profile Determination

As an application of the local dispersion relation (8) and of the instability criterion (9) for the \(\eta\)-mode it is possible to present a unique method for profile determination. The latter requires that the plasma persists in an marginal stable state and that locally that value of \(b\) is adopted which corresponds to the above mentioned minimum threshold. With other words, the plasma has adjusted a state in which the most dangerous mode has just become marginally stable. In the case of equal isotropic
temperatures, \( r = 1, T_{th} = T_{ii} \), the recipe can be formulated as follows:

For a given density profile \( n(z) \) and for a given position \( z \), (9) is used to get the values of \( b \) and \( \eta_{\text{crit}} \) corresponding to the minimum threshold. These values are then used to get the temperature profile by an \( z \)-integration:

\[
T(z) = T(0) \exp \left[ \int_0^z \frac{\eta_{\text{crit}} \min(z')}{r_n(z')} \, dz' \right]
\]

Formula (11) follows from the definition of the \( \eta \)-parameter. The result \( T(z) \) for a GAUSSian density profile (dashed line) is shown for two different values of \( k_x L \), where \( L \) is the inhomogeneity length, in Fig. 3.

Fig. 3: Calculated temperature profiles

As one notices, the obtained temperature profile has the qualitative feature of measured profiles without singularities or negative values. A similar concept can be developed for anisotropic plasmas with \( r_L = \frac{4}{k_x^2} \neq 1 / 2 \).

IV. The Convective and Absolute Character of Unstable ITG-Modes

In the isotropic limit \( T_z = T_{th} = T_{ii} \) the local dispersion relation (8) becomes:

\[
D_0(\omega, k) := \eta_r - \frac{1 + \tau(1 + S_0 Z(\zeta)) - Z(\zeta) \zeta S_0}{\zeta S_0 (\zeta + (C^2 - \frac{1}{2}) Z(\zeta) - b G Z(\zeta))} = 0
\]

An inspection of the second term shows that it is independent of \( \partial_\zeta T_i \). This implies that \( \eta_r \) can be regarded as a free parameter which is a necessary requisite to apply the criterion of Bers-Briggs /5/ for absolute instability in the modified form of McCUNE and CALLEN /6/. Hence, it is expected that an upper threshold for \( \eta_r \) exists characterizing the transition from convective to absolute instability.

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Instabilities of Gyro-Kinetic Plasmas with a Guiding Center Density Gradient, Finite Larmor Radius and Polarization Drift

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1 Introduction

Experimental and theoretical studies have revealed the importance of turbulence in the physics of the plasma edge in tokamaks. The theoretical research has concentrated on linking this turbulence to instabilities which involves a motion of the plasma along the field line as well as a gradient and a drift perpendicular to \( \vec{B} \), such as the ion-temperature gradient instability [1,2] or possibly other drift-wave instabilities [3].

In this work we show that a two-dimensional plasma with a guiding center density gradient and no variation along the magnetic field lines is already unstable with regard to disturbances located only in the plane perpendicular to the magnetic field, if the charge separation between ions and electrons due to the finite ion Larmor orbits in \( \vec{E} \times \vec{B} \) drifts is taken into account, and if the polarization drift (which gives rise to an additional charge separation in a time varying electric field) is also taken into account [4].

2 The Guiding Center Model

For simplicity, we consider a straight and constant magnetic field, leaving the treatment of a plasma in a sheared magnetic field to a later publication. In the finite Larmor radius guiding center equations, the finite Larmor radius of the ions is taken into account by a filtering operation [5]. It has been shown [5] that this system possesses three “rugged” invariants”, with the consequence that charges of equal sign tend to concentrate and form large vortices under certain conditions. In the present work, we extend these models by including the polarization drift, which has a different sign for ions and electrons and thus give rise to a charge separation in a time varying electric field. These equations have been recently presented [4] and are written as follows:

\[
\begin{align*}
\frac{\partial n_e}{\partial t} + (V_D + V_{Pe}) \cdot \nabla n_e + n_e \nabla \cdot V_{Pe} &= 0, \\
V_D &= -\nabla \phi \times \vec{\varepsilon}_z / B, \\
\frac{\partial n_i}{\partial t} + (\vec{V}_D + \vec{V}_{Pi}) \cdot \nabla n_i + n_i \nabla \cdot \vec{V}_{Pi} &= 0, \\
\vec{V}_D &= g \otimes V_D = -\nabla \phi \times \vec{\varepsilon}_z / B; \quad \vec{V}_{Pi} = g \otimes V_{Pi}, \\
\nabla^2 \phi &= -\frac{q}{\varepsilon_0} (\vec{n}_i - \vec{n}_e).
\end{align*}
\]

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\[ \bar{\phi} = g \otimes \phi, \quad \bar{n}_i = g \otimes n_i \quad (6) \]

\[ V_{Pe} = -\frac{1}{\Omega^2} \frac{q}{m_e} \frac{dE}{dt} \quad (7) \]

and
\[ \frac{dE}{dt} = \frac{\partial E}{\partial t} + V_D \cdot \nabla E \quad (8) \]

As described in [4,5], \( g \) is a convolution integral operator, so that we have, for instance
\[ \bar{n}_i = g \otimes n_i = \int g(\vec{r} - \vec{r}') \, n_i(\vec{r}') \, d\vec{r} d\vec{r}' \quad (9) \]

In the Fourier k-space, it becomes a filter operation, which is easily incorporated in the different Fourier modes. Each coefficient \( a_k \) of the mode \( e^{i k \cdot \vec{r}} \) is multiplied by a factor
\[ g = \exp\left(-\frac{1}{2} k^2 \tau^2_i\right), \]
where \( k \) is total wave vector and \( \tau_i \) is the ion Larmor radius. We emphasize that the densities are not particle but guiding center densities and this explains the absence of any pressure gradient terms in the present model.

### 3 Linear Stability Analysis

We will show now that as soon as there is a spatial gradient of the guiding center density in a neutral plasma, there will be an instability. It follows that a magnetized plasma with a guiding center gradient can never be in a stationary state, but rather it goes over into a turbulent state.

We consider a geometry of experimental interest, namely a plasma between two conducting plates, in the \( x \) direction \( d \) is the separation between the two plates. As boundary conditions we take \( n_i(d) = n_i(0) = N = \text{const}, n_i(0) = n_e(0) = 0 \), and for the potential \( \phi(d) = \phi(0), \phi(0) = 0 \). Initially we let the guiding center densities depend on \( x \) only. The system is periodic in the \( y \) direction, and is initially charge neutral, but we allow a potential difference across the plasma. Then the zeroth order potential becomes \( \phi_0 = \bar{\phi} \). We linearize the equations by expanding around a steady state and assuming only small deviations from it, i.e., \( \phi = \phi_0(x) + \delta\phi(x, y, t), n_{ic} = n_{ic,0}(x) + \delta n_{ic}(x, y, t) \), etc. The result is a linear system:

\[ \begin{align*}
\partial_t \delta n_e - \frac{1}{B} \partial_y \delta \phi \partial_x n_e^0 - \beta_e(\partial_t + \frac{1}{B} \partial_x \phi_0 \partial_y) \partial_x \delta \phi \partial_x n_e^0 + \frac{1}{B} \partial_x \partial_n \delta n_e & = 0 \\
\partial_t \delta n_i - \frac{1}{B} \partial_y g \delta \phi \partial_x n_i^0 - \beta_i(\partial_t + \frac{1}{B} \partial_x g \phi_0 \partial_y) \partial_x g \delta \phi \partial_x n_i^0 + \frac{1}{B} \partial_x g \phi_0 \partial_y \delta n_i & = 0 \\
\nabla^2 \delta \phi &= -\frac{q}{e_0} (g \delta n_i - \delta n_e) 
\end{align*} \quad (10) \]

where \( \bar{n}_i = g n_i \), etc. and \( q = q_i = |q_e| \).
All zero order gradients are constant; \( n_0^0 = n_0^0 = \alpha x \), with \( \alpha d = N \). Note further that \( d_z \phi_0 = -E_0 = Bv_D^0 \). As the equilibrium equations do not depend on \( y \) or \( t \), we make the ansatz \( \delta \phi(x, y, t) = \delta \phi(x) \exp (i k_y y - i \omega t) \), solve for the guiding center densities and insert them into Poisson’s equation. The resulting equation for the potential is given by

\[
\nabla^2 \delta \phi = -\frac{q}{\varepsilon_0} \left\{ \left(1 - \frac{g^2}{\omega^2}\right) \frac{k_y}{\omega B} \delta \phi + \frac{\alpha x}{\omega B} \delta \phi + \frac{\alpha x (\beta_i g^2 - \beta_e)}{\omega^2} \nabla^2 \delta \phi \right\}.
\]

Note that \( |\beta_e/\beta_i| = \frac{m_e}{m_i} \), which justifies to drop \( \beta_e \) in (11). In other words, the electron polarization drift is negligible. The coefficient of the last term in (11) can be written \( \frac{\alpha x}{\omega B} \beta_i g^2 = \frac{\alpha x}{\omega B} \frac{\omega_{pe}^2}{\omega_{ce}^2} g^2 \), where \( \omega_{pi}^2 = \frac{e^2 n_0}{m_i} \), and \( \Omega_i = \frac{2B}{m_i} \). If \( \Omega_i^2 \ll 1 \), this term can be neglected because it is small. (This is tantamount to neglecting the divergence of the polarization drift.) In the opposite case the coefficient of \( \nabla^2 \delta \phi \) becomes dependant on \( x \), but remains always positive, so that our results derived with (11) are still qualitatively valid.

With these approximations (11) can be written

\[
\frac{\partial^2 \delta \phi}{\partial x^2} + \left(\frac{\omega_{pi}}{\Omega_i}\right)^2 g^2 d^{-1} \partial_x \delta \phi - \left[ k_y^2 - \frac{\omega_{pi}^2 k_y (1 - g^2)}{\Omega_i d} \right] \delta \phi = 0
\]

(12)

The term \( \left(\frac{\omega_{pi}}{\Omega_i}\right)^2 g^2 d^{-1} \partial_x \delta \phi \) originates from the polarization drift (12) is an ODE of second order with constant coefficients, which is solved by \( \delta \phi \sim e^{ik_x x} \). The boundary conditions \( \delta \phi(x = 0) = \delta \phi(x = d) = 0 \) are satisfied for

\[
k_x = \pi n/d, \quad n = 1, 2, 3\ldots
\]

(13)

From Eq. (12) follows the dispersion relation

\[
\frac{\omega_{pi}}{\Omega_i} = \frac{k_y}{d} + \frac{\omega_{pi}^2 k_y d^{-1}}{k^2} \frac{1 + \frac{\omega_{pi}^2 k_y d^{-1}}{k^2} \left(1 - g^2\right)}{1 + \left(\frac{\omega_{pi}}{\Omega_i}\right)^4 \left(\frac{d^{-1} k_x}{k^2}\right)^2 (1 - g^2)},
\]

(14)

with \( g = \exp(-\frac{1}{2} k^2 r_i^2) \approx 1 - \frac{1}{2} k^2 r_i^2 \).

Without the finite Larmor radius correction, i.e. for \( g = 1 \), (14) describes a drift wave. The ion polarization drift is responsible for the imaginary term. Thus without the polarization drift but with finite Larmor radius correction, we would have a small correction to the drift wave only. It is the combination of both, finite Larmor radius and polarization drift which makes the plasma unstable. With \( \omega = \omega_{pi} + i \gamma \) we obtain

\[
\frac{\gamma}{\Omega_i} = \frac{\omega_{pi}^4 k_x k_y r_i^2 / d^2}{1 + \left(\frac{\omega_{pi}}{\Omega_i}\right)^4 \left(\frac{k_x d^{-1}}{k^2}\right)^2}
\]
The sign of $k_x$ and $k_y$ are arbitrary, so that there is always an instability. The thickness of the plasma sheath, $d$, is a measure of the density gradient which supplies the free energy for the instability. For a numerical example we take $k_x = k_y = \pi d^{-1}, k^2 = 2 \pi^2 d^{-2}$, and $r_i/d = 10^{-1}$. Then

$$\gamma = \frac{\Omega_i}{1 + \left(\frac{\omega_{pi}^2}{\Omega_i^2}\right)^2 \frac{1}{2} \cdot 10^{-2}}$$

The choice $\frac{\omega_{pi}^2}{\Omega_i} = \frac{1}{2}$ results in $\gamma_i = 1.56 \cdot 10^{-4}$. Within 100 ion gyrations the amplitude increases by 10%. This is a weak instability. The choice $\frac{\omega_{pi}^2}{\Omega_i} = 1$ violates an approximation in our derivation, however we may still expect qualitative results. The amplitude grows in 100 periods by a factor of 23.

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**References**

The UMIST linear quadrupole GOLUX is a steady-state device in which hydrogen plasma is continuously injected axially, at one end, from an external duoplasmatron source [1]. The magnetic field configuration is shown in cross-section in figure 1. The electron temperature in the drift wave region is about 1 eV, and the density about $10^{15}$ m$^{-3}$. Self-excited intrinsic drift modes are observed in the shared flux region of GOLUX, forming a broad band between 30 and 50 kHz. A typical power spectrum is shown in fig. 2.

Drift waves may also be launched into the system [2], by passing an AC current through the plasma between the two flag antennae shown in fig. 1. A typical launched wave at 60 kHz is shown superimposed on the intrinsic modes in fig. 2. These coherent launched waves present a powerful means of studying drift wave phenomena.

We obtain information about the launched wave by detecting both density and potential fluctuations, using a small cylindrical Langmuir probe and lock-in amplifier techniques. Scanning the probe in the longitudinal (z) direction yields the spatial variation of amplitude and phase with respect to the launching signal; the ratio of the wave potential, extrapolated back to the probe, to the launching current gives the launching impedance, a measure of the effectiveness of the launching process.

Waves can be launched in the frequency range 10 - 60 kHz. The upper frequency marks a sharp cut-off above which waves cannot be launched: the theory presented below shows that this is because the wavelength has become comparable with the z scale-length of the effective launching antenna, which is set by the disturbance in the plasma, not by the flag thickness. There appears to be no lower cut-off, and waves can be launched well below the frequency band of the intrinsic modes.

When waves are launched within the intrinsic band, we observe an exponential decay in z, the decay length being about 1.0 m. Below the intrinsic band, the observed wave exhibits strong interference effects, which we interpret as due to the simultaneous launching of the expected wave and also the second radial harmonic, which has a different wave-number [3]. The dispersion curves for both modes are shown in fig. 3. We have investigated the frequency dependence
of the launching impedance by measuring the wave potential amplitude as a function of distance $z$ from the flags, with a constant amplitude of the launching current. Results for two values of $z$, shown in fig. (4), clearly show the existence of two windows of good propagation. The upper one corresponds to the intrinsic band of unstable drift waves, while the lower one, corresponding to a mode apparently stable under present operating conditions, may be identified with the second radial harmonic [4].

This mode can be positively identified by measuring its radial co-variance in relation to its maximum radial amplitude. Figure 5 shows such results for both the observed modes. The co-variance of the second harmonic changes sign showing the presence of a radial node, in contrast with the fundamental mode. The additional evidence from the dispersion curve (fig. 3), which matches that predicted by theory [4], confirms the identification. We have performed a theoretical analysis of the wave launching mechanism, using the formulation introduced by Ashraf and Rusbridge [5]. The plasma response is given by

$$\dot{\phi}(k, \omega) = \frac{\rho_e(k, \omega)}{k^2 e(k, \omega) \varepsilon_0}$$

where $\phi$ is the wave potential, $\rho_e$ is the externally induced charge density, and $\varepsilon$ the dielectric function. The charge density $\rho_e$ is determined from the potential distribution in the plasma arising from the flow of current between the two launching electrodes. Although the two electrodes are linked by a common flux-tube, this is not capable of carrying the whole current, and the current channel therefore expands; the resulting cross-field current flow leads to a $j \times B$ force which drives the wave as suggested by Tessema et al [2]. The potential distribution in the current channel is given by the equation $d^2V/dz^2 = ik V$, with the acceptable solution

$$V = V_0 \exp \left[ \frac{\sqrt{2}}{2} \left( \frac{\sqrt{2} + 1 - i}{2} \right) \kappa z \right] \text{ where } \kappa \text{ is defined by } \kappa^2 = \frac{e^2 B^2}{2m_e \varepsilon_0 RLDv_D \omega}$$

where $R$ is the mirror ratio, $L$ is the length of a quadrant of a field line (i.e. from minimum to maximum field points), $D$ is the probe length, $v_D$ the plasma drift speed, and $\omega$ the launching frequency. $V_0$ is determined by the impedance of the current channel. From the potential distribution we obtain $\rho_e$ using Poisson's equation; this is then Fourier transformed to determine $\rho_e$. For the dielectric function we use a model form suggested by that of a drift wave in the "slab model", given by
\[ e = \frac{1}{k^2 \lambda_D^2} \left( 1 - \frac{\Omega(k)}{\omega} + i \frac{Y}{\omega} \right) \]

where \( \Omega(k) \) is the observed dispersion relation and \( Y \) the damping rate. For light damping, leading to a narrow resonance, we may approximate the dispersion curve by its tangent at the launching frequency, giving \( \Omega(k) = kv_g + C \) where \( v_g \) is the group velocity and \( C \) is a constant. Then, using eq.(1) and back transforming to obtain the wave potential \( \varphi \) in physical space, we finally obtain the launching impedance \( Z_L = \varphi/l \) in the form

\[ Z_L = \frac{\lambda_D^2 R^2 k^2 F}{2 \lambda m v_g} \left( \frac{2k^2 - (1 - i\sqrt{3})k^2}{k^4 - k^2 + k^4} \right) \]

which may be expressed as a function of \( \omega \) using the dispersion curve. This expression shows that \( \kappa \) represents the upper limit on the wave number that can be launched effectively; \( Z_L \) has a maximum at \( k = \kappa \sqrt{2} \), and becomes constant in the limit \( k \ll \kappa \). In this expression \( F \) represents the overlap integral between the potential distribution and the eigenfunction of the drift mode [3]. An approximate calculation gives \( F = 1 \) and we may ignore this factor. The dependence of \( Z_L \) on frequency (derived from \( Z_L(k) \) above) is shown in fig.(6). We use the following data: \( B = 14 \) mT; \( n = 1.0 \times 10^{15} \) m\(^{-3} \); probe dimensions 2 cm by 3 cm; \( T_e = 1.4 \) eV; \( v_g = 1500 \) m/sec. The largest uncertainty is in the density, about \( \pm 30\% \). Then the predicted value of \( Z_L \) in the limit \( k \ll \kappa \) is \( 10 \Omega \pm 50\% \). To estimate \( \kappa \), we also need the mirror ratio \( R = 7 \), and the plasma drift speed \( v_D = 1000 \) m/sec, giving \( \kappa = 480 \) m\(^{-1} \) at 60 kHz. Note that there are no fitted parameters in these estimates.

The experimental results for \( Z_L(\omega) \) are also shown in fig 6. In the intrinsic drift band, the observed value of \( \approx 15 \Omega \) is within the theoretical limits of uncertainty; however, there is no maximum in the impedance, and the high-frequency cut-off occurs at \( k = 120 \) m\(^{-1} \), much less than the predicted value \( k = \kappa = 400 \) m. The observed rise of \( Z_L \) at low frequencies does not appear in our theory, and has not yet been explained. Nevertheless, there is qualitative, and to some extent quantitative, agreement between theory and experiment.

References:
Fig 1: Magnetic field profile and launching configuration

Fig 2: Typical power spectrum of GOLUX with launched wave at 60 kHz

Fig 3: Launched wave dispersion curve

Fig 4: Wave amplitude versus frequency

Fig 5: Covariance of launched wave

Fig 6: Wave impedance versus frequency
STUDIES OF NON-LINEAR INTERACTIONS OF DRIFT WAVES IN THE
UMIST QUADRUPOLE "GOLUX"

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The UMIST linear quadrupole GOLUX is a steady-state plasma confinement system in which the plasma is injected along the null-line of the magnetic field and drifts along the vacuum vessel in the field; a detailed description is given in [1], and a cross-section of the magnetic field configuration is shown in [2]. Drift waves are spontaneously excited in the system, covering the frequency range 30 – 50 kHz; propagation is parallel to the symmetry axis, with wave-number about 100 m at 40 kHz. It has been shown [3] that the dispersion curve agrees well with theory. In previous work two possible non-linear saturation mechanisms have been considered: (1) the "frequency-doubling" process in which a basic drift wave \((\omega, k)\) drives another, the "f.d." mode at \((2\omega, 2k)\), on a different branch of the dispersion curve, and (2) a cascade mechanism in which energy flows through the drift wave spectrum towards smaller \(k\), mediated by the low frequency \((\approx 5\ kHz)\) flute-like mode which is also present [3]. Greb and Rusbridge [4,5] developed the theory of process (1) and showed experimentally that the energy flow in the process is insignificant, while Crossley et al [6] introduced a new approach to the analysis of experimental results, that of "amplitude correlations", and used it to confirm that process (2) is the dominant saturation mechanism.

In introducing the amplitude-correlation method, Crossley et al. [6] did not give a rigorous justification, but relied on its coherence with other evidence to provide confirmation. In this paper, we extend the work of Greb and Rusbridge [4] to provide a rigorous derivation of the method for the frequency-doubling process, and show that it can give estimates of the damping rate of the driven mode and of the rate of energy flowing into it through non-linear interactions.

The basic equation is that for the slow time variation of the potential amplitude \(\phi_a\) of a mode \((\omega_a, k_a)\) driven by the non-linear interaction of modes 1 and 2:

\[
i \frac{\partial \phi_a}{\partial \omega} \frac{\partial \phi_3}{\partial t} + \epsilon \phi_a = A \phi_1 \phi_2 \delta(\omega_3 - \omega_1 - \omega_2) \delta(k_3 - k_1 - k_2) \tag{1}\]

where \(A\) is the coupling constant, \(\epsilon = \epsilon(\omega_a, k_a)\) is the dielectric response function of the driven mode, and the \(\delta\)-functions enshrine the selection rules. We formally expand \(\phi_a = \phi_{a0} + \phi_a(t)\), where \(\phi_{a0}\) is the steady state solution. Multiplying eq.(1) by \(\phi_a^*\), substituting \(\phi_{a0}\) for \(\phi_a\) in the r.h.s. and dropping the subscript 1, we obtain

\[
i \frac{\partial \phi_a}{\partial \omega} \frac{\partial \phi_a^*}{\partial t} + \epsilon \phi_a^* = \frac{AA^* \phi_1^* \phi_2^* \phi_1 \phi_2}{\epsilon} \tag{2}\]
where the delta-functions have been absorbed into $A$ for convenience. By taking the complex conjugate of this equation and separating out the real and imaginary parts we can obtain an equation for $W_3 = \phi_3 \phi_3$ in the form

$$
\frac{\partial W_3}{\partial t} + 2W_3 = \frac{2A^* W_2 W_1}{\varepsilon^* x}
$$

(3)

To proceed further we need an explicit form for $\varepsilon$, and we use one modelled on that for drift waves in the slab model:

$$
\varepsilon = \frac{1}{k^2 \lambda r^2} \left[ 1 - \frac{\omega_0(k)}{\omega} + i \frac{\gamma}{\omega} \right]
$$

(4)

where $\omega_0(k)$ is the dispersion relation and $\gamma$ the damping rate. From this it is straightforward to obtain the final equation in the form

$$
\frac{\partial W_3}{\partial t} + 2\Gamma W_3 = \frac{2A^2 \Gamma W_1 W_2}{\varepsilon^2}
$$

(5)

where $\Gamma = \omega^2 / (\omega_0^2 + \gamma^2)$, or for light damping $\Gamma = \gamma (\omega / \omega_0)^2$.

The steady state solution to this equation leads to the power ratio [4]; it is governed by the quantity $(A/\varepsilon)^2$, so that neither $A$ nor $\varepsilon$ can be determined separately from it. In the time-dependent form above, the equation describes the response to fluctuations in $W_{1,2}$; we see that the response always lags the driving terms, and from the phase shift we can determine $\Gamma$ and thus obtain additional information specifically about the damping rate of the driven mode. Thus, if the product $W_1 W_2$ oscillates at a frequency $\Omega$, the phase lag is $\theta$ where $\tan \theta = \Omega / 2\Gamma$, and $0 < \theta < \pi / 2$.

The experiments are carried out by recording a series of records of the fluctuating density signal from GOLUX, using a transient recorder. Individual records are $1024$ $\mu$sec in length, digitised at $2$ $\mu$sec intervals. Records are Fourier-transformed using an FFT routine; the resulting power spectrum is shown in fig.(1). Frequency bands around the drift peak ($\approx 40$ kHz) and the frequency doubled peak ($\approx 80$ kHz) are isolated, squared, and again filtered to pass identical low-frequency (LF) bands, typically $1 - 10$ kHz. The resulting signals are then correlated in the usual way. The use of $50 - 100$ independent records is sufficient to give a random level of correlation of $\pm 0.1$, as can be seen at large time delay in the result shown in fig.(2).

It can be seen that this result agrees qualitatively with the theory presented. The maximum correlation is positive, and the phase lag is less than $\pi / 2$. In quantitative terms, the average frequency determined from the number of visible maxima is $6.9$ kHz, consistent with the $1 - 10$ kHz LF band limits: we identify this with $\Omega / 2\pi$. The delay of the maximum is $21$ $\mu$sec, equivalent to a phase lag of $0.93$ rad and leading to $\Gamma = 1.6 \times 10^4$ sec$^{-1}$. 
With $\omega/2\pi = 80$ kHz we obtain $\Gamma/\omega = 0.032$, consistent with weak damping of the f.d. mode; in fact, if the interaction is no more than 20% off resonance, i.e. $0.8 < \omega/o < 1.2$, then $0.020 < \gamma/\omega < 0.046$.

The maximum correlation is consistent with the level of background shown in fig.(1). We estimate the background by interpolation between the power levels above the f.d. band and below the drift band, as shown by the dashed curve; averaging over the upper frequency band, we obtain the signal-to-background ratio $S/N \approx 1.62$. Assuming that the background is unrelated to the drift waves, we find that the maximum possible value of the correlation is $S/(S+N) = 0.62$. The maximum observed correlation shown in fig.(2) is almost as large as this; we interpret this as confirming that the f.d. mode is completely driven by the non-linear coupling from the drift mode [5]. The high value also effectively rules out the possibility that the apparent correlation could be due to the modulation of both drift and f.d. modes by the same signal (e.g. the low frequency flute mode which is also seen in the system).

For greater precision, we need to determine experimentally the f.d. mode dispersion curve $\omega_o(k)$. We expect to do this through an application of the wave launching technique described in refs [2,3]. With this information available, we expect to be able to determine the dielectric function and thus obtain an experimental measurement of the coupling constant for comparison with theory.

In conclusion, we have shown that the intuitively appealing approach of the amplitude correlation method can be rigorously justified for the case of a non-linear interaction of the "frequency-doubling" type, and we have shown that the theory agrees well with observations made in the GOLUX Quadrupole. The results give a direct experimental measurement of a quantity related to the damping rate of the driven mode; we now require only a determination of the driven mode dispersion curve to obtain complete information about the interaction including a measurement of the coupling constant.

References
**POWER SPECTRUM**

$\log P(\omega)$

**DRIFT PEAK**

**FREQUENCY DOUBLED PEAK**

$\omega/2\pi$ KHz

**fig 1**

**DRIFT FD AMPLITUDE CORRELATION**

**CORR COEFF**

$-408 -306 -204 -102 102 204 306 408$

$\tau \mu s$

**fig 2**
Abstract Experiments on electron cyclotron wave excitation are carried out. Mode conversion of a left-hand polarized wave into an electron cyclotron wave is examined in detail. When small inhomogeneity exists in the magnetic field, the trapped electrons chaotically accelerated to a few tens of KeV range. The experimental results on the onset of chaos are presented.

1. Introduction

Excitation of electron cyclotron wave (ECW) /1/ is experimentally studied with a 2.45GHz microwave. The TE$_{10}$ rectangular mode is converted into the TE$_{11}$ circular mode, and launched from an open end of the field line. By controlling the microwave polarization, we excite a ECW (R-wave).

One of the most important application of ECW is to produce overdense plasmas. In the case of R-polarized microwave injection, ECW generates a plasma density $1 \times 10^{13}$ cm$^{-3}$ at 5 KW input. While, we use the L-polarized microwave, mode conversion process occurs; a L-wave is excited in the initial traveling distance and converted into a ECW as it propagates. The detailed results on the mode conversion process including the waveform and the polarization are presented.

When there is small inhomogeneity in the applied magnetic field, some of electrons (initially $\approx 20$ eV) are accelerated up to a few tens of KeV range. To examine the mechanism of acceleration to such the high energy region, we have measured the wave propagation and the soft X-ray emission from the accelerated electrons. It is found that this phenomenon is understood as a Hamilton chaos of mirror trapped electrons interacting with ECW.

2. Experimental Apparatus

The experiments are carried out in a cylindrical chamber with 8 magnetic coils. The chamber dimension is 16 in diameter, and 150 cm in axial length. Microwave source with a frequency 2.45 GHz and 5KW output is used to excite ECW. Magnetic fields are applied to satisfy the high field side injection at the launching point, $\omega / \omega_{ce} < 1.0$, where $\omega_{ce}$ is the electron cyclotron frequency and $\omega$ the microwave frequency. The TE$_{10}$...
rectangular mode from a magnetron oscillator is transformed into the circular TE$_{11}$ mode, and then polarized by a dielectric-plate-type polarizer. The rotation direction of the electric field is selected by the polarizer. The circularly polarized microwave is introduced from an open end of the field line through a tapered waveguide and a quartz window. Filling gas is Argon, and the operation pressure is $1 \times 10^{-4} - 1 \times 10^{-2}$ torr. The electron temperature and density measured with a Langmuir probe inserted from a radial port are 4 - 20 eV and $5 \times 10^{11} - 1 \times 10^{13}$ cm$^{-3}$, respectively. An axially movable dipole-antenna, covered with ceramic for thermal insulation, is introduced for the wave detection. A Si(Ge) solid state detector is connected to the radial port located between the third- and the fourth coil, and the output signal is processed by a Multichannel Analyzer. Figure 1 shows the experimental setup.

3. Mode Conversion of L-wave into ECW

In our experimental condition, two wave modes can propagate in a plasma along the magnetic field line. Right-hand wave (ECW) is accessible to any high dense plasmas and strongly interacts with electrons through the electron cyclotron resonance, while left-hand wave is only accessible to low density cases, whose interaction is negligible.

When we use the R-polarized microwave to produce a plasma, an ECW is excited in the plasma and a density $1 \times 10^{13}$ cm$^{-3}$ is achieved at 5 KW microwave input /2/. This is 140-times higher than the cutoff density for the ordinary wave of the same frequency. When the L-polarized microwave is injected into the evanescent plasma, mode conversion process is observed. Figure 2 shows the perpendicular component of the electric field, $E_y$, detected by the dipole antenna. As seen in this figure, a long-wavelength mode is excited in the initial propagation distance (broken line) and decays within the one period of oscillation. During this process, a short-wavelength mode grows from $z \approx 5$ cm, and dominates after $z \approx 15$ cm, showing the mode conversion.

Rotating the dipole antenna by 90 degrees, we have measured the $E_x$ component to determine the polarization of the long- and the short-wavelength modes. Figure 3 shows the polar plot of the observed electric fields. It is found that the long-wavelength mode is a left-hand polarized wave (Fig.3(a)), and the short-wavelength mode a right-
hand polarized one (Fig.3(b)). These waves are identified as the L-wave and the ECW, respectively. Since the conversion into ECW is complete, the plasma density achieved in this case is almost the same as obtained with the R-polarized microwave. It is interesting that, in the growing phase, the ECW is almost linearly polarized.

Fig.2 Interferometer trace of $E_y$ component (upper left).

Fig.3(a) Polar plot of $E_\perp$ for long-wavelength mode indicating left-hand polarization (upper right).

Fig.3(b) Polar plot of $E_\perp$ for short-wavelength mode indicating right-hand polarization (lower right).

4. Hamilton Chaos of Electrons Interacting with ECW

It is well known that an oscillating particle kicked by a force exhibits chaos. Such the situation is realized in a plasma with trapped ions moving in the potential well of an electrostatic wave /3/. We present another example of chaos in a plasma; electrons trapped in the weak inhomogeneity of the applied magnetic field suffer chaotic acceleration by an electromagnetic wave (ECW), reaching a few tens of keV range.

The mirror ratio is less than 1.3, and the field intensity at the bottom of the mirror is kept constant to provide the constant interaction with the ECW. The accelerated electrons radiate soft X-rays through the bremsstrahlung, which is measured with a Si(Ge) solid state detector. Changing the inhomogeneity of the magnetic field and the input microwave power, we have carefully measured the onset of X-ray emission.

The X-ray emission doesn’t occur with the straight magnetic field, but, at a finite mirror ratio, it does /4,5/. The observed spectrum is shown in Fig.4. The energy covers from 2 keV, the lower limit of the detector, to 15 keV, and the peak at 3 keV is the characteristic X-ray form Argon (K-shell). This means that electrons are accelerated to
the same range of energy. The count rate rises with increasing the field inhomogeneity, indicating that the mirror trapped electrons are responsible for the X-ray radiation. Figure 5 shows the X-ray total count as a function of mirror ratio. The onset of emission is so sudden that only a 3% change in mirror ratio triggers this phenomenon. The X-ray emission is also sensitive to the input microwave power. Beyond the threshold value between 1 KW and 2 KW, the total count rapidly increases. These results strongly suggest that the mirror trapped electrons are chaotically accelerated by the ECW, and that our system is equivalent to a harmonic oscillator (trapped electron) periodically kicked by an external force (ECW).

Fig.4 Soft X-ray energy spectrum. Input power: 3 KW; mirror ratio: 1.08 (upper left).

Fig.5 X-ray total count as a function of mirror ratio. Input power: 3 KW (upper right).

Reference

AN EULERIAN VLASOV PLASMA SIMULATION CODE
FOR ION PHASE SPACE PHENOMENA

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1 INTRODUCTION

A number of papers have been devoted during the last years to the nonlinear saturation of various instabilities. One proposed mechanism is the formation of vortices in phase-space. P.I.C. simulations have shown [2], [3] the occurrence of such structures in the nonlinear saturation regime of the ion beam driven acoustic instability. The need of a low noise level and a very good phase-space resolution to study the long time behaviour of these structures leads us to prefer an eulerian Vlasov simulation code. Thus we have modified a one-dimensional electron Vlasov code [1] to model ion beam-plasma systems.

In this paper we shall discuss the first results obtained with this code. Namely we shall report detailed comparisons between the P.I.C. and the eulerian Vlasov simulation approaches.

2 THE THEORETICAL MODEL AND SIMULATION METHODS

We consider a one-dimensional collisionless unmagnetized plasma in which ions are described by the Vlasov equation and electrons are assumed to be in a Boltzmann equilibrium. The ion distribution function $F(x,v,t)$ and the normalized self-consistent electrostatic potential $\Phi(x,t)$ are thus described by the following dimensionless Vlasov-Poisson system:

\[
\frac{\partial F}{\partial t} + v \frac{\partial F}{\partial x} + \frac{\partial \Phi}{\partial x} = 0
\]

\[
\frac{\partial^2 \Phi}{\partial x^2} = -\int F dv + \exp(\Phi / T)
\]

Here $T = T_i / T_e$ and $x \in [0,L]$, $v \in (-\infty, +\infty)$ and $t$ are the space, velocity and time variables respectively measured in units of ion Debye length, ion thermal velocity and inverse plasma period. Periodic boundary conditions are used.

Eulerian Vlasov simulation code: This code is based on the splitting scheme algorithm described by Cheng & Knorr (1976) [4], or Gagné & Shoucri (1977) [5]. It consists of alternately integrating the free streaming term and the acceleration term in Vlasov equation, using cubic spline interpolations.

But, contrary to the problem of electron dynamics in a background of static ions, our Poisson equation is nonlinear. To solve it, a regular grid with grid spacing $\Delta X$ and $N_G$ grid points is introduced. Thus the Poisson equation can be written:

$\Phi_{j+1} - 2 \Phi_j + \Phi_{j-1} = (\exp(\Phi_j / T) - \rho_j) \Delta X^2$, where $\rho(x) = \int F(x,v) dv$ is the ion charge density. The resulting set of equation in matrix form, i.e. $A\Phi = B + C(\Phi)$, can be solved iteratively by the formula $A\Phi^{(n)} = B + C(\Phi^{(n-1)})$. 


The $x$-$v$ phase-space is discretized as a 256$\times$128 grid.

**P.I.C. simulation code:** The ion distribution function is discretized using $N_p=512000$ superparticles and the leap-frog scheme is chosen for the solution of the simulation particle trajectories. The self-consistent electric field is obtained from the Poisson equation solved with the same iterative method as above, with a grid of $N_G=256$ points. The ion charge density at a grid point is calculated according to the formula:

$$
\rho_j(X_j) = \sum_{i=1}^{N_p} \frac{q_i}{\Delta X} W(x_i - X_j) \quad \text{where} \quad W(x) \text{ is the quadratic spline interpolation function.}
$$

3 RESULTS

**Initial data:** We consider the following initial distribution function:

$$
F(x,v,t=0) = (2\pi)^{-1/2} \alpha(T_{th}/T_e)^{-1/2} \exp \left( -\frac{(v-v_b)^2}{2T_{th}/T_e} \right) + (1-\alpha) \exp \left( -\frac{v^2}{2} \right) \times \left( 1 + \sum_n a_n \sin(k_n x + \phi_n) \right)
$$

where $a_n$ (resp. $\phi_n$) is the amplitude (resp. phase) of the initial perturbation of the mode with wavevector $k_n$, with $k_n = n(2\pi / L)$, and $v_b$ is the beam velocity.

The following plasma parameters are chosen: $T = 30$, $T_{th} / T_e = 0.05$, $v_b = 6$, $\alpha = 0.1$.

For this data set the $n=4$ mode (four wavelengths in a box of length $L=150$) is the most linearly unstable, as predicted by the linear dispersion relation [2].

All the simulations are run with a time step $\Delta t = 0.25$.

P.I.C. simulations need no initial perturbation and thus $a_n = 0$, $\forall n$. On the contrary the intrinsically noiseless Vlasov code needs some $a_n \neq 0$. We will discuss the results obtained with the following data sets:

Vlasov1: $a_1 = a_2 = 10^{-3}$, $a_3 = 10^{-3}$

Vlasov2: all modes initially excited with amplitudes $a_n$ distributed according to the thermal levels in a maxwellian plasma and randomly distributed $\phi_n$.

![Fig. 1 Time evolution of the first Fourier modes for P.I.C. a) and Vlasov2 b) simulations](image-url)

In the corresponding simulations we have observed the growth and the nonlinear saturation of the ion acoustic instability. The growth rates for the various Fourier modes supported by
the box have been obtained from their time evolution (fig. 1) and the main results are summarized in the following table:

<table>
<thead>
<tr>
<th>growth rates</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$\gamma_4$</th>
<th>$\gamma_5$</th>
<th>$\gamma_{n&gt;5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear theory</td>
<td>0.081</td>
<td>0.101</td>
<td>0.103</td>
<td>0.048</td>
<td>= 0</td>
</tr>
<tr>
<td>P.I.C.</td>
<td>0.079</td>
<td>0.100</td>
<td>0.102</td>
<td>0.044</td>
<td>$\approx 0.2$</td>
</tr>
<tr>
<td>Vlasov1</td>
<td>0.081</td>
<td>0.101</td>
<td>0.18</td>
<td>0.19</td>
<td>= 0.2</td>
</tr>
<tr>
<td>Vlasov2</td>
<td>0.081</td>
<td>0.101</td>
<td>0.103</td>
<td>0.044</td>
<td>= 0.2</td>
</tr>
</tbody>
</table>

It is seen that in Vlasov simulations only the growth rates of the modes initially excited are in agreement with the predictions of the linear theory. Now the number of vortices observed in the early saturated regime is fixed by the dominant mode. Thus, four vortices are observed in the P.I.C. (see fig. 2) and in the Vlasov2 simulations, instead of three in the Vlasov1 simulation. Nonetheless the modes not present in the initial perturbation are excited by nonlinear coupling processes taking place before the saturation is reached. Thus higher growth rates are observed (see table). The great efficiency of this mode coupling process can be explained by the nearly linear $\omega-k$ dependance for the ion beam-plasma system, so that the resonance condition is satisfied for the excited modes.

Finally it appears that the actual evolution of the system depends strongly on the choice of the initial perturbation.

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**Fig. 2** Phase-space plot showing the formation of vortices (P.I.C. simulation)

**Fig. 3** Phase-space plot showing detrapped particles above the separatrix

**Fig. 4** Time evolution of $E_z$ showing large amplitude side-bands
Finally the excellent phase-space resolution of the Vlasov simulation code enables to study the long time behaviour of the trapped particles. It has been observed (fig. 3 and 4) as previously reported for the electron beam-plasma instability [6] that growing side-bands can destroy the trapped particle oscillations which follow the saturation of the ion acoustic instability. The phase velocity of the side-bands oscillations is in agreement with the velocity of detrapped particles.

4 CONCLUSION

A one-dimensional eulerian Vlasov simulation code has been developed to model ion beam-plasma systems. The simulations are performed using spatial boundary conditions. The results obtained on unstable ion beam-plasma systems exhibit the ion phase-space vortices formation previously observed in P.I.C. simulations. Moreover careful comparisons of the Fourier modes in the two approaches have been done. This allows to point out the crucial importance of the choice of the initial perturbation, particularly in the case of linearly unstable systems. Indeed, in the case of P.I.C. simulations, unstable modes grow spontaneously from the numerical noise, while Vlasov simulations are noiseless and then require some initial perturbation. When this initial perturbation is distributed among the different modes according to the thermal levels in a maxwellian plasma the results obtained with Vlasov simulations are in close agreement with those of P.I.C. simulations. However the Vlasov code displays more complex phase-space correlations and permit to observe the long time behaviour of trapped particle structures. We thus have observed that the trapped particle oscillations which follow the saturation of the ion acoustic instability can be destroyed by growing side-bands.

Improvements of this code are under way and will permit to study the long time behaviour of more realistic non periodic ion beam-plasma systems.

Références:
An Eulerian Code for the Study of the Gyro-Averaged Kinetic Vlasov Equation

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1 Introduction

Strong magnetic fields are commonly used for plasma confinement in large devices. In many of these devices electric field $\vec{E}$ exists which results in an $\vec{E} \times \vec{B}$ drift. Any inhomogeneity in these electric fields results in a shear in the plasma flow and can lead to the appearance of low-frequency instabilities. The basic equation to describe such a system is the gyro-averaged kinetic Vlasov equation which couples the $\vec{E} \times \vec{B}$ motion across a magnetic field to the motion parallel to the magnetic field. We also note the recent interest in ion-temperature-gradient instabilities, also known as $\eta$ modes [1], coupled with $\vec{E} \times \vec{B}$ shear. An important problem in the study of these instabilities is their small growth rate and the relatively low saturation level of their potential, which makes their simulation using particle codes difficult [1]. An obvious way to improve the understanding of these instabilities is to use an eulerian Vlasov model where, unlike particles in cell codes, we have a good description of vortex structures and phase space resolution and very low numerical noise.

2 The drift-kinetic Vlasov model

2.1 The basic equations and the numerical code

The basic equations that govern the plasma dynamics in the case of a uniform magnetic field $\vec{B} = (B_x, 0, B_z)$ tilted in the $xz$ plane have been previously presented [2]. We use the guiding center drift approximation. The electron and ion velocity can be written in the following form

$$\vec{v}_{e,i} = \vec{v}_{\parallel e,i} + \vec{a} \quad \text{with} \quad \vec{a} = \frac{\vec{E} \times \vec{B}}{B^2}$$

(1)

The drift-kinetic Vlasov equation is written:

$$\frac{\partial f_{e,i}}{\partial t} + \vec{v}_{\parallel} \cdot \frac{\partial f_{e,i}}{\partial \vec{r}_{\parallel}} + \frac{\vec{E} \times \vec{B}}{B^2} \cdot \frac{\partial f_{e,i}}{\partial \vec{a}} + \frac{e}{m_{e,i}} \vec{E}_{\parallel} \cdot \frac{\partial f_{e,i}}{\partial v_{\parallel}} = 0$$

(2)

The electron and ion distributions functions are reduced to a 3D phase space function $f_{e,i} (x, y, v_{\parallel e,i})$ and $f_{i} (x, y, v_{\parallel})$ where $v_{\parallel}$ is the velocity variable along the magnetic field. We assume periodic boundary condition in the $x$-direction and finite-boundary in $y$-direction. Thus we perform the numerical experiment in a rectangular domain with $0 \leq x \leq L_x$ and $-L_y \leq y \leq L_y$. 

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The electric field are then given by:

$$\vec{E} = -\nabla \varphi$$

where the electric potential $\varphi$ obeys Poisson’s equation:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{e}{\varepsilon_o} [n_e(x,y,t) - n_i(x,y,t)]$$

where $n_e$ and $n_i$ denote respectively the electron and ion density given by

$$n_{ei}(x,y,t) = \int f_{ei}(x,y,v|_{ei}) \, dv|_{ei}$$

The numerical integration of Eq. 2 is performed by using a splitting scheme in which we separate the integration in both directions successively according to the sequence of operators $\hat{X}, \hat{Y}, \hat{V}_\parallel, \hat{V}_\perp$ where $\hat{X}, \hat{Y}$ denote the shift in $x$, $y$, and $V_\parallel$ direction for both particle species, and over a full time step $\Delta t$. (The term 1/2 denotes that the shift is effected over a half time step) [2]. The shifts are calculated using a cubic spline interpolation. This sequence allows us to compute the electron distribution function $f_e$ (or $f_i$ in a similar way) at each grid points at time $t_n+1$ from the known values of $f_e$ (or $f_i$), and this scheme is correct in the second order in $\Delta t$.

3 The ion temperature gradient instabilities

These are basically very low frequency electrostatic waves, which can become destabilized when the parameter $\eta_i = d(lnT_i)/d(lnn_i)$ exceeds a critical value. The Vlasov code presents a powerful tool for the investigation of these instabilities, because of their very low noise levels. We present in this section the result of a simulation with the following initial equilibrium profiles:

$$T_i = T_{io} \left(0.2 + 0.8 \times e^{-\beta y^2}\right) \quad \text{with} \quad T_{io} = 1$$

$$n_o = \sqrt{\frac{\alpha}{\pi}} \, e^{-\alpha y^2}$$

with $\alpha = 0.025$, $\beta = 7\alpha$. This corresponds to a maximum value $\eta_i = 5.2$.

The initial ion distribution function is then given by

$$f_i (x,y,v|_{i}, t = 0) = \frac{n_o}{\sqrt{2\pi T_i}} e^{-\frac{v^2}{2T_i}} \left(1 + \varepsilon \sin k_ox + \varepsilon \sin 2k_ox + \varepsilon \sin 3k_ox\right)$$

where we have introduced a perturbation term with $\varepsilon = 0.001$ to start up the instability. In this simulation the space variables $x$ and $y$ are normalized to $\rho_s = C_s/\omega_{pi}$ where $C_s$ is the ion acoustic velocity, the velocity $v_\parallel$ is normalized to $C_s$ and the time is normalized to the inverse ion plasma frequency $\omega_{pi}^{-1}$. The electrons are taken to follow an adiabatic law [1]:

$$n_e(y) = n_o(y) \left(1 + \frac{\varepsilon \phi}{T_s}\right)$$
We take $T_e = T_{io}$ and we assume at each time step $\Delta t$ the quasi-neutrality of plasma $< n_e > = < n_i >$ by computing the value of the potential constant. We note that the ion gyroradius effects are neglected in the present drift kinetic approximation. (Work is in progress to include the gyro-averaging effect in the previous equations.) The simulation was effected with $\omega_{ci}/\omega_{pi} = 0.50$. Figure 1 shows on a logarithmic scale the growth and saturation of the potential (growth rate $\frac{1}{\omega_{pi}} = 7 \times 10^{-4}$) at the center of the plasma for the mode $k_0 = \frac{2\pi}{L}$. Contours plots of the potential at a time $t\omega_{pi} = 9100$ and $t\omega_{pi} = 9500$ at saturation are shown in Figs. 2 and 3. They show vortices traveling at what appears to be the phase velocity of the wave. Figure 4 and figure 5 show the corresponding $E \times B$ flow at the same time.

The present results are effected with a time step of $\Delta t\omega_{pi} = 1$ and we use a mesh of $N_x N_y N_{eq} = 64 \times 128 \times 32$ which is equivalent to 262144 particles. This simulation took about 4 hours (CPU time) on the CRAY - 2 computer for $10^4$ time steps. The present results show how the solutions obtained from a Vlasov code are noiseless, and an instability with weak growth rate is described until saturation.

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**References**


RELATIVISTIC VLASOV-MAXWELL EQUILIBRIA OF ANISOTROPIC CHARGE-SEPARATION-FREE PLASMAS

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Introduction

The behavior of laboratory plasma containment devices is often modelled by so-called magnetic solutions of the Vlasov-Maxwell system (describing a collisionless plasma). For these magnetic solutions charge neutrality prevails at all times implying that electric fields are negligible or even completely absent. In the present contribution we investigate why charge neutrality prevails in a confined plasma and under which conditions the charge-separation electric field is indeed negligible. More specifically we demonstrate that with fully-relativistic anisotropic plasmas, equilibria can be realized only if the distribution functions of the electrons and ions are related to each other in a special manner provided that their dependence on the constants of motion is separable. This relation is non-local in momentum space and reduces to a local one in the non-relativistic limit. Furthermore, as an example we analyze a relativistic model for the plasma-magnetic field boundary and demonstrate how to choose distribution functions that lead to neutral steady states in which electric fields are completely absent.

Basics

Consider a one-dimensional static configuration that consists of a plasma, which is inhomogeneous in the $x$-direction and is permeated by a static magnetic field in the $z$-direction (laboratory-frame). In this case the electric and the magnetic fields can be derived from the potentials $V$ and $A$ respectively, in the following manner:

$$
\vec{E}_0 = -\frac{dV}{dx} \hat{e}_x, \quad \vec{B}_0 = \frac{dA}{dx} \hat{e}_z.
$$

(1)

The relativistic Vlasov-Maxwell system for this stationary situation is given by

$$
\frac{c^2 \rho_x}{\gamma} \frac{\partial f_s}{\partial x} - \frac{e}{m_e c} \left( \vec{E}_0 + \frac{c \vec{B}_0}{\gamma} \right) \cdot \nabla \rho f_s = 0, \quad (2a)
$$

$$
\frac{c^2 \rho_x}{\gamma} \frac{\partial f_i}{\partial x} + \frac{\gamma_i}{m_i c} \left( \vec{E}_0 + \frac{c \vec{B}_0}{\gamma} \right) \cdot \nabla \rho f_i = 0, \quad (2b)
$$

$$
-\frac{d^2 A}{dx^2} = \mu_0 (J_x + J_z), \quad (3a)
$$

$$
-\frac{d^2 V}{dx^2} = \frac{1}{\epsilon_0} (q_i N_i - e N_e). \quad (3b)
$$

Here, $f_s$ and $\vec{J}_s$ denote the distribution function and the current density respectively, of particles of species $s$ and

$$
\vec{\rho} = \frac{\gamma}{c} \vec{\varrho}, \quad \gamma(\rho) = \sqrt{1 + \rho^2}. \quad (4)
$$

The other symbols have their usual meaning. The relation between the particle density $N_s$ and $\vec{J}_s = J_{sy} \hat{e}_y$ on the one hand and $f_s$ on the other hand is given by

$$
N_s(x) = N_{s0} \int d^3 \rho \; f_s(x, \rho), \quad J_{sy}(x) = N_{s0} g_{sc} \int d^3 \rho \frac{p_y}{\gamma(\rho)} f_s(x, \rho), \quad (5)
$$
where \( N_{s0} \) is the density of particles of species \( s \) when there are no fields. For the equilibrium distribution functions \( f_s \) we choose the following functions of the constants of motion that resemble relativistic Maxwellians (and that coincide with them when \( A = 0 \)):

\[
\frac{\mu_s}{4\pi K_2(\mu_s)} \exp\left(\frac{\mu_s H_s}{m_e c^2}\right) \times h_s(\text{sgn}(q_s) P_{s_y}),
\]

where the Hamiltonians \( H_s \) and the canonical momenta in the \( y \)-direction \( P_{s_y} \) are given by

\[
H_s = H_s(\rho, x) = (\sqrt{\rho^2 + 1} - 1) m_e c^2 + q_s V(x),
\]

\[
P_{s_y} = P_{s_y}(\rho, x) = \rho_y + \frac{q_s}{m_e c} A(x)
\]

and where \( \mu_s = m_e c^2 / k_B T_s \). \( K_2 \) is the modified Bessel function of the second kind and of order 2. The functions \( h_s \) are meant to model the departure from global thermodynamic equilibrium (if one ignores the gyro-radiation field) due to the presence of the inhomogeneous (confining) magnetic field.

**Analysis**

If we introduce the kernel function

\[
K_s(\rho_y) = \frac{2\pi}{K_2(\mu_s)} \left( \sqrt{1 + \rho_y^2} + \frac{1}{\mu_s} \right) \exp\left( -\mu_s \sqrt{1 + \rho_y^2} \right)
\]

and the parameters

\[
Z = \frac{q_t}{c}, \quad \alpha = \frac{T_i}{T_e}, \quad \beta = \frac{m_i}{m_e},
\]

then, from (5), (6) and (7) we can deduce

\[
N_e(x) = N_{e0} \exp\left\{ \Phi_e(x) \right\} g_e(\Psi(x)), \quad N_i(x) = N_{i0} \exp\left\{ -\frac{Z}{\alpha} \Phi_i(x) \right\} g_i(\Psi(x)),
\]

\[
J_{ey}(x) = \frac{N_{e0} e c}{\mu_e} \exp\left\{ \Phi_e(x) \right\} g'_e(\Psi(x)), \quad J_{ix}(x) = \frac{\beta N_{i0} e c}{\mu_i} \exp\left\{ -\frac{Z}{\alpha} \Phi_i(x) \right\} g'_i(\Psi(x)),
\]

where the functions \( g_e \) and \( g_i \) are defined by

\[
g_e(\Psi(x)) = \int_{-\infty}^{\infty} d\rho_y \ K_e(\rho_y) h_e(\Psi(x) - \rho_y),
\]

\[
g_i(\Psi(x)) = \int_{-\infty}^{\infty} d\rho_y \ K_i(\rho_y) h_i\left\{ \frac{Z}{\beta} \Psi(x) + \rho_y \right\}
\]

and \( \Phi_e \) and \( \Psi \) are dimensionless potentials according to

\[
\Phi_e(x) = \frac{e V(x)}{k_B T_e}, \quad \Psi(x) = \frac{e A(x)}{m_e c}.
\]

The primes in equations (11) denote differentiation with respect to \( \Psi \). Finally upon introducing

\[
\Phi(x) = \frac{1}{2} \left( 1 + \frac{Z}{\alpha} \right) \Phi_e(x), \quad \delta = \frac{Z - \alpha}{Z + \alpha}, \quad \bar{g} = \frac{g_e + g_i}{2}, \quad g_d = \frac{g_e - g_i}{2},
\]

(14)
Maxwell’s equations (3) may be written as follows:

\[
\frac{d^2 \Phi}{d \xi^2} = \left(1 + \frac{Z}{\alpha}\right) \mu_e \exp\{-\delta \Phi(\xi)\} \left[\sinh\{\Phi(\xi)\} \tilde{g}\{\Psi(\xi)\} + \cosh\{\Phi(\xi)\} g_d\{\Psi(\xi)\}\right], \quad (15a)
\]

\[
\frac{d^2 \Psi}{d \xi^2} = -\frac{2}{\mu_e} \sqrt{\frac{\alpha}{Z}} \exp\{-\delta \Phi(\xi)\} \left[\cosh\{\Phi(\xi) + \phi_\delta\} \tilde{g}'\{\Psi(\xi)\} + \sinh\{\Phi(\xi) + \phi_\delta\} g_d'\{\Psi(\xi)\}\right], \quad (15b)
\]

where

\[\phi_\delta = \text{arctanh} (\delta), \quad \xi = \left\{\frac{\mu_0 N_{ee} e^2}{m_e} \right\} \right.\]

(16)

In the electrostatic situation (\(\Psi \equiv 0\)) the functions \(h_e\) and \(h_i\) become identically one. The equations (15a) and (15b) then reduce to a single equation for \(\Phi\) only, i.e.

\[
\frac{d^2 \Phi}{d \xi^2} = \left(1 + \frac{Z}{\alpha}\right) \mu_e \exp\{-\delta \Phi(\xi)\} \sinh\{\Phi(\xi)\}.
\]

(17)

This situation essentially reviews the solutions of the pure Vlasov-Poisson problem for the Ansatz (6) (a homogeneous magnetic field may still be present). The coupled second-order equations (15a) and (15b) may be combined into a single differential equation of fourth order. The highest derivative of this fourth-order equation is multiplied by \(1/\mu_e\). For the fusion-relevant case of a weakly-relativistic plasma, \(1/\mu_e \ll 1\). This means that (15) is a candidate for boundary-layer theory. A boundary layer is a narrow region where the solution of (15) changes far more rapidly than outside of it. By definition, the thickness of such a boundary layer must approach zero as \(\mu_e \to \infty\). Outside these isolated regions of rapid variations it is justified to neglect the second derivative of the potential \(\Phi\) (divided by \(\mu_e\)) in (15a) which in fact tantamount to the commonly employed quasi-neutrality approximation. Thus we arrive at the conclusion that the quasi-neutrality assumption is closely related to the non-relativistic limit relevant to fusion plasmas.

In the absence of an electric potential (\(\Psi \equiv 0\)) the plasma must be locally neutral. In that case equation (15a) is necessarily trivial which leads to the condition that \(g_d = 0\). So the functions \(h_e\) and \(h_i\) must satisfy the equation

\[
\int \int \rho_y K_e (\rho_y) h_e \{\Psi(x) - \rho_y\} \, d\rho_y = \int \int \rho_y K_i (\rho_y) h_i \left\{\frac{Z}{\beta} \Psi(x) + \rho_y\right\}. \quad (18)
\]

The integrals in this expression are of the convolution type. This fact offers the possibility to obtain the following integral relation between the functions \(h_e\) and \(h_i\) by employing the Fourier transform:

\[
h_e (\Psi) = \int \int d y K(y) h_i \left\{\frac{Z}{\beta} (\Psi - y)\right\}, \quad (19)
\]

where

\[
K(y) = \frac{\mu_0^2 K_2 (\mu_e)}{\mu_e^2 K_2 (\mu_i)} \sqrt{\frac{2}{\n}} \int \int \, dk \cos (k y) \frac{K_2 (\sqrt{\beta^2 Z^2 / k^2 + \mu_0^2})}{K_2 (\sqrt{k^2 + \mu_e^2})}. \quad (20)
\]
In the non-relativistic limit \( (c \to \infty) \) \( K(y) \) reduces to the Dirac delta function and relation (19) then becomes a local one in momentum space.

**Application**

The previous results may be employed to simulate various relativistic equilibria in which the plasma is everywhere exactly neutral \( (\Phi = 0) \); deviations from exact neutrality due to finite gyro-radius effects are ignored. More specifically we will now use our approach to construct a fully-relativistic steady-state model for the plasma-magnetic field boundary in which the plasma is free of charge separation and becomes relativistic Maxwellian for \( |z| \to \infty \) where the confining magnetic field becomes homogeneous. To that end we prescribe \( \tilde{g}(\psi(\xi)) \) as follows:

\[
\frac{d\tilde{g}}{d\psi} = \frac{1}{\cosh^2(\xi)}. \tag{21}
\]

Since we assume exact neutrality everywhere in the plasma, equation (15b) reduces to

\[
\frac{d^2\psi}{d\xi^2} = -\frac{2}{\mu_e} \sqrt{\frac{\alpha}{Z}} \cosh(\phi_\xi) \frac{d\tilde{g}}{d\psi}, \tag{22}
\]

from which we easily deduce

\[
\psi(\xi) = C\xi - \frac{2}{\mu_e} \sqrt{\frac{\alpha}{Z}} \cosh(\phi_\xi) \log(\cosh(\xi)), \tag{23}
\]

where \( C \) is an arbitrary integration constant to be determined by the boundary conditions. The function \( \tilde{g} \) (and herewith the particle density profile \( N = N_e = N_i \)) is established by using (21) and (22) and integrating. This yields

\[
\tilde{g}(\xi) = \int_{-\infty}^{\xi} d\eta \frac{C\mu_e - 2\sqrt{\alpha/Z} \cosh(\phi_\xi) \tanh(\eta)}{\mu_e \cosh^2(\eta)} = C \tanh(\xi) + \frac{\sqrt{\alpha/Z} \cosh(\phi_\xi)}{\mu_e \cosh^2(\xi)} + C. \tag{24}
\]

The Fourier transform of this expression and of the convolution integrals (12a) and (12b) finally leads to the following result for the function \( h_i \):

\[
h_i\left( \frac{Z}{\beta} \psi \right) = -\frac{4\mu_i^2}{ZK_2(\mu_i)} \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{dk}{k} \sin(k\xi) \exp\left\{ ik\sqrt{\frac{\alpha}{Z}} \cosh(\phi_\xi) \log(4)/\mu_e \right\} \frac{K_2(\sqrt{\frac{\beta^2 k^2}{Z^2} + \mu_i^2})}{K_2(\sqrt{\frac{\beta^2 k^2}{Z^2} + \mu_i^2})} \times \left( \frac{\beta^2 k^2}{Z^2} + \mu_i^2 \right) B \left\{ 1 + ik \left( \frac{\sqrt{\alpha/Z}}{\mu_e} \cosh(\phi_\xi) + \frac{C}{2} \right), 1 + ik \left( \frac{\sqrt{\alpha/Z}}{\mu_e} \cosh(\phi_\xi) - \frac{C}{2} \right) \right\}, \tag{25}
\]

where \( B \) is the beta function. Finally we note that \( h_i \) is found with (19) after which the particle and current densities may be found with (10), (11) and (12).

Note that the steady-state model of the plasma-magnetic field boundary considered above is by no way unique. Different choices of \( \tilde{g} \) in (21) are possible. Moreover the problem at hand is non-unique because of the presence of trapped particles somewhere in the plasma, which could never reach the relativistic Maxwellian part of it.

For further details and references see /1/ where we treated the special case of \( Z = 1 = \alpha \).

A NEW ACTION PRINCIPLE FOR THE VLASOV–MAXWELL SYSTEM

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Abstract: Everyone interested in action principles for the Vlasov–Maxwell system has probably noticed the difficulty to use the particle distribution function as a field to be independantly varied. A solution to this old problem is presented in this paper.

1. Introduction

The purpose of this paper is to present new action principles for the VP equation (i.e. the Vlasov–Poisson equation) and the VM system (i.e. the Vlasov–Maxwell system). These results will be useful for performing standard reduction procedures (like linearization, geometric optics, gyro-kinetics) in a manifestely Hamiltonian structure preserving way. In this short paper we will however concentrate on the relations to previous theory on the Lagrangian–Hamiltonian structure of these equations and more details will be published elsewhere.1

The VP equation and the VM system have the structure of a Lagrangian field theory found by Low (1958).2 The corresponding Hamiltonian structure follows in the usual way using the Legendre transformation. We then have field theories in the material (i.e. Lagrangian) coordinaters such that the solutions monitors the motion of each particle. One would like use Eulerian coordinates instead so that all this redundant information is eliminated and thus the particle distribution function give all information about the plasma particles. The needed reduction procedures make necessary generalizations of standard Hamiltonian mechanics. It is instructive to consider the relation to standard formulations of classical mechanics. We have

(a) Lagrangian action principle: \( \delta I(q) = 0, \ I(q) = \int L(q, q') \, dt \)
(b) Hamilton's canonical equations: \( \dot{q} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial q} \)

(c) Poisson-bracket formulation: \( \frac{dA(q,p)}{dt} = \{A,H\}(q,p) \)

(d) Hamiltonian action principle: \( \delta I(q,p) = 0, I(q,p) = \int [p \cdot \dot{q} - H(q,p)] \, dt \)

This reduction program have been carried out on the level (c) for the VP equation and the VM system.\(^3\,4\,5\) The necessary generalization to noncanonical field variables results in new Poisson-brackets which still possess the usual algebraic properties including the Jacobi identity. The corresponding results on the other levels, i.e. (a), (b) and (d), seems to be lacking. An underlying reason for the difficulties is the singular structure of the non-canonical Poisson brackets (i.e. the existence of Casimirs).\(^5\)

A basic novel feature underlying the new theory is that we consider the dynamics of extended systems so that the VP equation or the VM system are "embedded" in larger systems. The results are generalizations on the levels (b) and (d).

2. The Vlasov-Poisson equation

For notational brevity we now consider the VP equation. Let \( f = f(t,E,Y) \) denote the particle distribution function. As discussed in the Introduction there is no known action principle \( I(f) \) such that \( \delta I(f) = 0 \) produces the VP equation. A starting point for finding the new action principle presented below was the idea that it might be possible to find an phase-space action principle of the form \( I(f,S) \) where the function \( S = S(t,E,Y) \) is a canonically conjugate field to \( f \). This approach was further modified by the introduction of a continuous parameter \( \epsilon, 0 < \epsilon < 1 \), such that \( \epsilon = 0 \) corresponds to a reference state of the particles and \( \epsilon = 1 \) to the true physical state. We then consider functions \( \tilde{f} = f(\epsilon,t,E,Y) \) and \( \tilde{S} = S(\epsilon,t,E,Y) \) and an action principle \( I(\tilde{f},\tilde{S}) \). The function \( \tilde{S} \) is the Hamiltonian for the \( \epsilon \) motion of the particles from
their reference position in \((r, y)\) space to their physical position in this space. The new action principle is

\[
I(f, S) = \int \left[ \int \int \varpi \, \partial_t \frac{\partial}{\partial \varpi} f \, d^3 r \, d^3 v - H(f, S) \right] \, dt
\]

where

\[
H(f, S) = -\int \frac{1}{2} \nabla^2 \{f, S\}_r \, d^3 r \, d^3 v + \frac{1}{2} \int [\partial_t \phi(f, S)]^2 \, d^3 r
\]

with \(\{ , \}_{r/v}\) as the Poisson-bracket for the plasma particle motion in \((r, y)\)-coordinates and with \(\phi(f, S)\) defined from

\[
\partial_t^2 \phi(f, S) = -\int \tilde{f}(\varpi=1) \, d^3 v + \int \varepsilon \, (\partial_t \tilde{f} + \{f, S\}) \, d^3 v
\]

The Euler-Lagrange equations from (1) are Hamilton's field equations

\[
\partial_t \tilde{f} = -\delta H/\delta \tilde{S}, \quad \partial_t \tilde{S} = \delta H/\delta \tilde{f}
\]

The following diagram displays the relation between (4) and the VP equation:

\[
\begin{array}{ccc}
f(t_0) & \xrightarrow{\text{VP equation}} & t \sim f(t) \\
\downarrow \tilde{S}(t_0) \quad & & \uparrow f(\varepsilon=1) \\
\downarrow [\tilde{f}(t_0), \tilde{S}(t_0)] & \xrightarrow{\text{eq. (4)}} & t \sim [\tilde{f}(t), \tilde{S}(t)]
\end{array}
\]
In the upper left of the diagram an initial condition for the VP equation is given. The upper arrow towards the right shows that the VP equation together with the initial condition determine a solution. The arrow pointing downwards indicate that a corresponding initial condition $\tilde{f}(t_0), \tilde{S}(t_0)$ for (4) may be found for an arbitrary choice of $\tilde{S}(t_0): \tilde{f}(t_0)$ is determined by the requirements (a) $\tilde{f}(t=t_0) = f(t_0)$ and (b) $\frac{\partial}{\partial t} \tilde{f}(t_0) + \{\tilde{f}(t_0), \tilde{S}(t_0)\}_r = 0$. The lower arrow towards the right shows that eq. (4) have a unique solution with this initial condition. Finally by the upward arrow in the diagram we again get the solution to the VP equation.

3. The Vlasov-Maxwell system

A corresponding action principle for the VM system is

$$I(\tilde{f}^c, \tilde{S}^c, E, A) = \int \left[ \int \tilde{f}^c \tilde{S}^c \, d^3 p + E \cdot \partial_t A \right] \, d^3 r - H(f^c, S^c, E, A) \] \, dt$$

where the superscript $c$ indicates the use of canonical coordinates and

$$H(f^c, S^c, E, A) = \int \left[ - f^c \tilde{H}^c \right]_{rp} + f^c \tilde{H}^c \] \, d^3 r \, d^3 p - \int \left[ \frac{1}{2} (E^2 + B^2) \right] \, d^3 r$$

where $\tilde{H}^c(\epsilon, t, E, B) = \frac{1}{2} [E - \epsilon A(t, E)]^2$ and $B = \partial_r \times A$ (but not $E = - \partial_t A$, since this relation is now obtained as a dynamical equation).

References
I. Introduction

Considering arbitrary perturbations of general Vlasov-Maxwell equilibria, Morrison and Pfirsch [1, 2] derived expressions for the second variation of the free energy and concluded that negative-energy perturbations (which are potentially dangerous because they may become nonlinearly unstable and cause anomalous transport [4, 3]) exist in any Vlasov-Maxwell equilibrium whenever the unperturbed distribution function $f^{(0)}$ of any particle species $\nu$ deviates from monotonicity and/or isotropy in the vicinity of a single point, i.e. whenever the condition $(v \cdot k) \left( k \cdot \frac{\partial f^{(0)}}{\partial v} \right) > 0$ holds for any particle species $\nu$ for some position vector $x$ and velocity $v$ and for some vector $k$. The proof of this result is based on infinitely strongly localized perturbations. This raises the question of the degree of localization actually required for negative-energy modes to exist in a certain equilibrium. Studying a homogeneous Vlasov-Maxwell plasma with constant magnetic field, Correa-Restrepo and Pfirsch [5] showed that negative-energy waves exist for $v \cdot y$ deviation of the equilibrium distribution function of any of the species from monotonicity and/or isotropy, without having to impose any restricting conditions on the perpendicular wave number $k_\perp$, i.e. without requiring large $k_\perp$. These investigations are extended in the present paper to the more interesting case of an inhomogeneous, force-free equilibrium with a sheared, $y$-dependent magnetic field. The method of investigation consists in evaluating the general expression for the second-order wave energy derived by Morrison and Pfirsch [1, 2] in the form given by Correa-Restrepo and Pfirsch [5]. Although the calculations are considerably more involved than in the case of the homogeneous magnetic field, substantial simplification of the problem is achieved by the introduction of appropriate coordinates in $v - y$ space and by a convenient representation of the perturbations. It is concluded that negative-energy modes exist in the inhomogeneous configuration as well whenever any of the equilibrium distribution functions deviates from monotonicity and/or isotropy (in fact, owing to the inhomogeneity, the equilibrium distribution function for at least one particle species must be anisotropic!), and that large perpendicular wave numbers are not required in this case either. If there is only anisotropy, the presence of shear merely requires that the perturbations have a characteristic variation.
length in the direction perpendicular to the equilibrium magnetic field $\mathbf{B}^{(0)}$ smaller than (or of the order of) the shear length $a^{-1}$, which is not an important restriction, i.e. negative-energy modes persist without any major modification in the presence of shear, a feature which enhances their importance.

II. Equilibrium Electromagnetic Field and Distribution Functions

It is assumed that there is no equilibrium electric field, and that the equilibrium magnetic field has constant magnitude and straight field lines which twist as one proceeds in a given direction. In Cartesian coordinates $x, y, z$ with unit basis $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$, the equilibrium magnetic field is given by $\mathbf{B}^{(0)} = B^{(0)} \mathbf{e}_B = B^{(0)} (\sin \alpha \mathbf{e}_x + \cos \alpha \mathbf{e}_z)$. In this case, there is an electric current parallel to the magnetic field, and the charged particles of any species belong (according to the values of their constants of the motion) to either one of two essentially different groups, either to the group of gyrating particles (the overwhelming majority in all cases of interest), which move around the field lines, their motion being confined to a certain $y$-region around $y = Y_0$, where $Y_0$ is a constant of the motion, or to the group of swinging particles, which move freely in the $y$-direction. The two groups of particles must be investigated separately. Owing to the presence of the electric current associated with nonvanishing $a$, the equilibrium distribution function of at least one particle species $\nu$ must be anisotropic, unlike in the homogeneous case. For the inhomogeneous, force-free equilibrium, the distribution functions for particles of charge $e_\nu$ and mass $m_\nu$ are of the form $f^{(0)}_\nu = \frac{1}{m_\nu} \left( \frac{e_\nu}{2} (\dot{z}^2 + \dot{y}^2 + \dot{z}^2) \right), \mathcal{U}_\nu = \frac{a}{2\omega_\nu} (\dot{y}^2 + \dot{z} \sin \alpha \dot{y} + \dot{z} \cos \alpha \dot{y})$, where $\omega_\nu \equiv \frac{e_\nu B^{(0)}}{m_\nu c}$. The energy $\mathcal{H}_\nu$ and the velocity $\mathcal{U}_\nu$ are constants of the motion.

III. Second-Order Wave Energy

Within the framework of Maxwell-Vlasov theory, Morrison and Pfirsch [1, 2] derived expressions for the free energy $\delta^2 H$ available upon arbitrary perturbations of an arbitrary equilibrium. For electrostatic perturbations with $\delta \mathbf{B} = 0$, and in the absence of an equilibrium electric field, $\delta^2 H$ can be expressed as [5]

$$
\delta^2 H = \sum_\nu \int \frac{d^3x \, d\mathbf{v}}{2m_\nu} \left[ \frac{\partial f^{(0)}_\nu}{\partial \mathbf{v}} \cdot \left\{ - \left( \mathbf{v} \cdot \frac{\partial G_\nu}{\partial \mathbf{x}} \right) \frac{\partial G_\nu}{\partial \mathbf{x}} - \left( \mathbf{a}_\nu^{(0)} \cdot \frac{\partial G_\nu}{\partial \mathbf{v}} \right) \right\} \right.
\left. + \frac{\partial f^{(0)}_\nu}{\partial \mathbf{x}} \cdot \left\{ - \left( \frac{\partial G_\nu}{\partial \mathbf{x}} \cdot \frac{\partial G_\nu}{\partial \mathbf{v}} \right) \mathbf{v} + (d_\nu G_\nu) \frac{\partial G_\nu}{\partial \mathbf{v}} \right\} \right] + \frac{1}{8\pi} \int d^3x (\delta E^2),
$$

where $G_\nu(x, \mathbf{v})$ is a generating function for the perturbation of the particle position and velocity, and $\delta E^2/8\pi$ is the perturbation in the electric energy density. The operator $d_\nu$ is the equilibrium Vlasov operator, i.e. $d_\nu = \frac{\partial}{\partial \mathbf{x}} + a_\nu^{(0)} \cdot \frac{\partial}{\partial \mathbf{v}}, a_\nu^{(0)} = \frac{e_\nu}{m_\nu c} \mathbf{v} \times \mathbf{B}^{(0)}$. 

(in the absence of an equilibrium electric field). Since the equilibrium is independent of \( x \) and \( z \), an appropriate ansatz for the generating function \( G_\nu(x, v) \) is
\[
G_\nu(x, v) = \frac{1}{i} \left[ g_\nu(y, v) e^{i k_{xz} \cdot x} + g_\nu^*(y, v) e^{-i k_{xz} \cdot x} \right],
\]
with \( k_{xz} = k_x e_x + k_z e_z \). Setting \( g_\nu(y, v) = \Psi_\nu(y, v) e^{i \Gamma_\nu(y, v)} \), where \( \Psi_\nu \) and \( \Gamma_\nu \) are real functions, \( d_\nu = \dot{d}_\nu + \dot{z} \frac{\partial}{\partial x} + \ddot{z} \frac{\partial}{\partial z} \), \( \mathcal{D}_{f_\nu}^{(0)} = \frac{\partial f_\nu^{(0)}}{\partial \mathcal{H}_\nu} + \frac{a}{\omega_\nu} \frac{\partial}{\partial \mathcal{H}_\nu} \frac{\partial f_\nu^{(0)}}{\partial y^2} \), \( w = v - y e_v, \ s = 4 \pi^2 / k_x k_z \), the second-order wave energy can be expressed as
\[
\delta^2 H = \sum_\nu \frac{s}{4m_\nu} \int d^3 v \, dy \left\{ - \left[ \mathcal{D}_{f_\nu}^{(0)} \right] \left[ (\dot{d}_\nu \Psi_\nu)^2 + \Psi_\nu^2 \left[ \dot{d}_\nu \Gamma_\nu + (v \cdot k_{xz}) \right]^2 \right] + \Psi_\nu^2 \frac{\partial^2 f_\nu^{(0)}}{\partial y^2} \right\}
\]
which is the general expression for the second-order energy of electrostatic perturbations of the equilibrium considered. Note that \( \delta^2 H \) is a functional of \( \Psi_\nu \), which appears as \( \Psi_\nu \) and \( \dot{d}_\nu \Psi_\mu \) and of \( \Gamma_\nu \), which appears only as \( d_\nu \Gamma_\nu \). Taking \( \Psi_\nu \) as a function of the constants of the motion only (which can be shown to have no influence on the results), and carrying out the minimization with respect to \( \Gamma_\nu \), which is straightforward, yields
\[
\delta^2 H = - \sum_\nu \frac{s}{4m_\nu} \int d^3 v \, dy \left\{ \frac{a}{\omega_\nu} \mathcal{W}_\nu \left[ \Psi_\nu \right] \mathcal{D}_{f_\nu}^{(0)} \left[ N_\nu + a_\nu k_{||0} \cos a_\nu \right] \left[ N_\nu + b_\nu k_{||0} \cos a_\nu \right] \right\}
\]
Here, \( \omega_\nu \neq 0 \) is a certain frequency, and the angles represent mean values along the particle orbits. \( b_\nu = \langle \tilde{w}_\nu \rangle / \langle \tilde{\omega}_\nu \rangle \), with \( \langle \tilde{w}_\nu \rangle \) a mean value of a parallel velocity \( \tilde{w}_\parallel \), and \( a_\nu = \frac{1}{\langle \tilde{\omega}_\nu \rangle} \left\langle \mathcal{W}_\nu \sqrt{\mathcal{D}_{f_\nu}^{(0)}} \right\rangle \mathcal{W}_\nu \mathcal{D}_{f_\nu}^{(0)} \) \langle e_B(Y_\nu) \rangle \mathcal{D}_{f_\nu}^{(0)} \) \langle e_B(Y_\nu) \rangle \mathcal{D}_{f_\nu}^{(0)} \), with \( e_B(Y_\nu) \) a unit vector in the direction of \( \mathcal{B}^{(0)} \) at the plane \( y = \mathcal{Y}_\nu \). \( N_\nu \) is an arbitrary integer (positive or negative) for gyrating particles, and \( k_{||0} = k_z \) if one sets \( k_{xz} = k_x e_x \), which can be done without restriction. The electrostatic energy term \( (1/8\pi) \int d^3 x \delta E^2 \) has been dropped since the perturbed charge density can be made zero by appropriate choice of the signs of \( \Psi_\nu \), which do not influence \( \delta^2 H \) (3).

If \( k_{||0} = 0 \), there is wave propagation only in the \( y \)-direction, perpendicular to \( \mathcal{B}^{(0)} \). Then, \( \delta^2 H < 0 \) if \( \mathcal{D}_{f_\nu}^{(0)} = 2 \frac{\partial f_\nu^{(0)}}{\partial y^2} > 0 \) for some \( \mathcal{H}_\nu, \mathcal{U}_\nu \). It suffices to localize \( \Psi_\nu \) to the region in \( \mathcal{H}_\nu, \mathcal{Y}_\nu \) where \( \mathcal{D}_{f_\nu}^{(0)} > 0 \). Outside this region \( \Psi_\nu \) vanishes. All other \( \Psi_\mu \) are set equal to zero. The sign of \( \delta^2 H \) is determined only by the sign of the integrand in the region of localization.

If \( k_{||0} \neq 0 \) and \( \mathcal{D}_{f_\nu}^{(0)} > 0 \) for some \( \mathcal{H}_\nu, \mathcal{U}_\nu \), one again localizes \( \Psi_\nu \) around these values and sets all other \( \Psi_\mu \) equal to zero. It is easy to show that \( N_\nu \) can be chosen such that \( \delta^2 H < 0 \) without imposing any conditions on either \( k_{||0} \) or the spatial dependence of \( \Psi_\nu \) perpendicular to \( \mathcal{B}^{(0)} \). This is also the case if \( k_{||0} \neq 0 \), \( \mathcal{D}_{f_\nu}^{(0)} < 0 \) but \( \mathcal{D}_{f_\nu}^{(0)} \rangle \mathcal{W}_\nu \mathcal{D}_{f_\nu}^{(0)} \rangle > 0 \).

If \( k_{||0} \neq 0 \), and both \( \mathcal{D}_{f_\nu}^{(0)} \) and \( \mathcal{D}_{f_\nu}^{(0)} \rangle \mathcal{W}_\nu \mathcal{D}_{f_\nu}^{(0)} \rangle < 0 \), \( \delta^2 H \) can also be made negative in a way similar to that in the preceding cases, localizing \( \Psi_\nu \) in \( v \) space and exploiting the freedom in the choice of \( N_\nu \). Here, however, the possible values of \( k_{||0} \)
are limited to a certain interval (as is also the case for a homogeneous equilibrium with \( a = 0 \)), and the spatial variation of \( \Psi_\nu \) perpendicular to \( B^{(0)} \) is not completely arbitrary, but has a characteristic length \( \lesssim a^{-1} \), which is not an important restriction.

IV. Conclusions

In the case of an inhomogeneous, force-free Vlasov-Maxwell plasma with sheared magnetic field, waves of negative energy (\( \delta^2 H < 0 \)) exist for any local deviation from monotonicity (i.e. if \( Df^{(0)} := m_\nu \frac{\partial f^{(0)}}{\partial H_\nu} + \frac{a}{\omega_\nu} \frac{\partial f^{(0)}}{\partial U_\nu} - 2 \frac{\partial f^{(0)}}{\partial y^2} > 0 \) for some \( H_\nu, U_\nu \) for any wave number \( k \), irrespective of its magnitude and orientation.

If \( \frac{\partial f^{(0)}}{\partial y^2} < 0 \), only the waves with a component \( k_{\parallel 0} \) of \( k \) in the direction \( B^{(0)}(y_0) \) can possess negative energy.

If \( \frac{\partial f^{(0)}}{\partial y^2} < 0 \), but \( k_{\parallel 0} \left< \omega_\parallel \right> \left< e_B(y_0) \cdot \frac{\partial f^{(0)}}{\partial \nu} \right> > 0 \), negative-energy waves also exist, with no restriction imposed on either \( k_{\parallel 0} \) (other than \( k_{\parallel 0} \neq 0 \)) or the spatial variation of the perturbation perpendicular to \( B^{(0)} \). This result agrees with, and closely resembles, that obtained for a homogeneous plasma by Pärsch and Morrison [6], Eq.(144.b), within the frame of drift-kinetic theory.

If both \( \frac{\partial f^{(0)}}{\partial y^2} < 0 \) and \( k_{\parallel 0} \left< \omega_\parallel \right> \left< e_B(y_0) \cdot \frac{\partial f^{(0)}}{\partial \nu} \right> < 0 \), negative-energy modes also exist. In this case, the characteristic length for the variation of the perturbation \( \Psi_\nu \) perpendicular to \( B^{(0)} \) is of the order of the shear length \( a^{-1} \) (or smaller), and there is generally a restriction on the possible parallel wave numbers, which are limited to a certain interval, this also being so in the homogeneous case. The results of that case are of course regained by taking the limit of vanishing shear, \( a \to 0 \). The new results show that large perpendicular wave numbers are not necessary for the existence of negative-energy waves in the system under consideration, a feature which enhances the relevance of these modes.

References

ELECTRONS PASSING THROUGH A NEGATIVELY BIASED GRID DUE TO COLLECTIVE TRAPPING

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Grids are widely used in plasma experiments for various purposes. For example, the various plasma waves are either excited /1/ or detected /2/ by grids. Grids of different mesh size are also used as electromagnetic screen of an electron plasma, wave exciter /3/. In electrostatic energy analyzers /4/ and ion sources grids serve also for the separation of one charge carrier species from the other. In both double /5/ and triple /6/ plasma devices the grids are used in order to cut out the electron coupling between the chambers.

Usually it is assumed that a negatively biased grid reflects all electrons if two conditions are fulfilled: (i) the mesh size is smaller than Debye length and (ii) the potential of the grid (\(V_G\)) has to be enough negative so that: 

\[ |V_G| > \frac{\varepsilon}{e}, \]

where \(\varepsilon\) is the kinetic energy of the most energetic electron in the system.

In this paper, experimental results are presented which show that a considerable number of electrons is able to pass through a grid in spite of both of the above conditions being fulfilled.

The experiments were performed in both the Innsbruck /7/ and the Iasi DP-machines /8/ (Fig.1). In both cases an argon plasma was produced only in one chamber (the driver) with a density of less than \(10^5\) cm\(^{-3}\) and effective electron temperature larger than about 1.5 eV, i.e. the Debye length was
larger than about 0.7 mm. The most energetic electron was considered to have \( E = eV_a \) where \( V_a \) is the cathode potential with respect to the anode. The argon pressure was \( 10^{-4} \) mbar. The separation grids (G) between two chambers of the DP-machines have a transparency of 81% and a mesh width of 90 \( \mu m \) for Innsbruck DP-machine and a transparency of 60% and a meshwidth of 70 \( \mu m \) for Iasi DP-machine. The plasma parameters were measured using plane probes (LP) and electrostatic analyzers (EA) with two grids. Plasma density \( n \) in the driver chamber and bias of the separation grid \( (U_a) \) were used as parameters.

a) Driver chamber grounded.

In this case target chamber \( (T) \) is used as a large "collector" of an electrostatic analyzer and the grid \( (G) \) of the machine as first grid of the analyzer (fig.1). Plasma was not produced in this chamber. In fig.2a the current voltage characteristic of the "collector" is presented. As long as the "collector" is biased \( (U_a) \) negatively with respect to plasma potential of the driver chamber \( (D) \) (where \( n = 2.10^7 \text{ cm}^{-3} \); plasma potential \( V_p = 1.5 \text{ V} \); \( V_a = -55 \text{ V} \)), an ion beam is passing the G into the T chamber. When the \( U_a \) is positive with respect to the driver plasma a small electron current is still present in the T chamber. A clear hysteresis of the characteristic is present when the ion beam has to be formed. Thus, the ion beam is still flowing towards the T chamber until the \( U_a \) is close but still negative with respect to driver plasma, while this beam is set-up only then when the \( U_a \) becomes a few volts negative with respect to the driver plasma potential. The hysteresis might be related with a process of space charge forming around the grid which regulate the flux of particles between the chambers. Moreover strong oscillations of both
curtrents of the grid and of the "collector" and oscillations of plasma parameters into the driver are registered as long as the ions are reflected into the driver.

The presence of the electrons in the target chamber in spite of the negative bias of the grid ($U_g = \pm 75$ V and $< 55$ eV) but when the ion beam is there, could be proved by saturation current of the probe. Thus when the Langmuir probe is biased negative with respect to the "collector" ( $-10$ V curve b and $-0.6$ V curve c, Fig.2) the probe current ($I_p$) versus $U_g$ shows a similar dependence as that of the $I_C$ (curve a, Fig.2). The $I_p$ is an ion current which preserve also the hysteresis as the $I_C$ and practically does not depend on the probe potential. But, when the probe is biased positive with respect to the "collector" (4 V and 10 V for curves d and e, Fig.2) then constant and rather large electron current is measured as soon as ions penetrate into the target chamber but are reflected by the positive bias of the probe (limit A of the curves d and e of Fig.2). When the probe potential becomes also negative with respect to the plasma potential of the driver chamber (limit B of the curves d and e, fig.2) then the probe collects also ions. Moreover, probe characteristics registered in the target chamber for fixed $U_g$ value are presented in fig.3. They show that as long as the ion beam penetrates in the chamber, then rather large number of electrons is present (curves a and d registered for $U_g$ bias indicated in fig.2 a).

When ions are reflected into the D chamber, then probe characteristics registered in fig.3 (curves c and b) show that very few electrons are present in the T chamber. This result shows that in spite of the very negative bias of the grid ($-75$ V) comparing with the most energetic electron produced in the driver chamber ($E = 55$ eV), the electrons could pass that grid mainly when ions could also penetrate in the T chamber.

b) Target chamber grounded.

In that case the T chamber is used as a "collector" of the
electrostatic analyzer (EA) with a fixed bias. The G remains also as the first grid of the EA. By anode biasing of the D chamber with respect to the ground, an ion beam could be pushed into the T chamber as long as the potential \( V_p \) of the D plasma is positive with respect to the ground.

The probe characteristics recorded in the target chambers are presented in fig. 4 for \( U_e = -80 \) V, \( V_p = +5 \) V, \( V_a = 60 \) V, \( p = 10^{-4} \) mbarr, and plasma density in the driver chambers as parameter. A large electron current is also measured by the probe as long as probe bias is positive with respect to the potential of the D chamber which gives the energy of the ion beam. The electron current increases with both the plasma density and the \( U_e \) because in both cases more electrons could penetrate the grid.

It must also be stressed that PRI-like instabilities \( /9/ \) are excited when probe bias exceed a positive value so that a decreasing of the electron saturation current is registered.

EXPERIMENTAL ANALYSIS OF A LOW FREQUENCY INSTABILITY IN A MAGNETIZED DISCHARGE PLASMA

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1 Introduction

The Potential Relaxation Instability (PRI) [1], [3] appears in a laboratory plasma if an electrode is inserted into the magnetized plasma column perpendicular to the axis and electron current is drawn to the electrode. The diameter of the electrode must be comparable to the diameter of the plasma column. The instability appears as a moving double layer that travels in the axial direction along the plasma column from the plasma source towards the electrode. Travelling double layer is followed by a potential depression on the low potential side that limits electron current to the electrode. The instability is accompanied by coherent oscillations of the electrode current. The frequency \( \nu \) of the instability is proportional to \( \nu \propto c_s / L \). Here \( L \) is the distance between the plasma source and the electrode. The ion sound velocity \( c_s \) is given by \( c_s = \sqrt{kT_e / m_i} \), where \( k \) is Boltzmann's constant \( T_e \) is electron temperature and \( m_i \) is the ion mass. The frequency of the PRI described above is determined only by the axial motion of the positive ions so that it can be called 1-D PRI.

In this paper we report the observation of a low-frequency (30 - 100 kHz) instability in a weakly magnetized discharge plasma. The instability is excited by a disc electrode that is immersed in the magnetized plasma column perpendicular to the axis. Coherent oscillations of the current \( I_a \) collected by the electrode appear when the electrode is biased \( U_a \) is just above the plasma potential \( V_p \). It is found that the frequency of the observed instability is determined by the axial and radial motion of the ions. In section 2 the experimental procedure is described. In section 3 the experimental results are presented. In the final section the observed phenomenon is tentatively discussed as a 2-D PRI type of oscillations and some conclusions are given.

2 Experimental procedure

The experiments were performed in a linear magnetized plasma device 1.5 m long 18 cm inner diameter. The basic pressure in the vacuum chamber is below \( 10^{-4} \) Pa. Working pressure of argon is between \( 10^{-3} \) and \( 10^{-1} \) Pa. The plasma is produced by a discharge from hot tungsten filaments. The electron temperature in our plasma machine is \( kT_e = 1 - 3 \) eV and ion temperature is approximately \( kT_i \approx 0.1 \) eV.

The magnetic field density is \( B = 0.02 \) T so that the Larmor radius for ions is approximately \( r_{Li} \approx 1 \) cm and for electrons about \( r_{Le} \approx 0.2 \) mm. If we compare this to the diameter of the plasma column which is about 7 cm we can conclude that electrons are strongly magnetized, while ions are weakly magnetized.
For the plasma diagnostics Langmuir and emissive probes are used. The plasma density is proportional to the ion saturation current $I_n$, collected by a negatively biased cylindrical (0.15 mm diameter, 5 mm length) Langmuir probe. The plasma potential $V_p$ is determined as the floating potential $V_f$ of an emissive probe.

The instability is excited by a disc electrode. The diameters of the exciter electrodes range from $2\pi = 2.5$ mm to $2\pi = 35$ mm so that $r_{Le}$ is always much smaller than the electrode diameter, while $r_{Li}$ is comparable to the electrode diameter. Time resolved measurements are performed using the boxcar sampling and averaging technique. The data are analyzed by a computer.

Periodic signal from the collector is used as a trigger for the boxcar integrator (SR250 Stanford Research Systems) for the time resolved measurements. When the electrode potential $U_a$ exceeds the plasma potential $V_p$, the electrode current $I_n$ starts to oscillate periodically. Frequency $\nu$ of these oscillations is usually between 30 and 100 kHz. The amplitude of the oscillations can achieve up to 30% of the DC value of the anode current $I_a$.

### 3 Experimental results

Dependence of the frequency $\nu$ on various parameters was measured. The oscillations appear just after the electrode bias $U_a$ exceeds the plasma potential $V_p$. If $U_a$ is further increased, the frequency $\nu$ at first increases and when $U_a$ exceeds $V_p$ for roughly 2 V $\nu$ becomes constant (figure 1 a).

The frequency $\nu$ is inversely proportional to the electrode diameter (figure 1 b). It does not depend much on the plasma density and on neutral gas pressure. It is interesting that the frequency decreases with increasing magnetic field density $B$. In figure 2 axial (figures b and c) and radial (figures d and e) profiles of the plasma potential $V_p$ and plasma density that is proportional to the ion saturation current $I_n$, to a Langmuir probe in different phases of the electrode current oscillation are shown. Different phases are marked by numbers that are
explained in figure 2a. It can be observed that during the phase of the current decrease (phases 6, 7, 8 and 9 in figure a) a potential hill of the height $\Phi_H$ followed by a potential depression of the depth $\Phi_D$ travels towards the electrode in the axial direction. The travelling potential structure is followed by a plasma with higher density that penetrates into a region with lower density. During the phase of current increase (phases 2, 3 and 4 in figure a) the plasma potential increases rapidly in the whole experimental region. The phase of current increase is followed by

![Figure 2: Axial (b and c) and radial (d and e) profiles of the plasma potential (b and d) and density (c and e) in different phases of the electrode current oscillations. Phases are marked by numbers that are explained in figure a.](image)

a phase of almost constant electrode current. During that phase the plasma density decreases while the plasma potential still increases. Then the potential structure that limits the electrode current is formed again and the whole cycle repeats.
The plasma density was measured by a Langmuir probe and the plasma potential by an emissive probe. In both cases the probe was fixed in the center of the electrode while the electrode was moved in the axial direction away from the probe and from the source. It must be emphasized that the axial distance between the electrode and the source does not affect the frequency.

In figures 2 d and e radial profiles of the plasma potential and density are shown. The electrode was fixed at the distance 4 cm from the probe while the probe was moved in the radial direction. On the density profiles one can see that during the phases of the current increase and of the constant current the ions are pushed out of the current channel in the radial direction. In the same time the plasma potential in the current channel increases rapidly.

4 Conclusions

We observed a low frequency instability excited by a positive electrode in a magnetized discharge plasma. The oscillation of the electrode current is caused by a travelling potential hill followed by a potential dip that limits the electron current to the electrode. The double layer is formed a few cm from the electrode. We assume that the position of the potential structure formation is determined by the collection length L of the electrode. This distance is proportional to the diameter of the electrode and it increases with increasing magnetic field [4], [5]. A more detailed presentation of the model that explains the observed phenomenon as a 2-D PRI will be given elsewhere [6].

The plasma in front of the electrode oscillates between two quasisteady states. In the phase of maximum electrode current the plasma is in the state of high potential and low density and in the phase of minimum electrode current the plasma is in the state of low potential and high density. The observed phenomenon can be explained as a relaxation type of oscillation similar to the PRI observed in a Q machine [1], [3] or in a double plasma machine [2]. But there are two important differences. First in our experiment the distance L that the double layer has to travel is determined by the diameter of the exciter electrode and the magnetic field and not by the position of the plasma source [1], [3] or separation grid [2]. Second the radial motion of the ions is also important for the determination of the frequency as the axial motion.

References

ION CYCLOTRON OSCILLATIONS INDUCED BY AN EMISSIVE ELECTRODE FLOATING IN A MAGNETIZED COLLISIONLESS PLASMA COLUMN

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Abstract

Measurements have been performed on ion cyclotron oscillations induced by a small emissive electrode (electron gun) floating in a magnetized collisionless plasma column. The oscillations are observed only when the emission is so large that the electrode potential becomes higher than the plasma potential. This means that the oscillations are induced without dc electric current passing through the plasma column if the electrode potential is positive with respect to the plasma potential. Main features of the oscillations are the same as those of well-known electrostatic ion cyclotron oscillations induced by applying positive potentials to a small cold disc electrode immersed in a magnetized collisionless plasma column.

1. Introduction

The first clear-cut observation of electrostatic ion cyclotron oscillations in a laboratory plasma was reported by D'Angelo and Motley in 1962 /1/. In their experiment, the electrostatic ion cyclotron oscillations were driven by applying positive potentials to a small cold disc electrode immersed in a single-ended Q machine with uniform magnetic field. Under this configuration, an electric current is generated in the plasma along the magnetic field. Thus, the oscillations observed were ascribed to current-driven electrostatic ion cyclotron instability predicted by Drummond and Rosenbruth /2/. Many subsequent measurements of the electrostatic ion cyclotron oscillations and related phenomena, which were made under almost the same con-
configuration, were discussed also on the basis on this "current-driven" mechanism.

On the other hand, Hatakeyama et al./3/ made measurements on the electrostatic ion cyclotron oscillations in nonuniform magnetic fields under the same configuration as in Ref./1/ except the axial profile of the magnetic field. The oscillation frequency was observed around the ion cyclotron frequency defined by a local magnetic field near the position of the electrode biased positively to drive the oscillations. According to the observation, Sato and Hatakeyama pointed out an importance of two-dimensional potential structure near the electrode and have proposed a new mechanism for the generation of these oscillations /4/. The mechanism is based on a direct plasma response to the applied potential, which gives rise to a two-dimensional relaxation oscillation in a magnetized plasma column. Details of the related investigations on the electrostatic ion cyclotron oscillations were reviewed by Rasmussen and Schrittwieser /5/ although there was no clear conclusion for generation mechanism.

In this report, the ion cyclotron oscillations are induced by a small emissive electrode (electron gun) floating in a magnetized plasma column. Under this configuration, there is no dc electric current passing through the plasma column. The oscillations are observed when the emission is so large that the electrode potential becomes higher than the plasma potential, being consistent with the "potential-driven" mechanism proposed by Sato and Hatakeyama.

2. Experimental Methods and Results

The experiment has been performed on a single-ended Q machine at Tohoku University, which is schematically shown in Fig. 1. A potassium plasma column of 150 cm long [density=(5-10)x10^8 cm^-3, electron temperature=0.2 eV (ion temperature)] is produced by surface ionization at a hot tungsten plate of
52 mm in diameter and is terminated by an endplate of 80 mm in diameter under a strong magnetic field B. A background gas pressure is about \(1 \times 10^{-6}\) Torr. Under our experimental condition, we can neglect particle collisions.

An electron gun of 10 mm in diameter, consisting of oxide cathode and acceleration grid (50 mesh/inch), is set at a distance of about 110 cm from the hot plate as shown in Fig. 1. The cathode can provide high electron emission, the amount of which is larger than the plasma electron flux. But, in our experiment, the gun is kept floating electrically and then there is no dc electric current between the gun and the plasma. The gun emits electrons into the plasma just to cancel out the electric current from the plasma into the gun. With an increase in the acceleration potential applied between the grid and cathode of the gun, the grid potential increases and becomes positive with respect to the plasma in order to prevent the excess electron emission into the plasma. Thus, we can make the grid potential positive with respect to the plasma without dc electric current passing through the plasma column, being in contrast with the situation where the positive dc potential is externally applied to the cold electrode in the plasma.

According to the measurements, only when the grid potential is higher than the plasma potential, there appears a coherent oscillation a little above the ion cyclotron frequency in the plasma column. The measured spectra of the oscillations are shown with the magnetic field B as a parameter in Fig. 2. A dependence of the frequency on B is shown in Fig. 3 which also includes the oscillation frequencies measured when the positive potentials are applied to the grid without heating of the cathode just as in case of cold disc plate, where the grid is biased at the same positive potentials as the gun (grid).
floating potentials. We can find a good agreement between the frequencies obtained under different situations. The oscillation amplitudes are smaller by an order of magnitude in case of the electron gun than in case of the cold plate. When the magnetic field is changed only in the region around the gun, the frequency is observed to follow the ion cyclotron frequency near the gun position as in case of the cold plate.

4. Summary

Measurements have been performed on ion cyclotron oscillations induced by using a small emissive electrode (electron gun) kept floating electrically in a magnetized collisionless plasma column. There appear the oscillations only when the electrode potential is positive with respect to the plasma. This means that the oscillations are generated even if there is no electric current passing through the plasma column. The result is consistent with the "potential-driven" model of Sato and Hatakeyama. Even if electron and ion velocity distributions are taken into account under our experimental situation, we can find no ion cyclotron instability based on nonequilibrium velocity distributions.

ON THE DOUBLE LAYER PHENOMENOLOGY

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Space charge double layer (DL) appears frequently both in collisional and in
collisionless considered plasmas /1/. As a consequence, the up-to-date dominant theoretical
explanations start with the assertion that elementary processes like excitations and ionizations
are not essential for understanding of the DL phenomenology. However, many experimental
physicists /2/ evidence the fact that ionizations within the DL are the dominant processes in
spite of the fact that the typical ionization mean free path for electrons is much greater than the
thickness of the DL.

In this paper we present experimental results that evidence the essential role played
both by the excitation and the ionization processes in the DL phenomenology. To investigate
the causes which determine the profile of the electron concentration and also of their energy
distribution function, we have performed spectral measurements in the region where the DL is
formed (figs. 1 and 2).

Figs. 1 and 2
Spectral lines which evidence the
direct connection between the excitation processes and the allure of the electron
concentration n_E and of the energy
distribution function F(E) and consequently of the DL potential profile

The results presented in figs. 1 and 2 prove that the DL is formed and
maintained only in a region where electrons are accelerated so that they achieve kinetic
energies for which the excitation cross section increases suddenly. Such a situation is present
near a positive electrode immersed in a plasma /3/ or in a constricted region of a current
carrying plasma column. The DL formation process is usually started near the positive
electrode but in certain condition /1/, in the further evolution, the DL is shifting off from the
positive electrode and fixed at a certain distance from this (electrode free DLs). In such
situations the space charge layer formed of positive ions in the high potential side of the DL
plays the role of a virtual anode /5/ which locally assures the electric field necessary to
maintain the electron acceleration inside the DL. Evidently in the region where the kinetic energy of the electrons, accelerated inside the DL, has values for which the neutral excitation cross section increases suddenly, the intensity of spectral lines is maximal and, as a consequence of inelastic collisions, a great number of the electrons are accumulated. Because of their Maxwellian distribution, at first are accumulated the electrons with high kinetic energies and afterwards the electrons with small energies are trapped at the negative barrier already formed. In a first phase the profile of the electron concentration which corresponds to a negative potential barrier is determined by both phenomena. The part of electrons with sufficiently high energy, able to penetrate the negative potential barrier so formed, gain by acceleration in the present electric field, a kinetic energy sufficient to produce positive ions. Taking into consideration the energy dependence of the ionization cross section, the maximum rate of the ion production occurs at a certain distance from the well localized negative space charge. Because of their high mobility, the electrons producing ionization and the resulting electrons are quickly collected by the anode so that between the negative space charge and the anode, a plasma enriched with positive ions appears. The potential drop $V_{DL}$ developed between the plasma enriched with electrons (placed at the low potential side of the DL) and that enriched with positive ions (placed at the high potential side of the DL) is maintained by the current by a dynamical equilibrium between the accumulation of electrons after excitation processes and different electron loosing processes as recombinations and diffusion. The final arrangement of the negative and positive space charges inside of a DL strongly depends by a self-confinement mechanism determined by the electrostatic forces which act between the well localized negative space charge and the positive ions. These forces determine the formation and the maintenance of two adjacent layers with opposite electric sign, having properties similar to cell membranes. Since the electrostatic forces also determine an important increase of the local concentration of electrons and positive ions, the recombination probability is very high in this region so that the neutral concentration necessary to assure the excitation and ionization rate required for the DL maintenance is locally also present in collisionless considered plasmas. The well localized light phenomena observed for example in Tokamaks [6] confirm that the neutral concentration in the region where a DL is formed is different from those existent in the region where usually this is measured. This assertion is confirmed by the experimental results presented in fig.3 which evidence that inside a globular DL (plasmoid) the neutral pressure is higher than in the surrounding plasma.

As it was shown [4,7], the DL becomes unstable when the DL potential drop $V_{DL}$ is equal with the ionization potential of the gas $V_i$. We have already discussed [7] the
connection between different time sequences of an unstable DL and different elementary processes.

Fig. 3.
The difference between the inside and outside pressures (\(P_m\) and \(P_w\)) in a plasmoid for different gas pressure.

Here we mention only the fact that a direct connection between \(V_i\) and the \(V_{DL}\) exists only under conditions for which the ionization rate at the high potential side of the DL is sufficiently high to determine an important increase of the positive ion concentration so that the DL turns into a moving one and finally disrupts. When this condition is not satisfied the \(V_{DL}\) is not related with \(V_i\) and depends only on the external imposed potential differences although the DL generation mechanism does not differ qualitatively from those above described. In this case the DL disruptions process does not appear.

The described self-organization process (sustained by the current) during which the spatial non uniform distributions of electrons and positive ions themselves assure the necessary rates of the mentioned quantum processes (local potential drop and neutral concentration) explains the property of a DL to act as an autonomous body \cite{11}. Such bodies initially appear in the region of a gaseous conductor where external imposed causes determine a local acceleration of thermalized electrons.

Time resolved measurements \cite{4,7} prove that the DL becomes moveable (expands) when \(V_{DL} = V_i\). This is a consequence of the increase of the positive ion concentration at the high potential side of the DL so that this region acts as a virtual anode with increasing potential. Under such conditions the region where the electrons are accumulated after excitation processes and, consequently, also the adjacent sheath of positive ions is shifted toward the plasma enriched in electrons. This moving phase of the DL takes place as long as the production rate of the positive ions is sufficient to locally assure the conditions required for the DL maintenance. When such conditions are present along the entire plasma column, the observed phenomenon is known as soliton. When the positive ion production rate decreases during of the DL displacement after covering of a certain distance, this disrupts. In this case an internal triggering mechanism \cite{4,5,7} assures the periodical formation and disruption of the moveable DLs. The described phenomenology explains the ability of DLs to act as stimulators for different instabilities and chaotic phenomena usually present in all plasma devices \cite{9}.
A very interesting phenomenon is the appearance in low voltage arcs of electrode-free globular space charge structures (fireballs FB) surrounded by a spherical DL /10/. More frequently the FB behaves in a pulsatory state /4,10/ during of which the potential inside it, in respect of the surrounding plasma, reaches Vp. Pulsatory FBs were also observed in thermionic converters, so that very probably, their maintenance can be assured, without external dc power supply, only by thermal energy /10/.

The knowledge of the phenomenology implied in the DL dynamics can contribute to solve problems related with different instabilities observed in modern applications of the plasma physics (Tokamaks, plasma focus, etc.) and also to understand the phenomena taking place in the high region of the polar Earth atmosphere and also in extraterrestrial plasmas. The described phenomenology is, in our opinion, also able to explain the appearance of ordered structures in semiconductors and electrolytes (wandering domains) and the transition from a disordered to an ordered (cellular) state of the matter, in general /9,11/. Evidently, spectral investigations are able to give essential information on the role played by quantum processes in the DL phenomenology. Because DLs appear in all plasma devices as visible images /6/, the results of such investigations are decisive answers whether a collisionless plasma theory is appropriate to describe such phenomena.

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A PROBE SHEATH INSTABILITY IN A DOUBLE PLASMA DEVICE

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Introduction:
It is well known that the sheath regions of a plasma can generate electrostatic instabilities [1,2]. One such ion–beam driven instability at a disc probe has been investigated recently in a Double Plasma (DP) device [3]; it appears to be related to the formation of a double layer in front of the probe when the probe potential exceeds the beam energy. This double layer is also understood to be responsible for the appearance of a second knee in the probe characteristic: its presence increases the effective collecting area of the probe and thus leads to a further increase in electron saturation current.

Here we report on an instability that is observed, as in [3], at frequencies somewhat less than the ion plasma frequency \(\omega_{pi} = 660 \text{ kHz}, \omega_{pe} = 180 \text{ MHz}\) when an ion beam is directed at a positively biased disc probe. However, in our case, it is destabilized at much lower beam energies \(V_b \approx \kappa T_e\), for certain values of grid potential. In particular we investigate how the onset of the instability depends on beam energy, probe bias and the negative potential of the separator grid of the DP.

Fig. 1: Experimental set up. Dimensions and parameters of the Innsbruck (Durban) machine: \(L_n = 40 \text{ cm} (30 \text{ cm}), L_t = 50 \text{ cm} (60 \text{ cm}), \theta = 44 \text{ cm} (60 \text{ cm}), d = 1 \text{ cm} (1.8 \text{ cm})\). Typical range of variables: \(V_b = 2 - 10 \text{ V} (4.5 - 20 \text{ V}), V_g = -40 - -100 \text{ V} (-40 - -60 \text{ V}), V_p = -10 - 30 \text{ V} (-10 - -50 \text{ V})\).

Experimental set-up:
Experiments were performed in DP devices at the University of Innsbruck (Austria) and the University of Natal, Durban (South Africa). Fig. 1 shows the experimental set-up.

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Fig. 2: Set of probe characteristics \(-10 \leq V_p \leq 50 \) for values of \(4.5 \leq V_b \leq 14\) V at \(V_g = -30\) V.

which is essentially the same in both machines, the main difference being the size of the tungsten disc used (\(d = 1\) cm diam. in Innsbruck, 1.8 cm diam. in Durban) and the availability of a weak magnetic field perpendicular to the ion beam in the Durban machine. In both experiments fast or arbitrarily slow Langmuir sweep traces were captured on a digital oscilloscope or displayed on an XY recorder. Thus the onset and quenching of probe current oscillations could be detected and the shape of the probe characteristics checked for changes due to the formation of a double layer in front of the disc. The disc position could be altered between \(z = 0\) cm and \(z = 40\) cm from the grid.

Experimental Results:

Fig. 2 shows a set of Langmuir sweeps \((-10 \text{ to } +50 \text{ V})\) for variable beam energy (source chamber bias \(4.5 \text{ V} \leq V_b \leq 14\) V) and a grid bias \(V_g = -30\) V. Note that the zero of probe current has been shifted for clarity. The two curves marked 11 V and 14 V show the formation and shifting of the "second knee" discussed in [3]. However, there is no instability for a probe bias well above this knee, and the electron saturation current increases with the ion beam voltage for all curves.
Fig. 3 shows some similar traces for \( V_g = -40 \) \( V \), with a certain value of \( V_b \) resulting in the occurrence of oscillations (typical \( f \approx 100 \) kHz) without visible change of the (time averaged) shape of the characteristics. The subsequent quenching of the instability for higher \( V_b \) values is also apparent. This behaviour was more noticeable when time averaged Langmuir traces were plotted on the XY recorder, in the presence of oscillations as displayed on the digital oscilloscope. For particular values of beam energy and probe bias different modes could be observed, as displayed in Fig. 4. Fig. 4 (a) shows a mode destabilized at low probe bias, while Fig. 4 (b) shows a mode that is quenched for low probe potential \( V_p < V_b \). Although captured separately both modes appeared in rapid succession, i.e. the system could switch between them intermittently.

![Graphs showing two modes at different probe biases](image)

**Fig. 4:** Two modes at \( V_b = 7.2 \) \( V \), destabilized at low and high \( V_p \) respectively. Although captured separately, the system continously switched between both modes intermittently.

**Discussion:**

In both (Innsbruck and Durban) experiments, although for different grid bias and discharge parameters, the current oscillations reported here occurred long before a deformation of the disc probe characteristic became evident. Their frequency was sensitive to changes in the beam voltage (it decreased with increasing beam energy) and, to a much lesser degree, to probe bias. In both machines the onset (and quenching) of the instability depended on grid and probe bias, ion beam energy and the potential of the separator grid. This, together with the fact that a typical scale length computed from \( \lambda_{De} \approx \)
\((\kappa T_e/\lambda_\perp)^{1/3}/f\) is in the order of a few millimeters \((\lambda_\perp \approx 0.5 \text{ mm})\) makes us suspect, that we are here, as in [3], dealing with a sheath instability, related to the sheath in front of the disc (e.g. a double layer) and in the vicinity of the grid. This is in agreement with the observed quenching of the instability by an extremely weak magnetic field \((B < 0.2 \text{ mT})\) perpendicular to the ion beam which affects the electron behaviour over a sheath width but should leave the ions unmagnetized.

In contrast to [3] there was no obvious frequency dependence on probe position in the Innsbruck experiment, which was confirmed in the Durban machine. However, in both machines a frequency dependence on plasma density, base pressure of the Argon plasma \((p = 10^{-4} \text{ mbar})\) and electron temperature \((\kappa T_e \approx 1 - 2 \text{ eV})\) was noticeable. The formation of a double layer at low ion beam energies and its consequent collapse at higher beam energies would explain the destabilization and quenching of different modes, the increase in electron saturation current for beam voltages \(V_b < \kappa T_e\) and the deformation of the characteristics for higher beam voltages, \(V_b \approx \kappa T_e\). We are thus led to extend the explanation offered in [3] to lower beam energies: a double layer, forming in front of the disc when the beam ions can no longer reach it, increases the effective collecting area of the probe and may under certain conditions drive the electron current unstable.

**Conclusion and Acknowledgements:**

These experiments provide further evidence for the destabilization of low frequency electron current oscillations, which we attribute to the presence of an ion beam, with energies comparable to the probe bias, affecting the sheath conditions in front of the disc and in the vicinity of the grid.

The work at the Innsbruck machine was undertaken while one of us (P.K.) was on leave from the University of Natal, Durban. He would like to express his sincere gratitude for the warm hospitality experienced and the financial assistance rendered. Part of this work was supported by the "Österreichische Forschungsgemeinschaft" under grant No. 06/1619, the "Auslandsabteilung der Universität Innsbruck", the "Jubiläumsfonds der Universität Innsbruck zur Förderung der Forschung und Lehre" and the "Fonds zur Förderung der wissenschaftlichen Forschung" (Austria) under project P8709-PHY. Finally, we wish to thank P.J. Barrett for stimulating and helpful discussions.

**References:**

Progress in the kinetic modeling of collisionless two-dimensional bounded plasma systems

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1. Introduction and summary. In Ref. 1, a two-dimensional slab model was developed for calculating self-consistent dc states of a collisionless single-ended Q-machine in which the potential distribution decreases monotonically in the axial direction (i.e., the direction of the constant applied magnetic field). This method combines trajectory integration of the Vlasov equation for each particle species (electrons and ions) and a well-known Poisson solver into a numerical iteration scheme yielding the two-dimensional self-consistent potential and space-charge distributions. Typical results for "short" systems (just a few Debye lengths) were presented in Refs. 1, 4 and 5.

In the present contribution, the method is used to calculate the dc states of a "long" system (i.e., several hundred Debye lengths), and typical potential and space-charge distributions are shown. In addition, related ion and electron velocity distribution functions are presented for a position near the plasma-vacuum transition region, where the largest potential and electric-field gradients occur. These results demonstrate that our method is well capable of describing the complex fine structure of the distribution functions. Finally, a comparison is made between electron trajectories obtained in the guiding-center approximation used by us and the numerically exact electron trajectories, which shows that the approximation is good even in the plasma-vacuum transition region.

2. Model and assumptions. Figure 1 shows our two-dimensional model of the single-ended Q-machine. Although the model is still cartesian, the y-coordinate simulates to some extent the radial coordinate of the cylindrical Q-machine geometry. Both the ions and the electrons leave the emitter with half Maxwellian distribution functions corresponding to the
hot-plate temperature $T$, but their emission densities may be different. All particles reaching the collector, the emitter or the surrounding wall are absorbed. A uniform magnetic field $B$ is applied in the $x$-direction, and the plasma is assumed to be sufficiently rarefied ($n_p \leq 10^8$ cm$^{-3}$ for typical Q-machines) to be tractable as collisionless. The physical parameters are chosen such that the potential is monotonically decreasing in the axial direction, but due to the vastly different particle Larmor radii and our fairly realistic boundary conditions the potential changes in the normal direction are still nontrivial. The system is specified by prescribing the following parameters: axial length $L$, emitter "radius" $R_e$, collector "radius" $R_c$, wall "radius" $R_w$; emitter potential $\Phi_e \equiv 0$, collector potential $\Phi_c$, wall potential $\Phi_w$; emitter temperature $T$, electron emission density $n_{e_0}$, ion emission density $n_{i_0}$ (or, equivalently, the "neutralization parameter" $\alpha \equiv n_{i_0}/n_{e_0}$).

3. Potential and space-charge distributions. Solving the time-independent Vlasov equation for each particle species ($a = e,i$),

$$\left[ v \cdot \nabla + \frac{q_a}{m_a} (E(x) + v \times B) \right] f_a(x,v) = \frac{d}{dt} f_a(x,v) = 0, \quad (1)$$

and Poisson's equation,

$$
\nabla^2 \Phi(x) = -\frac{\rho(x)}{\varepsilon_0} = \frac{\varepsilon}{\varepsilon_0} [n_e(x) - n_i(x)],
$$

(2)

together with the boundary conditions

$$
\begin{align*}
\Phi(0,0 \leq |y| \leq R_w) &= \Phi_w = 0, \\
\Phi(0,R_c < |y| \leq R_w) &= \Phi_e + \frac{(\Phi_w - \Phi_e) (|y| - R_c)}{(R_w - R_c)} \\
\Phi(L,0 \leq |y| \leq R_w) &= \Phi_c, \\
\Phi(L,R_c < |y| \leq R_w) &= \Phi_e + \frac{(\Phi_w - \Phi_e) (|y| - R_c)}{(R_w - R_c)} \\
\frac{\partial \Phi}{\partial y}(x,0) &= 0, \\
\Phi(x,R_w) &= \Phi_w,
\end{align*}

(3)
$$

by a numerical-iterative scheme as described in Refs. 1 and 4, we find the self-consistent dc potential distribution, the density distributions for both particle species and, hence, also the space-charge distribution. In Fig. 2, the potential and space-charge distributions are shown for a system of length $L = 5$ cm (which is short but still easily realizable) and $R_e = R_c = 1.5$ cm, $R_w = 6.5$ cm, $\Phi_e = \Phi_w = 0$, $\Phi_c = -3$ V, $T = 2000$ K, $n_{e_0} = 5.12 \cdot 10^{14}$ m$^{-3}$, and $\alpha = 0.1$. With these parameters we have a plasma density $n_p = 1.3 \cdot 10^{13}$ m$^{-3}$ and the associated Debye length becomes $\lambda_{Dp} \approx 8.8 \cdot 10^{-4}$ m, so that the plasma region is about sixty $\lambda_{Dp}$ long. In the center of the plasma ($y \approx 0$), the results are in good agreement with one-dimensional theory. The plasma region, which is determined by $\rho(x) \approx 0$, is seen to be much more extended than both the emitter and the collector sheaths. In the normal direction, the potential and space-charge distributions are flat near the center but jump quite sharply at the plasma-
vacuum transition. This is where the largest potential gradients in the normal direction occur, and we will show that even in this region our method is able to resolve the complex structure of the electron and ion distribution functions very well.

Fig. 2. Potential and space-charge distributions

4. Velocity distribution functions. Integrating the appropriate particle trajectory back in time until it intersects with some boundary, we are able to calculate, from Eq. (1), the ion and electron velocity distribution functions for a given phase-space point. Discretizing velocity space (i.e., the parallel velocity $v_x$, the normal velocity $v_n$, and the polar angle $\theta$), we calculate the velocity distribution functions for each particle species as a function of all three velocity coordinates. Due to the nontrivial structure of the electric field and the boundary conditions, these distribution functions have a
complex form which cannot be calculated analytically. Figures 3 and 4 show typical normalized electron and ion velocity distribution functions near the plasma—vacuum transition for the above parameters at the position \((x = 2.5 \text{ cm}, y = 1.4995 \text{ cm})\). In each of these plots one velocity variable is fixed, so that the velocity distribution functions shown depend only on two velocity variables.

5. Particle trajectories. In view of the high gyration frequency of the electrons and practical limitations on computing time, we are treating electron motion in a guiding—center approximation. To convince ourselves that this approximation is indeed good we have made a comparison with the numerically exact guiding—center motion. These calculations show that even in the plasma—vacuum transition region, where the steepest field gradients are located, our approximation is excellent (cf. Ref. 6).

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ANISOTROPIC CONDUCTIVITY OF THE COLLISIONLESS MAGNETIZED PLASMA AND ITS APPLICATION TO THE CYLINDRICAL PLASMA COLUMN

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1. INTRODUCTION

The effective conductivity perpendicular to the magnetic field of the collisionless plasma is shown to be very low, by the classical macroscopic equation without adding any anomalous effect and assumptions. This highly anisotropic conductivity is clearly understood also by the microscopic picture of the individual particle motions in the magnetic fields and is the common characteristic of the high temperature magnetized plasma in most of the nuclear fusion devices such as in tokamaks. The consideration based on the anisotropic conductivity may give the good explanation for the specific and common behavior of the plasma, such as the self organization, the fast relaxation and the various anomalous transport phenomena which have been considered to be the results of the various turbulences and instabilities.

2. EFFECTIVE TRANSVERSE CONDUCTIVITY

In the case where the stepped electric field perpendicular to the magnetic field is applied abruptly to the magnetized uniform plasma, the electric currents can be analytically described by the macroscopic equation derived from the Boltzmann equation. If the electron to ion collision frequency is low as compared with the electron cyclotron frequency, the electron current approaches rapidly to a steady value after a few electron cyclotron oscillations, and the ion current which is carried by the ion accelerated directly to the electric field, becomes dominant after a half period of the ion cyclotron motion. However, this ion current and the Hall current may also vanish in a few ion cyclotron periods, and the transverse electron conduction current parallel to the electric fields will remain after this phase.

The electron conduction current density \( j \perp \) perpendicular to the magnetic field and parallel to the perpendicular electric field \( E \perp \), and the effective perpendicular electrical conductivity \( \sigma_{\perp \text{eff}} \) parallel to \( E \perp \) are expressed by [1],

\[
\sigma_{\perp \text{eff}} = \sigma_s \cdot \frac{1}{1 + (\omega_e / v_e)^2} \cdot \frac{v_e}{\omega_e} \tag{1}
\]

Here \( \sigma_s \), \( v_e \), \( \omega_e \) are the Spitzer conductivity, the electron ion collision frequency and the electron cyclotron frequency, respectively. In the final state both ions and electrons drift in the same direction so as to eliminate the electric fields and all the transverse currents except the diamagnetic current may vanish. However, if the particle diffusion speed is same as the drift velocity, the steady density profiles might be sustained, as usual the case in most of the experiments. In any case, the transverse conduction current should be less than the value given by the equation (1). It should be noted that this relation may universally hold in the real plasma with space and time variation of the electromagnetic field and the plasma pressure.

3. CONDITIONS FOR THE HIGHLY ANISOTROPIC CONDUCTIVITY

The highly anisotropic conductivity may realize, if \( (v_e / \omega_e)^2 \) is much less than unity. In Fig.1, the criteria for the condition \( (v_e / \omega_e)^2 \) are shown as the function of \( T_e \) and \( n_e \) for the characteristic magnetic field \( B \) as a parameter. In the region below each line, the anisotropy may appear more extremely as the distance from the line is longer. The typical experimental
conditions in the common devices are also shown. In the large tokamak, \((v_e/\omega_m)^2 \leq 10^{-16}\) and then the plasma may behave as the dielectric substance. Even in most of the small devices, \((v_e/\omega_m)^2 \leq 10^{-3}\) and then the perpendicular conduction current can be neglected. As for the diffusion of the magnetic fields, the ideal anisotropic conductivity approximation that the parallel conductivity is infinite and the transverse conductivity is zero, is applicable over the very wide time scale. In the large tokamaks, this approximation may be valid in the time scale from \(0.1 \mu s\) to \(1 s\).

The highly anisotropic conductivity may give the relaxation dominant character of the plasma, which is caused by the frequent reconnection process of the magnetic fields. The relaxation process combined with the diamagnetic current provides the self-organized configurations in high \(\beta\) regime. In the next two examples with very simple configuration, the specific phenomena may be clearly explained by the consideration based on the anisotropy.

4. APPLICATION TO THE CYLINDRICAL PLASMA COLUMN

The highly anisotropic characteristics are applied to the implosion phase of the \(\theta\)-pinch and the force-free configuration in a cylindrical geometry, and the skin depth of the magnetic fields and the flux generation mechanism are discussed, respectively.

(1) Magnetic Skin Depth in the Implosion Phase of \(\theta\)-pinch

In the cylindrical configuration of the fast \(\theta\)-pinch geometry, the diffusion of the axial magnetic field into the plasma is much faster than the value derived from the classical isotropic conductivity. It has been observed that the skin depth of the axial magnetic field in the implosion phase is about \(c/\omega_m\) (\(c\) is the light speed and \(\omega_m\) is the ion plasma frequency), and consequently the many theoretical considerations and computer simulations were devoted to the subject [2]. These discussions mostly employed the anomalous transport which is caused by the microscopic instabilities or turbulences to explain the high resistance in the skin region. However, in the well preheated plasma of the fast \(\theta\)-pinch devices, the condition \((v_e/\omega_m)^2 \ll 1\) was already satisfied and then the transverse resistance should be estimated to be very high without any anomalous transport. In the ideal simplified process, the electrons simply behave so as to keep the quasi-charge neutrality and cannot carry any conduction current. In this situation, each ion moves as the individual particle in the induced electric fields and the self-consistent magnetic fields. The fast ions which are accelerated by the induced electric fields, carry the diamagnetic-like current. An ion particle simulation was done on this simple model. The results well explain the whole structures in detail, which were observed in the fast \(\theta\)-pinch experiment [3]. In Fig. 2, the time dependent skin depth obtained by the computer simulation (a) is compared with the experimental one (b). Here, \(\omega_{pe}\) is the ion plasma frequency correspond to the initial ion density. In both experiment and simulation, the skin depth is approximately \(c/\omega_{pe}\) in the early stage and transits to \(c/2\omega_{pe}\) after a few ion gyration. A slight discrepancy is recognized only in very early stage (<50ns), where the electron conduction current may have some effects. The physical pictures of the magnetic
Fig. 2 Time dependent magnetic skin depth. (a) Computer simulation by the ion particle model, (b) Experimental results in the fast θ-pinch.

The plasma of this specific character may be represented by the linear cylindrical plasma column of radius \( a \) and the axi-symmetric magnetic field \( B_z \) and \( B_\theta \). Here, we choose a force free equilibrium configuration with the constant pitch magnetic field, which was used for the screw pinch stability analysis [4] and given in the following form,

\[
B_z(\mu r, t) = B_\theta(t) \frac{1}{1 + \mu^2 r^2}, \quad B_\theta(\mu r, t) = B_\theta(t) \frac{\mu r}{1 + \mu^2 r^2}
\]

Here, \( 2\pi/\mu \) is the pitch length and the pinch parameter \( \theta = \mu a \). This configuration is nearly same as the bessel function model in the regime \( \theta < 0.3 \) and therefore belongs to the energy minimum state \([5]\). This configuration is also similar to the low \( q \) tokamak configuration if \( \theta \) is small and tends to that of the stabilized pinch if \( \theta \) is large.

If the magnetic field changes keeping the similar profile as in the equation (2), it can be verified that the electric field is everywhere perpendicular to the magnetic field and the electric fields components \( E_z(a) \) and \( E_\theta(a) \) at the wall should be as,

\[
E_z(a, t) = \frac{1}{2\mu} \ln(1 + \mu^2 a^2) \frac{dB_\theta(t)}{dt}, \quad E_\theta(a, t) = -\frac{1}{2\mu^2 a} \ln(1 + \mu^2 a^2) \frac{dB_\theta(t)}{dt}
\]

In the ideal highly anisotropic conductor where the parallel conductivity is infinity while the
perpendicular conductivity is zero, the magnetic field can increase or decrease, keeping the similar configuration just as the equation (2) indicates, if the matched electric field given by equation (3) are applied at the wall. The condition for the matched electric fields may expressed as $E_z(a)/E_e(a)=\mu a=\theta$. There are two cases according to $\theta$, in which the interesting experimental results might be explained by this fundamental mechanism.

(A) $\theta<1$

The constant pitch screw pinch configuration can be definitely established in the range $\theta<0.3$, if the magnetic fields are ramped up keeping the pitch at the wall constant, that is, if the matched condition is kept at the wall. This fact was found at the early phase of the screw pinch research and it has been understood by the frozen-in magnetic fields and the compression of the highly conductive plasma under the MHD approximation. However, more recent experiment shows that the constant pitch configurations are established even in the case where any compression effect cannot be observed both in the fast and slow screw pinch experiment [6]. Therefore, in any case the consideration based on the anisotropic conductivity might be appropriate for the mechanism of the formation of the constant pitch configuration.

(B) $\theta \gg 1$

As $\theta$ increases, the configuration tends to the stabilized z-pincho type in which the axial magnetic fields is enhanced in the center, and the ratio $E_z(a)/E_e(a)=\mu a=\theta$ become very large. This means that the axial magnetic fields can be created by the axial electric field, that is, the axial magnetic flux can be generated by increasing the axial current which is excited by the axial electric field. This phenomena may be explained by the frozen-in magnetic field and the compression of the plasma column under the MHD approximation or $E\times B/B^2$ drift motion as well. However, in most experiment, the compression effects or the drift motions are scarcely recognized. Therefore, in any case except in the extremely high density and low temperature plasma, the mechanism based on the highly anisotropic conductivity might be responsible for the flux generation in the configuration with high $\theta$.

**REFERENCES**

1. Introduction and summary. In Ref. 1, an integral–equation method was developed for solving the general linearized perturbation problem for a one–dimensional, uniform collisionless plasma with thin sheaths, bounded by two planar electrodes. The underlying system of equations consists of (i) the Vlasov equations for all particle species involved, (ii) Poisson’s equation, (iii) the equation of total–current conservation, (iv) the particle boundary conditions at the left– and right–hand electrodes, and (v) the external–circuit equation. The method allows for very general equilibrium, boundary, and external–circuit conditions. Using Laplace transformations in both time and space, it is set up to handle the complete initial–value problem but also yields the solution to the eigenmode problem as a by–product.

In the present paper, this method is used to study, in a general and predominantly analytical manner, eigenmodes in a system modeling the negatively biased Q–machine. This problem was already considered in Refs. 2 and 3, though only by purely numerical means and in the "low–frequency approximation (LFA)", yielding discrepant results.

2. The collisionless plane–diode model and its linearized basic equations. We consider a one–dimensional diode as shown in Fig. 1, where the surfaces of the (ideally conducting) electrodes are located at x = 0 ("left–hand electrode") and x = L ("right–hand electrode"), and the electrodes are connected through an external circuit with specified properties. The intervening space is filled with a collisionless plasma consisting of n_a particle species, denoted by the species index σ. Each physical quantity Q involved is decomposed in the form Q(x,v,t) = Q(v) + ̃Q(x,v,t), with Q the given dc part and ̃Q the sought–for small–amplitude perturbation.

The basic equations, listed at the beginning of Sec. 1 and given in full in Refs. 1 and 2, determine the evolution of the perturbations ̃E(x,t) (electrostatic field), ̃f^σ(x,v,t) (velocity distribution function of species σ), ̃j_a(t) (external–circuit current density), and the global goal is to solve them for given initial, injection and external–circuit perturbations. To this end we follow the Laplace–transformation method formulated in Ref. 1, to which the reader is here and henceforth referred for full details.

The first step consists in transforming the basic equations into a set of (2+2n_a) coupled linear integral equations for the (2+2n_a) time Laplace transforms ̃j_a(ω),
\( E(x, w), I^0_1(v < 0, w), \) and \( I^0_1(v > 0, w) \), with \( I^0_1 \) and \( I^0_2 \) the injection distribution functions at the left- and right-hand electrodes, respectively. These "Laplace-transformed integral equations" [Eqs. (37)–(40) of Ref. 1] can be symbolically written in the form

\[
\mathcal{D}(x, v, [x'], [v'], \omega) \bar{u}([x'], [v'], \omega) = \bar{k}(x, v, \omega),
\]

where \( \bar{u}(x, v, \omega) \equiv [\tilde{J}_e, \tilde{E}_1, ..., \tilde{I}^0_1, \tilde{I}^1_1, ..., \tilde{I}^0_{13}]^T \) is the column vector of the unknown perturbations, \( \mathcal{D}(x, v, [x'], [v'], \omega) \) is a matrix of known linear operators acting on \( \bar{u} \) (with the variables of operation indicated by brackets), and \( \bar{k}(x, v, \omega) \equiv [-V_{eo}, \tilde{k}_8, \tilde{k}^1_1, ..., \tilde{k}^n_{10}, \tilde{k}^n_{13}, ..., \tilde{k}^n_{13}]^T \) is the column vector of the known perturbations \( -V_{eo} \) etc. The specific form of the system (1) which is appropriate to the physical situation considered here (Sec. 3) will be given in Sec. 4.

Basically, the solution of our linearized perturbation problem is found by solving the system (1) for \( \bar{u}(x, v, \omega) \),

\[
\bar{u}(x, v, \omega) = \mathcal{D}^{-1}(x, v, [x'], [v'], \omega) \bar{k}([x'], [v'], \omega),
\]

(with \( \mathcal{D}^{-1} \) the inverse to \( \mathcal{D} \)), and by then Laplace inverting the latter:

\[
\bar{u}(x, v, t) \equiv \left[ \frac{d\omega}{2\pi} e^{i\omega t} \bar{u}(x, v, \omega) \right] = \left[ \frac{d\omega}{2\pi} e^{i\omega t} \mathcal{D}^{-1}(x, v, [x'], [v'], \omega) \bar{k}([x'], [v'], \omega) \right],
\]

where, as usual, the original path of integration in the complex \( \omega \)-plane is a straight line with \( -\alpha \leq \text{Re} \omega \leq +\alpha \) and \( \text{Im} \omega \) a "sufficiently large" positive number. In the present context, we are only interested in the "natural eigenmodes" of the system, i.e., the contributions of the poles of \( \mathcal{D}^{-1} \) to \( \bar{u}(x, v, t) \), cf. Sec. 4.

3. Special case: longitudinal oscillations in the negatively biased single-ended Q-machine. For Q-machines under a wide range of operating conditions, the sheath regions are usually far less extended than the plasma region, so that the uniform–plasma, thin–sheath approximation is applicable. Assuming half–Maxwellian emission from the hot plate (whose temperature is \( T \)), the injection distribution functions at the left-hand plasma boundary are cut–off Maxwellians for both the electrons and the ions. At the right-hand boundary plane \( (x = L) \), on the other hand, all ions are absorbed, while all electrons are specularly reflected due to the cold–plate sheath, cf. Fig. 1.

In the present, rather new approach to the linear perturbation problem, we try to simplify our model as much as possible, while still keeping the essential physics. In particular, we approximate the electron velocity distribution function by the "waterbag" \( \tilde{f}^w(v) = n_p[U(v + \nu_{cw}) - U(v - \nu_{cw})] / (2\nu_{cw}) \) [with \( n_p \) the equilibrium plasma density and \( \nu_{cw} = \sqrt{3kT/m_e} \) the waterbag cutoff velocity], and the ion distribution by the cold beam \( \tilde{f}_i(v) = n_p \delta(v - \bar{v}_i) \) [with \( \bar{v}_i \) the ion average velocity]. It is assumed that the only externally imposed perturbations are initial ones.
4. Solving the eigenmode problem by means of the integral-equation method. With the specifications of Sec. 3, the Laplace-transformed integral equations (cf. Sec. 3) yield the explicit relations \( \tilde{I}_{\text{I}}^{\text{e}}(v>0, \omega) = \tilde{I}_{\text{I}}^{\text{e}}(v>0, \omega) = \tilde{I}_{\text{I}}^{\text{e}}(v<0, \omega) = 0 \) and the "reduced" system of coupled integral equations

\[
\tilde{Z}_e(\omega) \tilde{J}_e(\omega) + \int_0^L \tilde{\mathbf{E}}(x', \omega) \, dx' = -V_{\text{e}}(\omega), \quad \text{[external-circuit equation]} \tag{4}
\]

\[-k_5(x, \omega) \tilde{J}_e(\omega) + \tilde{E}(x, \omega) + \mathcal{J}_0(x, [x'], \omega) \tilde{E}([x'], \omega) + \mathcal{J}_e(v<0, [x'], \omega) \tilde{E}([x'], \omega) + \tilde{I}_p(v, \omega) = 0, \quad \text{[Poisson's equation]} \tag{5}
\]

\[
\mathcal{J}_e(v<0, [x'], \omega) \tilde{E}([x'], \omega) + \tilde{I}_p(v, \omega) = 0, \quad \text{[right-hand electron]} \tag{6}
\]

from which the remaining unknowns \( \tilde{J}_e(\omega), \tilde{E}(x, \omega) \) and \( \tilde{I}_p(v, \omega) \) must be determined. Here, \( k_5, k_6, \tilde{Z}_e, \) and \( V_{\text{e}} \) are known functions, \( \mathcal{J}_0 \) and \( \mathcal{J}_e \) are known \( x \)-space operators, and \( \gamma_e \) is a known \( v \)-space operator.²

By successively eliminating \( \tilde{I}_p \) and \( \tilde{E} \) via Eqs. (6) and (5), respectively, the system (4)–(6) can now be cast into a single equation for \( \tilde{J}_e(\omega) \):

\[
\tilde{J}_e(\omega) = \left[ \tilde{V}_{\text{d}}(\omega) - V_{\text{e}}(\omega) \right] / \tilde{Z}_{\text{tot}}(\omega) \tag{7}
\]

where \( \tilde{V}_{\text{d}} \) and \( V_{\text{e}} \) are expressions describing the effects of initial perturbations and

\[
\tilde{Z}_{\text{tot}}(\omega) \equiv \tilde{Z}_d(\omega) + \tilde{Z}_e(\omega) \tag{8}
\]

is the "total" impedance, with \( \tilde{Z}_d(\omega) \) the external-circuit impedance and

\[
\tilde{Z}_d(\omega) \equiv \int_0^L dx' \mathcal{B}'(x', [x'], \omega) k_6([x'], \omega), \tag{9}
\]

the diode impedance. The operator \( \mathcal{B} \) is defined by

\[
\mathcal{B}(x, [x'], \omega) \tilde{E}([x'], \omega) = \left[ 1 + \mathcal{J}_0(x, [x'], \omega) - \{ \gamma_e \mathcal{J}_e \} (x, [x'], \omega) \right] \tilde{E}([x'], \omega). \tag{10}
\]

According to the remarks following Eq. (3), the \( n \)th linear eigenfrequency is the \( n \)th pole of the integrand of the Laplace integral, which from Eq. (8) is seen to be the \( n \)th zero of \( \tilde{Z}_{\text{tot}}(\omega) \). Hence, we have the eigenfrequency equation

\[
\tilde{Z}_{\text{tot}}(\omega_n) = 0. \tag{11}
\]

In the present paper we restrict ourselves to solving this equation. While \( \tilde{Z}_d(\omega) \) is usually given as a simple analytic function of \( \omega \) and can hence be evaluated easily, \( \tilde{Z}_d(\omega) \) is defined by the integral (9), whose evaluation has hitherto been possible only by purely numerical means and only in the LFA.²³ By contrast, we have now obtained, without further approximations, the following analytic form of \( \tilde{Z}_d(\omega) \):
\[ \ddot{\zeta}_d(\omega) = I_1(\omega) - \frac{\tilde{\alpha}(\omega)}{A(\omega)} I_2(\omega), \]  

with \[ I_1(\omega) = \int_0^L dx' k_5(x', \omega), \quad I_2(\omega) = \int_0^L dx' k_5(x', \omega) \sin \left[ \frac{L-x'}{\nu_{cw}} \right], \]

\[ \tilde{\alpha}(\omega) = a_0(\omega) \int_0^L dx' k_5(x', -\nu_{cw}^p, \omega), \quad A(\omega) = a_0(\omega) \int_0^L dx' k_5(x', -\nu_{cw}^p, \omega) \sin \left[ \frac{L-x'}{\nu_{cw}} \right], \]

and \[ a_0(\omega) = -i \frac{(\omega^p)^2}{(\nu_{cw}^p)^2} \exp \left[ i \frac{\omega L}{\nu_{cw}^p} \right]. \]

The integrals occurring here can be represented by elementary functions, so that the eigenfrequency equation (11) has now become an analytically explicit transcendental equation which is valid for all frequency ranges.

5. Results and discussion. As previously,\(^3\) we consider the eigenmodes of a negatively biased single-ended Q-machine characterized by the following parameters: electron–K\(^+\) plasma; interelectrode distance \( L = 300 \) cm; hot–plate temperature \( T = 2200 \) K, electron emission density \( n_{e0}^* = 10^{10} \) cm\(^{-3}\), ion emission density \( n_{i0}^+ = 10^8 \) cm\(^{-3}\) (hence neutralization parameter \( \alpha = n_{i0}^+/n_{e0}^* = 10^{-2} \)), plasma density \( \tilde{n}_p = 2 \times 10^7 \) cm\(^{-3}\), plasma potential \( \Phi_p = -1.3 \) V, electron waterbag cutoff velocity \( v_{ew} = 2.7 \times 10^5 \) cm/s, electron plasma frequency \( \omega_p^e = 2.5 \times 10^8 \) s\(^{-1}\), ion–beam velocity \( v_i = 2.7 \times 10^8 \) cm/s, ion plasma frequency \( \omega_i^p \) is \( 9.4 \times 10^4 \) s\(^{-1}\), external short–circuit.

The eigenfrequencies corresponding to the first three eigenmodes are \( \omega_1 = (1.5862 - 3.1950i) \times 10^4 \) s\(^{-1}\), \( \omega_2 = (4.7975 - 3.2137i) \times 10^4 \) s\(^{-1}\), \( \omega_3 = (7.9967 - 3.2044i) \times 10^4 \) s\(^{-1}\), while in the "low–frequency approximation" the first two eigenfrequencies had been found\(^2\) to be \( \omega_1^L = (3.2204 \times 10^4 + 1.5235 \times 10^4) \) s\(^{-1}\) and \( \omega_2^L = (6.2512 \times 10^4 + 6.6399 \times 10^4) \) s\(^{-1}\). This discrepancy, which seems to indicate that the "low–frequency approximation" is inappropriate for bounded plasmas, will need further study.

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A COLLISIONLESS E x B INSTABILITY WITH LARGE ION ORBITS

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Introduction

A flute instability of a partially ionized, magnetized plasma with a transverse electric field was shown by Simon /1/ and Hoh /2/ to be responsible for the violent instability of reflex discharges which allows electron current to cross the magnetic field. The main cause of this instability is the difference between the electron and ion E x B drifts due to ion-neutral drag. This effect is not present in collisionless plasmas, but the instability is still found if the ion drift is slowed by the finite Larmor radius effect /3//4/. In a recent experiment by Sakawa /5/, clear evidence of this collisionless instability was found in a plasma which was much smaller than the ions' Larmor radii. The observed frequency was much higher than the ion cyclotron, diamagnetic drift, and E x B frequencies, thus eliminating both the E x B and the drift instabilities. The ions trapped in the potential well of the E-field had nearly straight-line orbits, with only a small curvature due to the Lorentz force. The ion fluid then has an E x B drift much smaller than that of the electrons, even though there are no collisions.

It is easy to show that such a fluid suffers from a strong flute (not drift) instability with a phase velocity close to that of the slow ion drift, as observed. The equilibrium, however, has to be treated kinetically, since the system is much smaller than the ion orbits; and it is the equilibrium which is difficult to treat. An expansion in a small Larmor radius parameter would not be suitable; rather, the curvature of the ion orbits is the small quantity. By choosing a particular functional form for the electric field profiles, we have found a simple way to model the equilibrium in both plane and cylindrical geometries. The cylindrical case is applicable to beam-plasma experiments /5/, while the plane case can be used to devise experiments to clarify the edge physics in tokamak H-mode boundary layers which are thinner than the ion Larmor radius.

Equilibrium: plane geometry

Consider a plasma confined between infinite planes at \( z = \pm L \), embedded in a uniform magnetic field \( B = B_0 \hat{z} \). A jet-like E x B velocity distribution in the y direction in the slab can be modeled by the following analytic functions:

\[
\phi = \phi_0 \tanh \xi \\
E = -(N/L) \phi' \hat{x} = -(N/L) \phi_0 \text{sech}^2 \xi \hat{x} \\
\rho = (N/L) \varepsilon_0 E'_x = 2\varepsilon_0 (N/L)^2 \phi_0 \text{sech}^2 \xi \tanh \xi ,
\]

where

\[
\xi = N \varepsilon / L ,
\]

\( \phi \) is the potential, \( \rho \) is the charge density, and \( N \) is a parameter for adjusting the sharpness of the profile. These profiles are shown in Fig. 1 for \( N = 2 \). We take \( \phi_0 > 0 \), so that \( E_x < 0 \),
and ions are driven to the left. The magnetic field is so weak that the ions bounce between their birthplace and the left wall in nearly straight lines, but there is a small curvature to the orbits, giving the ions a drift in the \( y \) direction. For a tokamak edge layer, the left boundary would be the main plasma, and the right boundary would be the first wall. For a cylindrical beam-plasma experiment, the left boundary would simulate the axis, and the right boundary would be the wall.

We next construct a density profile that is consistent with this potential profile. Let \( \mathbf{v}(\xi, \xi_0) \) be the velocity of an ion, located at \( \xi \), which was born (at rest) at \( \xi_0 \). Its kinetic energy \( \frac{1}{2} M \mathbf{v}(\xi, \xi_0)^2 \) is gained from the potential drop between \( \xi_0 \) and \( \xi \). Thus,

\[
\mathbf{v}^2(\xi, \xi_0) = (2e\phi_0 / M)(\tanh \xi_0 - \tanh \xi) .
\]

The \( y \) component of the ion equation of motion reads

\[
dv_y / dt = - (e / M) v_x B = - \Omega_c v_x ,
\]

where \( \Omega_c \) is the ion cyclotron frequency. The Lorentz force accelerates the ions in the \( y \) direction at the rate

\[
\frac{dv_y}{dt} = \frac{dv_y}{dx} \frac{dx}{dt} = \frac{dv_y}{dx} v_x = - \Omega_c .
\]

The velocity \( v_y \) of an ion at \( \xi \) which was born at \( \xi_0 \) is thus

\[
v_y(\xi, \xi_0) = - \Omega_c (x - x_0) = (L/N) \Omega_c (\xi_0 - \xi) .
\]

Note that this depends only on position; it is independent of the strength of the \( E \)-field. Combining Eqs. (7) and (10), we find for the \( x \) velocity component,

\[
v_x^2(\xi, \xi_0) = v_x^2 - v_y^2 = (2e\phi_0 / M)(\tanh \xi_0 - \tanh \xi) - (L \Omega_c / N)^2 (\xi_0 - \xi)^2 .
\]

Let the ions be created at a rate \( S(x_0) \text{ m}^{-3} \text{ sec}^{-1} \). Since the flux \( \pi w_0 \) is conserved, the density at \( x \) due to a source \( S(x_0) \text{ d}x_0 \) is proportional to \( 1 / |v_x(x, x_0)| \). The constant of proportionality is 0, 1, or 2, depending on whether the ion is turned around before reaching \( x \), is absorbed at the left boundary, or is reflected at \( -L \) or turned around at a radius between \( -L \) and \( x \). For simplicity we assume a reflecting boundary. Integrating over all source positions upstream of \( x \) and changing to the variable \( \xi \), we obtain

\[
n(\xi) = \frac{2L}{N} \int_{\xi}^{\xi_{\max}} S(\xi_0) \text{ d}\xi_0 / |v_x(\xi, \xi_0)| .
\]

Here, \( \xi_{\max} \) is the smaller of \( N \) and that value of \( \xi_0 \) beyond which \( v_x^2 \), given by Eq. (9), becomes negative, indicating that the ion turns around before \( \xi \). One needs only to end the integration when \( v_x \) becomes imaginary. The integrand is singular at two points: \( \xi = \xi_0 \), and \( v_x = v_y \), but these are obviously integrable singularities because the density physically has to be finite if the source \( S \) is finite. The experimental density profile can be matched by choosing a suitable function \( S(\xi_0) \). Fig. 2 shows sample \( n_1 \) profiles which are consistent with Eq. (1).

To obtain the ion fluid drift \( \mathbf{v}_y \) at any position \( x \), we need only to sum over all source positions the values of \( v_y(\xi, \xi_0) \), given by Eq. (8), weighted by the source strength \( S(\xi_0) \) and the residence time \( 1 / |v_x| \), and normalize:

\[
\mathbf{v}_y(\xi) = \frac{2}{n(\xi)} \int_{\xi}^{\xi_{\max}} v_y(\xi, \xi_0) \frac{S(\xi_0) \text{ d}\xi_0}{|v_x(\xi, \xi_0)|} .
\]

The corresponding profiles of \( v_0 \) are also shown in Fig. 2.
Equilibrium: cylindrical geometry

In the experiment in question /5/, a beam of electrons injected along the axis depresses the potential there and gives rise to a nonuniform radial electric field. The corresponding sheared \( \mathbf{E} \times \mathbf{B} \) drift is in the \( \theta \) direction and falls to zero at \( r = 0 \) and \( r = a \). This situation can be modeled by a consistent set of Bessel functions:

\[
\phi = -\phi_0 [J_0(Tr) - J_0(w_1)] \tag{12}
\]

\[
E_r = -\frac{\partial \phi}{\partial r} = \phi_0 \frac{\partial}{\partial r} J_0(Tr) = -\phi_0 T J_1(Tr) \tag{13}
\]

\[
\rho = \frac{\varepsilon_0}{r} \frac{\partial}{\partial r} (r E_r) = -\varepsilon_0 \phi_0 T^2 J_0(Tr) \quad \tag{14}
\]

where \( w_1 \) is a zero of \( J_1 \); for instance, the first zero is \( w_1 = 3.83 \). That Eq. (16) is true can be verified by substituting for \( E_r \) from Eq. (15), taking the \( r \)-derivative of \( \rho \) in both parts of the equation, and recognizing the differential equation that defines \( J_1 \). Setting \( T = w_1/a \) ensures that both \( \phi \) and \( E_r \) vanish at \( r = a \). These profiles are shown in Fig. 3.

The kinetic energy \( \frac{1}{2}Mv^2(\tau, r_0) \) of an ion at \( \tau \) which was born at rest \( r_0 \) has been gained from the potential drop between \( r_0 \) and \( r \). Defining \( z = T \tau \), we can therefore write

\[
v^2(z, z_0) = \frac{2e\phi_0}{M} [J_0(z) - J_0(z_0)] \quad . \tag{15}
\]

The Lorentz force curves the ion orbits, which would otherwise be straight lines through the axis, as the ions bounce in the dc potential well. The \( \theta \) component of velocity can be found from the conservation of canonical angular momentum:

\[
P_\theta = Mr v_\theta + \frac{1}{2} \varepsilon B_0 r^2 = \frac{1}{2} \varepsilon B_0 r_0^2 \quad , \tag{16}
\]

where \( \frac{1}{2}B_0r \) is the vector potential \( A_\theta \) in a uniform field, and we have taken (for the time being) \( v_\theta \) to be zero at \( r = r_0 \). Solving for \( v_\theta \), we have

\[
v_\theta(z, z_0) = \frac{\Omega_e}{2T} \frac{z^2_0 - z^2}{z} \quad . \tag{17}
\]

Eqs. (15) and (17) give

\[
v^2(z, z_0) = v^2 - v_\theta^2 = \frac{2e\phi_0}{M} [J_0(z) - J_0(z_0)] - \left( \frac{\Omega_e}{2T} \right)^2 \left( \frac{z^2_0 - z^2}{z} \right)^2 \quad . \tag{18}
\]
To calculate the density, we again integrate over a source distribution $S(z_0)$, weighted by the residence time $1/|v_r|$ and a geometrical compression factor $z_0/z$ due to the cylindrical geometry:

$$n(z) = 2 \int_{z_1}^{z_{\text{max}}} \frac{z_0}{z} \frac{S(z_0)}{|v_r(z, z_0)|} \, dz_0.$$  \hspace{1cm} (19)

Here, $z_{\text{max}}$ is the smaller of $v_1$ and the value of $z_0$ at which $v_r^2$, given by Eq. (18), is zero; beyond that, the ions do not reach $z$. Since all ions are "reflected", they contribute twice to the density at any radius. The factor 2 included for this is not important, since $S(z_0)$ is to be adjusted to fit the experiment anyway. The fluid drift $v_0$ in the $\theta$ direction is given by the weighted average of $v_\theta$, as given by Eq. (17):

$$v_0(z) = \frac{2}{n(z)} \int_{z_1}^{z_{\text{max}}} v_\theta(z, z_0) \frac{z_0}{z} \frac{S(z_0)}{|v_r(z, z_0)|} \, dz_0.$$  \hspace{1cm} (20)

Examples of $n$ and $v_0$ profiles are given in Fig. 4.

![Fig. 3](image1)

![Fig. 4](image2)

Although the ions are born with very little energy, a small amount of angular momentum at large radii could greatly affect the ions' turnaround radius, and thus the density at small radii. To examine this effect, we can add a thermal velocity $v_\theta(r_0) = \pm v_t$. A term $m v_\theta(r_0)$ is then added to Eq. (16), resulting in a term $v_\theta(r_0)$ added to Eq. (17), with corresponding changes in subsequent equations. The results, which we cannot show here, indicate that $v_\theta(r_0)$ has more effect when it is in the same direction as the Lorentz force, but that the effect is small unless $\frac{1}{2}MV_0^2$ is comparable to $e\phi_0$.

The instability

Since the instability is basically a fluid effect, we use the fluid equations. We confine ourselves here to plane perturbations of the form $\exp(i ky - \omega t)$. For the ions, the linearized equations are:

$$M \left( \frac{\partial v}{\partial t} + v_0 \cdot \nabla v + v \cdot \nabla v_0 \right) = e(E + v \times B_0)$$  \hspace{1cm} (21)

$$\frac{\partial n}{\partial t} + n_0 \nabla \cdot v + v \cdot \nabla n_0 + n \nabla \cdot v_0 + v_0 \cdot \nabla n = 0.$$  \hspace{1cm} (22)

Here, $v_0(z)$ is the averaged velocity computed above. From the electron equations, we obtain the modified Boltzmann relation /6/ appropriate for $k_\parallel = 0$ and finite $\nabla n_0$ and $E_0$:

$$\frac{n}{n_0} = \frac{e\phi}{K_T c} \frac{\omega_e(X)}{\omega - \omega_e(X)}.$$  \hspace{1cm} (23)

Here, $\omega_e$ is the electron diamagnetic drift frequency $-k(K_T/eB_0)n_0/n_0$, and $\omega_{E}$ is the $E \times B$ drift frequency $-kE_0/B_0$. Solving Eq. (21) for the first-order ion fluid velocity $v$, ...
substituting into Eq. (22), and eliminating $n$ with Eq. (23), we obtain a second-order differential equation for $\phi(x)$. We keep all $x$-derivatives ($'s$) in both zero- and first-order quantities. The result can be written in the form

$$\phi'' + f(x)\phi' + g(x)\phi = 0$$

Defining $\omega(x) = \omega - kv_0(x)$ and

$$\delta(x) = n'_0(x)/n_0(x), \quad \gamma(x) = \frac{2\omega k v'_0 + \Omega_c v''_0}{\omega^2 - \Omega_c (\Omega_c + v'_0)}$$

we can write the coefficients as

$$f(x) = \delta(x) + \gamma(x)$$

$$-g(x) = k^2 \left[ 1 + \frac{\Omega_c}{\omega} \frac{\delta + \gamma}{k} + \frac{\delta}{k} \frac{\omega^2 - \Omega_c (\Omega_c + v'_0)}{\Omega_c (\omega - \omega_E)} \right]$$

where all the quantities are $x$-dependent except $k$, $\Omega_c$, and $\omega$. Eq. (24) is to be integrated between $-L$ and $L$ using the self-consistent profiles of $n_0$, $\Omega_c$, and $v_0$ computed above to obtain the eigenvalues of $\omega$. By eliminating the velocity shear terms, one can study the Kelvin-Helmholtz effect on this instability.

The basic instability can be uncovered by making the local approximation $g(x) = 0$. This yields a cubic equation for $\omega$:

$$\omega^3 + \left( k/\delta \right) \Omega_c (\omega^2 - \omega_d \omega) - \Omega_c^2 \omega_d = 0$$

where

$$\omega_d = \omega_E - kv_0$$

is the difference between the electron and ion $E \times B$ drifts. The unstable root can be seen by taking the limit $\omega_E \gg \Omega_c$ and small $v_0$, so that $\omega \approx \omega_d \approx \omega_E$. The last term in Eq. (28) can then be neglected; and, for $n'_0 / n_0 < 0$, the resulting quadratic equation yields

$$\omega = kv_0 + k\Omega_c/2|\delta| + (k\Omega_c \omega_d / |\delta|)^{1/2}$$

The frequency is of order $kv_0$, and the growth rate depends on the difference between electron and ion drifts in the equilibrium.

INFLUENCE OF THE THERMOIONIC EMISSION ON THE CURRENT-DRIVEN ION-ACOUSTIC INSTABILITY


1. INTRODUCTION.

In this paper we analyze the conditions for the onset of the instability in an electron emitter cathode and plasma system in a DP machine. Three kinds of particles are considered: plasma ions, electrons and thermoionic electrons. The distribution functions of the first two species of particles is considered to be maxwellians. The distribution function of the thermoionic electrons is determined through the following steps: The starting point is the Fermi-Dirac distribution of the electrons inside the cathode. Then the quantum reflexion of these particles is considered and a suitable transmission coefficient found. This factor is considered together with the transformed distribution function originated by the electrons escaping from the metal to the plasma. In this way we arrive to the thermoionic distribution function. Now the dispersion relation for the ion acoustic waves is analyzed using the well known Landau treatment. The marginal stability is considered and a characteristic product $\bar{x}\bar{v}$ is found in terms of the critical density ratio $x$ and drift velocity $\bar{v}$. Finally using this density ratio, the critical current for the onset of the instability is determined. The theoretical values and compared with some experiments and good agreement is found.

2. THEORY.

The dispersion relation for the ion-acoustic wave is well known [1].

$$
\varepsilon(k, \omega) = 1 - \sum_{\alpha} \frac{\omega_{P\alpha}^2}{k^2} \int_{-L}^{L} \frac{\partial f_{\alpha}/\partial v}{v - w/k} dv
$$

(1)

where $k$, $\omega$ are the wave number and frequency and $\omega_{P\alpha}$, $f_{\alpha}$ are
the plasma frequency and distribution function for the species \(a\). \(L\) means the Landau contour of integration under the pole.

We consider a plasma composed of \(n_0\) ions at rest plus two kinds of electrons: \((1-x)n_0\) plasma electrons and \(xn_0\) thermoionic electrons. The temperatures are \(T_i = \frac{T_e}{\theta_i}\), \(T_e\), and \(T_f = \tau T_e\), respectively. a) Distribution function for the thermoionic electrons. Inside the cathode the electrons have a Fermi-Dirac distribution, which is approximated by a Maxwellian since \((E - E_F)/T_i\) is large,

\[
F(E) = A \left( e^{(E - E_F)/T_e} + 1 \right)^{-1} = A \exp\left( - (E - E_F)/T_e \right) \tag{2}
\]

where \(E = \frac{1}{2} m (v')^2\) is the kinetic energy, \(E_F\) the Fermi energy and \(T_e\) the cathode temperature. The quantum transmission coefficient \(T(E)\) of a plane wave incident on a potential barrier \(E_b = E_F + \phi\) \((\phi\) is the work function) is given by \([2]\).

\[
T(E) = \frac{4k_B}{(k + \beta)^2} \tag{3}
\]

where \(k = (2mE/\hbar^2)^{1/2}\) and \(\beta = (2m(E - E_b)/\hbar^2)^{1/2}\) are the propagation constants for the wave functions inside and outside the cathode. Using \(\frac{1}{2} m v^2 = E - E_b\) and \(E_b = \frac{1}{2} m v_b^2\), we obtain

\[
T(v) = 2v(v^2 + v_b^2)^{1/2}/(v^2 + \frac{1}{2} v_b^2 + v (v^2 + v_b^2)^{1/2}) \tag{4}
\]

Considering this transmission coefficient and the velocity change across the potential barrier we arrive at the distribution function \(g(v)\) for the thermoionic electrons just outside the cathode surface,

\[
g(v) = C_o T(v) \exp\left( - (\frac{1}{2} m v^2 + \phi) / T_f \right) \tag{5}
\]

where \(C_o\) is the normalization constant.

b) Dispersion Relation

Using the distribution function Eq. (5) and the dispersion relation Eq. (1) we obtain

\[
e(k, \omega) = 1 - \left( k^2 / k_e^2 \right) \left( (1-x)Z' (s\mu^{1/2}) + x/\tau G' (s\mu^{1/2}) + \theta Z' (s\theta^{1/2}) \right) = 0 \tag{6}
\]
Here $s = \omega/kcs$ is the phase velocity, $c_s = (T_e/m_e)^{1/2}$ is the ion acoustic speed, $\mu = m_e/m_i$, $\theta = T_e/T_i$ and $\tau = T_i/T_e$. $Z'$ is the derivative of the plasma dispersion function and $G'$ is defined by:

$$G'(s) = A_0 \int_0^\infty \frac{\partial}{\partial u} \left( T(u) \exp(-u^2) \right) (u - s)^{-1} du$$

We consider the problem of spatial excitation with real $\omega$ and complex $k$. For marginal stability we compute the principal part of the integral plus one half the residue from the pole. The integrals are evaluated in the usual way [3]: We expand the denominator of $Z'(s)$ in powers of $s/u < 1$ for the plasma electrons and in powers of $u/s < 1$ for the plasma ions. As for $G'(s)$ we consider $s < \bar{u}$ where $\bar{u}$ is the mean velocity of the thermoionic electrons and expand the denominator in powers of $(u - \bar{u})/\bar{u}$. Separating the real and imaginary part of Eq.(6) we get

$$\epsilon_r (k, \omega) \equiv 1 + (1 - x + ax/\tau) \frac{k_e^2}{k^2} - \frac{\omega_{pi}^2}{\omega^2} \quad (8)$$

$$\epsilon_i (k, \omega) \equiv (\pi/2)^{1/2} \left( \frac{k_e^2}{k^2} \right) (T_e/m_e)^{-1/2} \left[ (1 - x) \frac{\omega/k - 4(\pi/2)^{1/2} x \bar{v}_e^3}{v_{Te}^3} + \frac{m_i T_e^3/m e T_i^3}{(\omega/k) \exp(-\omega/k)^2/(2T_i/m_i))} \right] \quad (9)$$

Here $a = (v_{Te}^2/\bar{v}^2) (1 - 3(\bar{v}^2 - \bar{v}^2)/\bar{v}^2 - 4(\bar{v}^3 - 3\bar{v}^2 \bar{v} + 2\bar{v}^3)/\bar{v}^3)$.

From Eq.(8) we obtain the real part of the wave number

$$k_r = (\omega/\alpha c_s) (1 - \omega^2/\omega_{pi}^2)^{-1/2} \equiv \omega/(\alpha c_s) \quad (10)$$

where $\alpha \equiv (1 - x - ax/\tau)^{-1/2}$.

The threshold condition for instability, $\epsilon_i = 0$, gives the product $x\bar{v}$ of the critical density and drift velocity

$$x\bar{v}/c_s \equiv (\omega c_i/4(\pi/2)^{1/2}) (v_{Te}/v_{pe}) (m_i/m_e)^{1/2} \theta^{3/2} \exp(-\theta^2/2) \quad (11)$$

The term $(1 - x)\omega/k$ in Eq.(9) is small and has been neglected.
3. DISCUSSION.

Eq. (11) gives the critical value for the product $x\bar{v}$, which is a measure of the thermoionic current density from the cathode. The instability develops when the thermoionic electron current interacts with the plasma some Debye lengths from the cathode filament.

Consider the following Argon plasma $\theta = 10$; $\tau = 0.2$; $\phi \bar{v} = 4.5$ eV; $E_r = 8.9$ eV (for a Tungsten filament) then $\gamma_{\text{th}}/\gamma_{\text{B}} = 3.6$; $C_0 = 0.329$; $C_1 = 0.264$; $a = 1.43$ and we obtain $x\bar{v}/c_s = 3$. A simple model for plasma production in the DP machine has been given in Ref. [4]: The total electron current $I_e$ of the discharge is equal to the thermoionic current from the N filaments: $I_e = e \times n_0 \bar{v} N A_0$; $A_0$ is the surface where the beam impinges in the plasma. The loss ion current $I_i$ to the DP walls (area = A) is given by the Bohm diffusion: $I_i = 0.8 e n_0 c_s A$. The ions are produced by electron collisions: $I_i = N_0 \sigma l I_e$ (where $N_0$ = neutral density, $\sigma$ = ionization cross section, $l$ = mean free path for ionization). From these relations we obtain

$$(x\bar{v}/c_s)_{\text{exp}} = (0.8/N_0 \sigma l) (A/N A_0)$$

(12)

For typical DP parameters we obtain $(x\bar{v}/c_s)_{\text{exp}} = 8$, which is close to our value.

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Modulational instability of relativistic ion acoustic waves in a plasma with trapped electrons

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I. Introduction

Interplanetary space and the Earth's magnetosphere encompass a rich variety of plasma physical processes and nonlinear wave phenomena. In a high speed solar wind and the magnetosphere, the phenomena of trapped electrons are frequently observed [1]. When we consider such particles, the effect of trapped electrons has to be included in addition to the relativistic and ion temperature effects. Although the existence of the nonlinear modulation of electrostatic ion acoustic waves is suggested by space observations [2], relativistic ion acoustic waves with trapped electrons in space have not been so well investigated. The effect of ion temperature and relativistic effect of ion acoustic waves have been considered as energetic phenomena in space plasma [3]. Although the nonlinear modulation phenomenon due to trapped electrons has been considered by Dewer et al. [4], they have not shown the solution of the modified nonlinear Schrödinger equation. Moreover, nonlinear modulation of relativistic ion acoustic waves associated with trapped electrons have not yet been investigated. The purpose of this investigation is to show the modulational instability due to trapped electrons in a relativistic plasma.

II. Relativistic solitary waves

We consider the small but finite amplitude ion acoustic wave propagating in an unmagnetized plasma. The plasma is quasineutral. It is assumed that the relativistic plasma is composed of a mixed fluid with nonisothermal resonant electrons and hot and adiabatic ions. We assume that there exist high speed streaming ions in equilibrium state and the ions have a relativistic effect. The dynamics of the relativistic ion fluid in non-dimensional form can be written as

\[ \frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nv) = 0, \quad (1.1) \]

\[ (\frac{\partial}{\partial t} + v \frac{\partial}{\partial x}) \gamma n (\frac{\partial}{\partial x} + (\frac{\partial}{\partial x}) + \frac{\partial}{\partial x} \phi) = 0, \quad (1.2) \]

\[ (\frac{\partial}{\partial t} + v \frac{\partial}{\partial x}) p + 3p \frac{\partial}{\partial x} n = 0, \quad (1.3) \]

where the Lorentz factor is \( \gamma = \left[1-(v/c)^2\right]^{-1/2} \). Poisson's equation is
\[ \frac{\partial^2 \phi}{\partial x^2} + n - n_e = 0 \]

(1.d)

with the electron density in the small amplitude limit,

\[ n_e = \exp(\phi) - \frac{4}{3} b \phi^{3/2} \]  

(1.e)

The effect of trapped electrons is expressed in terms of \( b = \pi^{-1/2}(1 - \beta) \). \( b \) introduces the deviation from the isothermality due to the contribution of nonlinear resonant electrons (both free and trapped) to the electron density. \( \beta \) is defined by \( \beta = T_i/T_r \). \( T_r \) and \( T_i \) are the free and trapped electron temperatures. We assume \( \beta > 0 \) and \( T_i < T_r \). We expand

\[
\begin{bmatrix}
  n \\
  p \\
  \phi
\end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_1 \\ v_1 \\ p_1 \\ \phi_1 \end{bmatrix} + \varepsilon^{3/2} \begin{bmatrix} n_2 \\ v_2 \\ p_2 \\ \phi_2 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_3 \\ v_3 \\ p_3 \\ \phi_3 \end{bmatrix} + \ldots .
\]

(2)

We introduce a scaling \( \xi = \xi^{1/4} (x - \lambda t) \), \( \tau = \tau^{3/4} t \),

(3)

where \( \varepsilon \) and \( \lambda \) refer to the smallness parameter and the velocity of the moving frame, respectively. Inserting eqs. (2) and (3) into eqs. (1.a) - (1.e)', we easily obtain a MKdV equation

\[ \frac{\partial \phi}{\partial \tau} + \frac{b}{\gamma_1 (1 + 3 \sigma)} \phi^{1/2} \frac{\partial \phi}{\partial \xi} + \frac{1}{2 (\gamma_1 (1 + 3 \sigma))^{1/2}} \frac{\partial^3 \phi}{\partial \xi^3} = 0 , \]

(4)

with \( \lambda (\sigma, \gamma_1) = \nu_0 + [(1 + 3 \sigma)/\gamma_1]^{1/2} \), and \( \gamma_1 = 1 + 3/2 (\nu_0/\omega)^2 \).

---

III. Relativistic ion oscillation modes and modulational instability

III. A Envelope solitary wave type of oscillatory solution

In order to seek the oscillatory wave solution in the small wave number region, we apply the following expansion. \( \phi \) is to be expanded as

\[ \phi = \sum_{\mu = -\infty}^{\infty} \sum_{n=0}^{\infty} \phi_{n \mu} (\eta, \zeta) \exp (i l (k \xi - \omega \tau)) , \]

(5.a)

where the condition \( \phi_{-n} = \phi_{n} \exp (i \pi n) \) be satisfied. We select a scaling

\[ \eta = \mu^{1/4} (\xi - \lambda \tau) , \quad \zeta = \mu^{1/2} \tau , \]

(5.b)

and apply eqs. (5.a) and (5.b) to a MKdV equation (4), where \( \mu \) is the smallness parameter. Substituting (5.a) and (5.b) into eq. (4), we can obtain the corresponding sets of equations. To the third order in \( \mu \) the \( l = 1 \) term implies a modified nonlinear Schrödinger equation

\[
\frac{i}{\partial \zeta} \phi_{1}^{(1)} + \frac{3 k}{2 (\gamma_1 (1 + 3 \sigma))^{1/2}} \frac{\partial^2 \phi_{1}^{(1)}}{\partial \eta^2} + \frac{b k}{(\gamma_1 (1 + 3 \sigma))^{1/2}} (\phi_{1}^{(1)})^{1/2} \phi_{1}^{(1)} = 0 ,
\]

(6)

by using the first and second order equations in \( \mu \). Although Dewer et al. derived a similar type of (6) with the different coefficients, they didn't find the solution. In order to obtain the solution of eq. (6), we simplify the problem. We rewrite eq. (6) using the transformations
\[ \phi_{1}^{(1)} = \frac{\psi}{B^2}, \quad \eta = A^{1/2} \eta', \quad \zeta = \zeta'/C, \quad (7) \]

where \( A = 3k/2, \quad B = bk, \quad C = [\gamma/(1+3\sigma)]^{-1/2}. \)

Substitution of eq. (7) to eq. (6) yields

\[ i \frac{\partial \varphi}{\partial \zeta'} - \frac{\partial^2 \varphi}{\partial \eta'^2} - \frac{|\varphi|^{1/2} \varphi}{\varphi} = 0. \quad (8) \]

In order to obtain the solution of (8), we replace \( \varphi \) by \( \varphi = \Psi(\nu) \exp \left[ i(\kappa \eta' - \Omega \zeta') \right] \), where \( \Psi(\nu) \) is a slowly varying real amplitude and \( \nu = \eta' - s \zeta' \). Substituting \( \varphi \) into eq. (8), we obtain

\[ \frac{\partial^2 \Psi}{\partial \nu^2} - (K^2 + \Omega) \Psi + \Psi^{3/2} = 0, \quad (9) \]

and

\[ (s+2K) \partial \Psi / \partial \nu = 0, \quad (10) \]

for the real and the imaginary part, respectively. We have

\[ \left( \frac{\partial \Psi}{\partial \nu} \right)^2 - (K^2 + \Omega) \Psi^2 + \frac{4}{5} \Psi^{3/2} = 0. \quad (11) \]

When we set \( \Phi = \Psi^{1/2} \), eq. (11) reduces to

\[ \left( \frac{\partial \Phi}{\partial \nu} \right)^2 = - \frac{1}{5} \Phi^2 (\Phi - \Phi_0), \quad (12) \]

where \( \Phi_0 = 5/4(\Omega + K^2) \). We note that \( \Phi_0 \) has to be positive for the stable solution. Integrating eq. (12), we then get

\[ \Psi(\nu) = \Phi_0 \sech^{1/2} \left[ \frac{1}{2} \left( \frac{\Phi_0}{5} \right)^{1/2} (\nu - \nu_0) \right]. \quad (13) \]

where \( \nu_0 \) is the integration constant. Using \( \varphi \) and eq. (13), and returning to the original expressions, we obtain an envelope solitary wave

\[ \phi_{1}^{(1)}(\Psi_0, \nu) = \frac{\Psi_0}{(bk)^2} \sech^{1/2} \left\{ \left( \frac{\Psi_0^{1/2}}{30k} \right) \left[ \frac{\eta - s}{2 \gamma_1 (1+3\sigma)} \right]^{1/2} \zeta - \nu_0 \right\} \times \exp \left\{ i \left( \frac{2}{3k} \right)^{1/2} \left[ \frac{Kx - \frac{3k}{2 \gamma_1 (1+3\sigma)}}{2 \gamma_1 (1+3\sigma)} \right]^{1/2} \Omega \zeta \right\}. \quad (14) \]

Next, let us consider the velocity of this solution. Using \( \Phi_0 \), eq. (12) for the imaginary part requires

\[ s = -2 \left[ -\Omega + (4/5) \Psi_0^{1/2} \right]^{1/2}. \]

In the low frequency limit, the linear dispersion relation is obtained as

\[ \omega = k \left\{ \lambda(\sigma, \gamma_1) - \frac{k^2}{2 \gamma_1 (1+3\sigma)} \right\}^{1/2}. \quad (15) \]

### III. B The stability of the oscillatory wave solution

In order to consider the stability of the envelope solitary wave, we define the coefficients and the potential of the modified nonlinear Schrödinger equation (8) as

\[ p = -3k/2 \left[ \gamma_1 (1+3\sigma) \right]^{1/2}, \quad q = -(bk/ \left[ \gamma_1 (1+3\sigma) \right]^{1/2}) \phi_{1}^{(1)} \mid^{1/2}, \]

and

\[ \phi_{1}^{(1)} = \phi \exp(-i \Omega_0 \zeta) \]

and two small sidebands at \( k \pm \xi \) and \( \omega \pm \Omega \). Then \( \phi \) is represented as
\[ \phi = \phi_0(\zeta) \{ 1 + \phi_+ \exp \left[ i(K\eta - \Omega \zeta) \right] + \phi_- \exp \left[ -i(K\eta - \Omega \zeta) \right] \} \]  

Substituting eq. (16) into eq. (8), we find

\[
\begin{bmatrix}
\Omega - K \frac{\partial^2 \omega}{\partial K^2} - pK^2 - \frac{1}{2} |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} & - \frac{1}{2} |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} \\
\frac{1}{2} |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} & \Omega - K \frac{\partial^2 \omega}{\partial K^2} + pK^2 + \frac{1}{2} |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2}
\end{bmatrix}
\begin{bmatrix}
\phi_+ \\
\phi_-
\end{bmatrix}
= 0.
\]

We obtain the nonlinear dispersion relation

\[
\left( \Omega - K \frac{\partial^2 \omega}{\partial K^2} \right)^2 = p \left( - |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} + pK^2 \right) K^2.
\]

When the nonlinearity is stronger than the dispersion, that is \( |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} > pK^2 \), \( p \frac{\partial^2 \omega}{\partial |\phi_0|^2} \) is positive. In this case, since the right-hand side of eq. (17) is negative, then sidebands turn out to be unstable. When \( K \) and \( \Re(\Omega) \) satisfy the conditions \( K_m = |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} \times (1/2p) \) and \( \Omega_m = K_m(\partial \omega / \partial K) \), the maximum growth rate is obtained as \( \Gamma_m = (1/2) |\phi_0| \frac{\partial^2 \omega}{\partial |\phi_0|^2} \), where \( K_m \) and \( \Omega_m \) are the maximum wave number and maximum frequency, respectively. The relativistic ion acoustic envelope solitary wave is therefore modulationally unstable against the nonlinear self-modulation.

IV. Concluding discussion

We have found modulational instability due to trapped electrons of relativistic ion acoustic waves. The following facts are found by the present investigation. (i) We have shown for the first time that there is a relation between a \( MKdV \) solitary wave and a modulationally unstable envelope solitary wave of a modified nonlinear Schrödinger equation in the long wavelength region. (ii) The amplitude of the solitary wave increases as a number of trapped electrons decrease. The velocity of the envelope solitary wave is proportional to the amplitude and is inversely proportional to the relativistic and the ion temperature effects. Hence, the velocity of the solitary wave decreases as the ion temperature and the relativistic effects increase. Since these results are in quantitative agreement with several features seen in the observations [5], it appears reasonable to assume that the modulational instability of relativistic ion acoustic solitary waves can be described by the nonlinear self-interaction in the plasma. Therefore, these results are applicable to the study of nonlinear modulation phenomena of energetic ion acoustic waves associated with trapped electrons propagating in space.

References

Cross-field ion acoustic instability and turbulence in a weakly magnetized plasma

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ABSTRACT
In a discharge plasma with a weak axial magnetic field the plasma conditions can be appropriate for the generation of the cross-field ion acoustic instability by azimuthally drifting electrons. This is observed either as a single mode near the lower hybrid frequency or as broadband turbulence up to the ion plasma frequency.

INTRODUCTION
In this paper we are concerned with the cross-field ion acoustic instability which occurs when (i) the relative drift between ions and electrons exceeds the ion acoustic speed $c_A$, and (ii) the electrons are magnetized whereas the ions are not ($\rho_e \ll \lambda \ll \rho_i$ and $\omega_{ce} \gg \omega \gg \omega_{ci}$). In the present experiment the electron drift occurs as a combination of diamagnetic and ExB (and sometimes $\nabla T_e \times B$) drift in the azimuthal direction. When the magnetic field and electron temperature lie above certain onset values, the instability appears either as a single mode or as broadband turbulence, or in both forms simultaneously. It was observed during an attempt to set up a modified two-stream instability, which likewise occurs at about the lower hybrid frequency but requires the stringent condition $k_B/k\sim (m_e/m_i)^{1/2}$. The present ion acoustic instability appears to be the same as that reported recently by Stenzel who used similar plasma conditions.

EXPERIMENTAL LAY-OUT
The experiment is performed in a cylindrical discharge plasma device of length 65 cm and diameter 40 cm, shown in Fig 1. Two concentric arrays of hot filaments are located at one end of the chamber and biased at -60 V to form the cathode. A nearby grounded grid $A_1$ and similar grid $A_2$ at the far end share duty as anodes and make the intervening region almost free of axial electric field while allowing the existence of a weak radial field. Large Helmholtz coils provide an axial magnetic field that is uniform to about 5 per cent. Plasma and wave properties are measured with the usual electrostatic probes (including an emissive probe EP) and energy analyzer EA. The latter receives ions either axially or azimuthally but is not strongly directional. Particle drifts are obtained from differential flux measurements with one-sided plane probes. The wave number of a given spectral component of the turbulence can be measured by correlating the signals from say $P_1$ and $P_2$ in a spectrum analyzer as $P_1$ is rotated.

Typical plasma conditions are as follows: $n \approx 5 \times 10^8 \text{ cm}^{-3}$, $T_e \approx 2.5 \text{ eV}$ (but can range up to 8 eV), $T_i \approx 0.4 \text{ eV}$, $B \approx 30 \text{ G}$, $p (\text{argon}) \approx 1 \times 10^{-4} \text{ torr}$, with the electron gyroradius
several mm and $f_{pe}/f_{ce}$ ranging from 3 to 30.

**EXPERIMENTAL RESULTS**

**Steady state profiles:** With two cathode filament arrays available, $C_1$ and $C_2$, the radial profiles of $n$, $T_e$, $V_p$ (plasma potential) and electron drifts can be varied greatly. In Fig 2 two sets of profiles are presented: in (a) the inner cathode ring $C_1$ is dominant, while in (b) both $C_1$ and $C_2$ are at high power. In order to obtain a sufficiently large value of $T_e$, the argon pressure is set at $1 \times 10^{-4}$ torr, which makes for a marked tail of fast primary electrons. (At $2 \times 10^{-4}$ torr the instability is usually absent). The effective temperature $T_r$ of the tail is especially large at the radial locations $r_1$ and $r_2$ of $C_1$ and $C_2$ respectively; see Fig 2(b). It appears that the tail is responsible for the observed high frequency electron turbulence (not shown) around $r_1$ and $r_2$, and that this in turn creates the peaks in the bulk electron temperature $T_e$. Figure 2(b) also shows a particular profile of $V_p$ for which $\nabla n$ and $E_r = -\nabla V_p$ have the same sign; opposite signs can, however, be arranged. Generally, the combined effect of the gradients is to produce an azimuthal electron drift $v_\phi$ of order $5 \times 10^6$ cm s$^{-1}$, which greatly exceeds the ion acoustic speed $c_i$ ($\sim 2.5 \times 10^5$ cm s$^{-1}$). The $n_1$ profile in Fig 2(a) represents the relative amplitude of the single mode, labelled $S$, and of the low level turbulence $N$ in the region of $C_2$.

**Single mode properties:** At the lower values of the magnetic field ($< 20$ G) density and potential oscillations are observed near the lower hybrid frequency, as in the similar experimental conditions of /3/. In Fig 3 we show the dependence of the frequency on the magnetic field for a mode which has maximum amplitude at $r = 6.5$ cm, together with the corresponding azimuthal mode number $m$. (There is some uncertainty about the assignment $m=5$). There are discontinuities in $k_\phi$ ($= m/r$) as the mode maintains the parameter $k_\phi \rho_e$ fairly constant, within the range $0.2$ to $0.3$, i.e. somewhat lower than the theoretical value for maximum growth rate, $0.7$ /4/. The frequency lies close to $\omega_{LH}$ at the higher values. Also, throughout the frequency range the azimuthal phase velocity is appreciably greater than $c_i$. Phase measurements show that the single mode propagates in the direction of the azimuthal electron drift and is a standing wave axially, with $\lambda_z \geq 120$ cm.

In Fig 4 the amplitude of a single mode is observed as a function of $T_e$ and $B$, where $T_e$ is controlled by varying the argon pressure (between 0.2 and 0.08 mtorr). In both cases the onset is sharp. As found in /3/, $T_e$ needs to be sufficiently large to avoid ion Landau damping. Furthermore, it is clear from kinetic theory (see Fig 8 in /4/ for example) that the growth rate falls rapidly as $k_\phi \rho_e$ rises above the value for maximum growth rate.

**Turbulence properties:** The strongest noise occurs usually near the radial position of the outer filament ring but becomes more widespread radially as the single mode gives way to turbulence at the highest values of $B$ ($> 25$ G). Typical outer region noise spectra are shown in Fig 5. As $B$ is increased there is a rise in amplitude and frequency width, and in the frequency $f_m$ of the sharp peak at the low frequency end. From the inset, $f_m$ is close to the lower hybrid frequency. Also, as found by Stenzel et al /5/ in an experi-
ment on ExB driven turbulence, the spectrum extends from $f_{LH}$ to about the ion plasma frequency ($\sim 2$ MHz in this case). An initial noise correlation measurement at a very low frequency, 10 kHz, indicated an azimuthal phase velocity of $0.4c_s$.

DISCUSSION
The above single mode propagates mainly in the azimuthal direction, with $k_\parallel / k$ ranging from 0.2 to 0.02, i.e. $\gg (m_2/m_1)^{1/2}$. This suggests that it is an ion acoustic rather than a modified two-stream instability. The unduly high value of its phase velocity (several times $c_s$) may be due to the presence of an ion drift, but it is not clear what could create the latter. The question of electron heating by the turbulence is clouded by the presence of axial electron drift and its effects. Also, the energy analyzer shows no difference between axial and azimuthal ion energies; there appears not to be any ion heating, in agreement with /3/.

ACKNOWLEDGEMENTS
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Fig 1. Experimental lay-out. $C_1$, $C_2$ cathode filaments; $A_1$, $A_2$ anodes; P electrostatic probes, EP emissive probe, EA energy analyzer; B axial magnetic field.
Fig 2. Radial profiles for two cathode settings: (a) plasma density $n_p$ and oscillation amplitude $n_1$; typically $n_1/n_p \leq 10\%$ for single mode S and $\leq 50\%$ for noise N; (b) plasma potential $V_p$, electron temperature $T_e$ (bulk) and $T_f$ (tail).

Fig 3. Frequency versus magnetic field for single mode with azimuthal mode number $m$, peaked at $r = 6.5$ cm (circles); upper line represents lower hybrid frequency $(\omega_n, \omega_p)^{1/2}$. These data give the four $\omega/k_b > c_s$, $n_0 \sim 3 \times 10^9$ cm$^{-3}$, $T_e \sim 3$ eV, $p = 1 \times 10^{-4}$ torr, $v_p \sim 5 \times 10^6$ cm s$^{-1}$.

Fig 4. Amplitude of single mode as a function of electron temperature (squares) and of magnetic field (circles).

Fig 5. Noise spectrum at different values of magnetic field. The frequency $f_m$ of the low frequency peak is plotted in the inset where the line represents $(v_e k_b)^{1/2}$.
NUMERICAL STUDY OF POTENTIAL FORMATION AND NEGATIVE RESISTANCE IN A CURRENT-CARRYING ION SHEATH

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Abstract Potential formation of a current-carrying ion sheath on a negatively charged grid is numerically studied using the one-dimensional particle code. The imbalance of ion which are injected into the sheath region from two plasmas divided by the grid, that is, the difference in plasma density produces a potential difference between the two plasmas, which reflects some ions injected from the high-density plasma. The thickness of the sheath facing low-density plasma is obtained to be thinner than at the sheath facing the high density plasma. A negative resistance is also numerically demonstrated by applying a perturbed voltage to the grid, and found to be associated with the ion inertia in the ion sheath. These numerical results are in good agreement with those obtained experimentally.

1. Introduction

In our previous paper, the coherent instability has been observed in the current-carrying ion sheath on the negatively charged grid/1/. When the instability is excited, there are the density and potential differences between two plasmas divided by the grid, and the sheath thickness on the low-density plasma side is thinner than that on the high-density plasma side. On the basis of these experimental results, we have proposed the physical model of the instability; the instability is excited when a negative rf resistance associated with the ion inertia in the ion sheath is coupled to an ion resonance caused by a positive feedback due to the reflection of ions in the sheath region. However, the experimental observations mentioned above are not clearly understood and are still open to studies of their physics.

The present paper gives a numerical study of potential formation and negative resistance in a current-carrying ion sheath by using the one-dimensional particle code.

FIG. 1. Schematic of experimental apparatus. $R_0=50\Omega$. 
2. Experimental Results

The experiments are performed in a large, unmagnetized plasma device 70 cm in diameter and 120 cm in length, equipped with multidipole magnets for surface plasma confinements, as shown in Fig. 1. The argon plasma produced by the dc discharge between the filaments and the chamber wall is divided by a fine-meshed grid. Typical plasma parameters in the region designated P are \( n_e = 10^7-10^8 \text{ cm}^{-3} \), \( T_e \sim 1 \text{ eV} \) and \( T_i \sim 0.1 \text{ eV} \). The density of the plasma Q is chosen to be less than that of the plasma P by about one order of magnitude. The plasma densities are controlled by the heater currents. A dc voltage is applied between the grid and a movable 12 cm diam target located at a distance of 4-6cm from the grid. Plane Langmuir probes 6 mm in diameter are used to measure the plasma parameters and their fluctuations. The ion temperature and the plasma potential are measured with Faraday cups and emissive probes, respectively. The time averaged value \( I_o \) and fluctuating component of the current through the plasma are obtained from the voltage drop across the resistor \( R_0 \). The gas pressure is kept at about 2x10^4 torr.

Figure 2(a) shows the dependence of the time-averaged current \( I_o \) on the applied dc voltage \( V_o \). The distance between the grid and target is kept at 5 cm. This \( I_o - V_o \) curve is very similar to the current-voltage characteristic of the Langmuir probe. As shown in Fig. 2(b), a coherent instability appears when \( I_o \) begins to saturate. A typical example of the frequency spectrum is illustrated in the inset. The frequency \( f_o \) of the instability is found to decrease with an increase in \( V_o \), and this frequency change is explained by the dependence of \( f_o \) on the sheath thickness.

Figures 3(a) and (b) show the axial profiles of space potential \( \phi \) and ion saturation current \( I_s \), respectively. The ion saturation current \( I_s \) is known to be proportional to \( ne \) if \( T_e \) is constant. It is found that \( \phi \) of the plasma Q is higher than that of the plasma P and the sheath thickness in the plasma Q is thinner than that in the plasma P. The

![Figure 2: Dependences of time-averaged value \( I_o \) and fluctuating component \( I \) of the plasma current on the applied dc voltage \( V_o \): (a) \( I_o - V_o \) characteristic; (b) \( V_o \) dependence of the frequency and amplitude of \( I \). A typical example of the frequency spectrum of \( I \) is illustrated in the inset at \( V_o = -60 \text{V} \).]
3. **Numerical Procedures and Results**

In order to study the formation mechanism of the current-carrying ion sheath, we have used the one-dimensional particle code (P. I. C). The ion fluxes are injected towards the grid from \( d=10 \) and \(-10\), as shown in Fig. 3(c), where \( d \) is the coordinate normalized by the Debye length. Based on the experimental results, we make the following assumptions: (1) At \( d=10 \) and \(-10\), electrons obey the Boltzmann relation and ions have the drifting Maxwellian distribution whose averaged ion velocity is about the ion sound velocity and satisfies the Bohm sheath criterion; (2) The electron density at \( d=10 \) is below one tenth of that at \( d=10\); (3) The potential \( \phi \) at \( d=10 \) and \( d=0 \) are kept at \( 0 \) \( V \) and \(-30 \) \( V \), respectively and the \( \phi \) at \( d=10 \) is determined to keep charge neutrality; (4) The transmission rate of the grid is \( 0.71 \). The potential profile is obtained selfconsistantly from solving Poisson's equation. Figure 3(c) shows the typical profiles of potential \( \phi \), ion density \( n_i \) and electron density \( n_e \). The potential \( \phi \) at \( d=10 \) is found to be larger than that at \( d=10\), as expected. This potential difference results from the ambipolar potential due to the imbalance between the ion fluxes in opposite directions. Some ions injected from \( d=10 \) and passing through the grid are reflected by this potential difference back to \( d=10 \). The ion density has a maximum at the position where most ions are reflected. This makes the sheath thickness at the low-density plasma side thinner than that on the other side, although the plasma density in the region of \( d \leq 0 \) is smaller than that in the region of \( d \geq 0 \).

In order to confirm the existence of the negative resistance in such a current-carrying ion sheath, the relation between the current at grid and the sheath potential is obtained by applying a perturbed potential \( \phi(t)=V_0+V_1 \sin t \) to the grid. The phase difference between the fluctuating components of current and potential, \( I(t) \) and \( \phi(t) \), is obtained by using Fast Fourier Transformation. The frequency \( \omega \) is normalized by the bounce frequency \( \omega_0 \), which is defined to be the inverse

![FIG. 3. Axial profiles of (a) potential and (b) ion saturation current, obtained experimentally. (c) Axial profiles of \( \phi \), \( n_i \) and \( n_e \), obtained numerically.](image-url)
of time necessary for the ions to come back to $d=0$. Figures 4(a) and (b) show the time series of $I_1$ and $\phi_1$ at $\omega=0.7$. When the grid bias is minimum (~29.9V), the ion flux is found to be maximum, i.e., $I_1$ and $\phi_1$ are out of phase. The $\omega$ dependences of the phase difference between $I_1$ and $\phi_1$ and the differential conductance $G (=\partial I_1/\partial V)$ are shown in Fig. 4(c). Here, $G$ is calculated from the amplitudes of $I_1$ and $\phi_1$. Both $G$ and phase difference depend strongly on the frequency $\omega$ and $G$ is negative at $\omega$'s below $\omega=1.0$. These results indicate that such a current-carrying ion sheath can be used as element of oscillator. When this negative resistor cancels out all loss including the resistor of external electric circuit, the system oscillates spontaneously.

4. Summary

A potential profile and negative resistance in a current-carrying ion sheath, which are strongly associated with ion-sheath instabilities, are numerically analysed by using the P. I. C. code. It is shown that the density difference between two plasmas divided by a negatively charged grid produces the space potential difference which reflects ions injected from the high-density plasma. The current-carrying ion sheath is also found to work as a negative resistor for a certain frequency range.

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Reference

Asymmetric $\vec{E} \times \vec{B}$ Equilibria and Diocotron Adiabatic Invariants

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Recent results on theoretical and experimental studies of nonlinear collective modes and asymmetric equilibria in pure electron plasmas will be presented. The standard cylindrical shape of a pure electron plasma can be distorted by applying voltages on azimuthal sections of the confining cylindrical walls. Because of the applied perturbing electric fields, the asymmetric equilibria have guiding centers which $\vec{E} \times \vec{B}$ drift along non-circular closed paths. The resulting plasma is asymmetric, but stationary in the lab frame. Some of the equilibria are dramatically convoluted. The plasma is imaged in a single shot. The equipotential contours are found using both a vortex-in-cell simulation and an analytical model. As expected, these contours are in excellent agreement with the observed plasma shapes. By displacing the plasma off axis, a diocotron-like mode is excited. The center of charge of the plasma follows a circular path when there is no applied voltage on the azimuthal sections. Measurements indicate that the area enclosed by the center of charge is an adiabatic invariant when the external applied electric field is slowly varying. A simplified Hamiltonian model is used to study this phenomenon.

The pure electron plasma is confined inside a highly conducting cylinder. The radial confinement is provided by a strong axial magnetic field of 1.2 kG. The axial confinement is achieved by biasing the end caps with negative potentials. More extensive descriptions of this type of setup can be found elsewhere /1/. The confining cylinder has a radius of 1.9 cm and a length of 5.0 cm. A typical electron column confined has density $\geq 5 \times 10^7$ cm$^{-3}$, temperature 3 eV, and length $\sim 3$ cm, while the radius can be chosen to suit the purpose of the experiments. The experiments are performed in a repeated inject/manipulate/dump cycle. During the inject phase, electrons from a hot tungsten filament are trapped inside the conducting cylinder. The plasma is then manipulated and, finally, in the analyze phase, one of the negatively biased end caps is grounded, allowing the electrons to rapidly dump out along the magnetic field lines onto a phosphor screen. The resulting image is captured by a CCD camera. There are 360° azimuthal sections on the wall, any of which can be biased to different potentials individually, hence changing the boundary conditions of the cylinder. The changes in boundary conditions can be used either to manipulate the equilibrium shapes of the column or to change the shapes of the diocotron-like mode orbits.

Initially a circular electron column located on axis is created in the inject phase. The asymmetric equilibria are then obtained by applying voltages to the azimuthal sectors on the wall, hence changing the boundary conditions of the confining cylinder. The unperturbed electron columns are $\sim 1.0$ cm in radius. The applied voltages are turned on slowly ($\sim 1$ ms) compared to the l=1 diocotron period ($\sim 10$ ms), so that no diocotron oscillations are excited while the equilibrium shape of the column deforms slowly. Examples of the different shapes that can be formed are shown in Figure 1.

For the time scales studied here, which are slow compared to the gyro-frequency and fast compared to the plasma lifetime, the electrons can be described by their guiding center coordinates. Furthermore, the motion along the axial magnetic field is fast compared to any drift motions of the guiding centers. The electric field experienced by a particle can, therefore, be approximated by the bounce-averaged field. Consequently, the dynamics of the system is two-dimensional, being determined by the bounce averaged $\vec{E} \times \vec{B}$ drifts along the electrostatic potential lines. The time scale for these experiments is short enough so that collisional effects and any transport due to misalignment are negligible. In summary, the system is treated analytically as a two-dimensional system in which guiding centers drift along potential lines.
For a uniform density plasma to be in equilibrium in the laboratory frame, the electrostatic potential on its surface must be constant. In such a case, the guiding centers on the surface will drift along the surface, and the shape of the plasma column will, therefore, remain unchanged. Hence, the condition for the plasma column to be in equilibrium is:

$$\phi(r_p(\theta), \theta) = \phi_p(r_p(\theta), \theta) + \phi_a(r_p(\theta), \theta) = \text{constant}, \quad (1)$$

where $r_p(\theta)$ specifies the shape of the uniform density plasma, $\phi_a$ is the externally applied sector potential, and $\phi_p$ is the potential induced by the plasma itself, which can be found by integrating the Green function for the cylindrical geometry with zero potential on the wall. In general, the integral can only be evaluated numerically. Analytic insight can be gained by replacing the field induced by an asymmetric plasma column with that of a circular column with the same area as the asymmetric column and an appropriately calculated surface charge density. Thus,

$$r_p(\theta) = r_0 + \delta r(\theta) = r_0 + \sum_{i=1}^{\infty} (\delta r_i^c \cos(\theta) + \delta r_i^s \sin(\theta)), \quad (2)$$

where $r_0$ is the radius of the cylindrical plasma used in the approximation. The surface charge density is $\sigma(\theta) = n \delta r(\theta)$, where $n$ is the density of the plasma. This approximation is valid when $\delta r \ll r_0$. The potential field, $\phi_p(r, \theta)$, can then be easily solved for, both inside and outside of the cylindrical column. Gauss's law relates the discontinuity of the electric field on the surface with the surface charge:

$$\phi_p(r, \theta) = 2 \pi en \left\{ \begin{array}{ll}
\frac{r_0^2 \ln\left(\frac{r}{r_0}\right)}{2} + \sum_{i=1}^{\infty} \frac{1}{2} \left[ (\frac{r}{r_0})^i - (\frac{r_0}{r})^i \right] r_0 [\delta r_i^c \cos(\theta) + \delta r_i^s \sin(\theta)] & \text{for } r > r_0, \\
\frac{r_0^2 \ln\left(\frac{r}{r_0}\right)}{2} - \sum_{i=1}^{\infty} \frac{1}{2} \left[ (\frac{r}{r_0})^i - (\frac{r_0}{r})^i \right] r_0 [\delta r_i^c \cos(\theta) + \delta r_i^s \sin(\theta)] & \text{for } r < r_0, 
\end{array} \right. \quad (3)$$

where $R$ is the radius of the confining wall (assumed to be at zero potential). The externally applied electrostatic potential inside the confining cylinder, $\phi_a(r, \theta)$, which obeys the Laplace equation, can be written as:

$$\phi_a(r, \theta) = \phi_{a0} + \sum_{i=1}^{\infty} \left[ \phi_{ai}^c \cos(\theta) + \phi_{ai}^s \sin(\theta) \right] \left( \frac{r}{R} \right)^i, \quad (4)$$

where $\phi_{a0}$ and $\phi_{ai}^{c,s}$ can be expressed in terms of the applied perturbation on the boundary, $\phi_a(R, \theta)$. The total field, $\phi = \phi_p + \phi_a$, is now given by the applied perturbations, $\phi_a(R, \theta)$, and by the parameters that specify the shape of the plasma column, $\delta r_i^{c,s}$. Since the potential is constant on the plasma boundary in equilibrium configurations (Eq. 1), these parameters can be calculated in terms of the externally applied perturbations. Substituting $\phi_p$ in Eq. 3 and $\phi_a$ in Eq. 4 into Eq. 1 and keeping the lowest order term in $\delta r$ yields

$$\delta r_i^{c,s} = - \frac{\left( \frac{r_0}{R} \right)^i}{2 \pi en r_0 \left[ 1 - \left( \frac{r_0}{R} \right)^2 \right]} \phi_{ai}^{c,s}, \quad (5)$$

The shape of the asymmetric plasma column can then be plotted out. Figure 2 shows the good agreements between the analytic theory, simulations, and the experimental results shown in Figure 1(a). The simulations are performed with a two-dimensional vortex-in-cell electrostatic code in which the plasma is represented by a finite number of line charges. Typically 4096 line charges are used in the simulation. In one cycle of the simulation, the charges are weighted to a uniform polar grid, the Poisson equation is then solved, and the line charges are moved.
according to the $\vec{E} \times \vec{B}$ dynamics. The external applied field is also slowly turned on in the simulations.

The preceding experiments were conducted in a manner so as not to excite bulk motion of the plasma. The well studied $l = 1$ diocotron mode is excited when a centered electron column is displaced from the center of the grounded conducting cylinder. (The image charge on the cylindrical wall creates a radial electric field which causes the electron column to drift in the azimuthal direction.) In a second set of experiments, the boundary conditions on the cylinder wall are changed after a very strong diocotron oscillation has been excited. Both the shape of the diocotron orbit and the shape of the column are then distorted by the changes in the boundary conditions. Measurements indicate that if the perturbations are applied slowly, the area enclosed by the orbit of the center of charge of the column is invariant, even through the shape of the orbit is substantially distorted.

For these experiments, a typical electron column has radius 0.5 cm, length 3 cm, density $5 \times 10^7$ cm$^{-3}$, and temperature 3 eV. During the manipulate phase, the electron column is first displaced from the cylinder center by growing an $l = 1$ diocotron mode orbit to about 1 cm off center. The period measured is 6 $\mu$s. Perturbations are then applied to the boundary conditions by ramping the voltage on one of the azimuthal sectors up to the value $V_p$ over a time $T_p$. Finally, during the analyze phase, the shape of the orbit is ascertained. The plasma is then dumped through one end so that its location can be determined. This cycle is repeated fifty times, each time dumping the electron column at a slightly different orbital position, thereby obtaining a set of points which defines the diocotron orbit.

Figure 3 shows an undistorted orbit, and two diocotron orbits that have been distorted by perturbations. Figure 4 shows the area, $A_{\Phi}$, for a wide range of perturbation voltages. $A_{\Phi}$ is an invariant for $V_p > 0$. For $V_p < 0$, some electrons cross the separatrix and are lost to the wall. In that case, the invariant no longer holds.

In order to understand the origin of the $A_{\Phi}$ adiabatic invariant, a Hamiltonian treatment of the guiding center system is necessary. The plasma can be modeled as a distribution of line charges, each with charge per length, $q$, and mass per length, $m$. For the case in which magnetic field, $B$, is constant the Hamiltonian is simply the total electrostatic energy per unit length.

The dynamics described by an N-particle Hamiltonian is in general quite complicated. However the analysis is greatly simplified if the shape of the plasma is assumed. In the simplest case we can model the plasma as having circularly shaped throughout its motion. This assumption is appropriate when the plasma radius is small compared to the wall radius and the diocotron orbit radius. Under these assumptions, the motion of the center of charge is governed by

$$
H(P_c, \Theta_c) = -enA_p \varphi_e(r_c, \Theta_c) + (enA_p)^2 \ln(1 - \frac{r_c^2}{R^2}),
$$

(6)

where the coordinates of the center of charge are $(r_c, \Theta_c)$, with $P_c = (\frac{BenA_p}{2c})r_c^2$ being conjugate variable to $\Theta_c$. $A_p$ is the area of the plasma, and $n$ is the density of the plasma. Since the system is dynamically one-dimensional, the motion is always periodic in the phase space $(P_c, \Theta_c)$. Hence, the set of orbits is a nested set of non-intersecting loops, each with a unique orbital area and a total electrostatic energy associating with it. When the externally applied perturbation is slowly varying, the area enclosed by the phase space orbit is an adiabatic invariant $\beta$:
Thus, the area enclosed by the center of charge orbit is an adiabatic invariant in the circular plasma approximation. A more realistic elliptical model has also been developed, where the plasma shape is assumed to be elliptical but the aspect ratio and the orientation are allowed to change.


\[ = \left( \frac{BenA_p}{c} \right) A_\phi. \]  

Fig. 1: (a) \( V_1 = V_2 = V_3 = -30V \),
(b) \( V_1 = V_2 = -30V \), and \( V_3 = 0V \).

Fig. 2: (a) Analytic theory, (b) simulation.

Fig. 3: Orbits for perturbations \( V_p = 0, 30, 50V \).

Fig. 4: \( A_\phi \) vs. \( V_p \), for \( T_e = 2ms \).
Compression of multiple ion acoustic solitons at the plasma sheath edge

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Behavior of multiple ion acoustic solitons at the plasma sheath edge is experimentally investigated. We present experimental results which show the evidence of a new mechanism for the compression phenomenon of multiple ion acoustic solitons. These characteristics are similar to the calculated results which employed a particle model for the pulse.

1. Introduction

When the multiple ion acoustic solitons propagate obliquely towards the reflector which is biased deeply negative potential, the velocity of each soliton is changed just before the plasma sheath edge with smaller change of the first pulse, but with greater change of the second and/or third pulse. In this result, the multiple solitons are once compressed into one soliton at the sheath edge, and it breaks into the multiple solitons again such as the incident ones after going through certain sheath area. This phenomenon is similar to the recurrence phenomenon \cite{1,2,3} of the plane soliton. In the present experiment recurrence seems to take place at the edge of the plasma sheath. The velocity ratio of the second pulse of the transmitted solitons, which goes through the plasma sheath edge, to that of the incident soliton becomes smaller than unity. The change of this velocity ratio could clarify the reflection and/or transmission phenomena \cite{4,5,6}.

2. Experimental set up and Results
The experiments are carried out in a double plasma device with multidipole magnets described elsewhere. The typical parameters in an argon plasma at a pressure of $3 \sim 4 \times 10^{-4}$ Torr are the plasma density $n_p = 5 \times 10^8$ cm$^{-3}$, the electron temperature $T_e = 2$ eV, and the ion temperature $T_i = T_e/(10 \sim 12)$ eV. The reflector, $(350 \times 300 \text{ mm}, 8 \text{ mesh/cm})$, can be rotated around its axis within an angle $(0 < \theta < 60')$, and be biased at the desired potential $V_b$ with respect to the plasma potential (floating potential $= -25 \text{V} < V_b < -500 \text{V}$). The ion acoustic solitons are picked up with cylindrical probes (0.1mm diameter by 1mm length). It was biased to collect electron saturation current. The wave is excited by a double plasma action by applying a positive pulse to the driver plasma with a typical pulse width $\Delta t \approx 10 \mu s$.

The typical wave forms are shown in Fig.1(a), where an incident (I), reflected ($R_1$, $R_2$), and transmitted (T) pulses are seen. The incident multiple solitons are compressed to be piled up near $z=1.5$ cm the sheath edge, and multiple solitons become a single soliton. After passing through this point, it again separates into multiple solitons (T).

Loci of the maximum amplitude of each pulse are figured in Fig.1(b). From this figure, we can obtain the velocity of each pulse. As also seen from Fig.1(a) Typical wave forms of the excited ion acoustic soliton. (b) Loci of the maximum amplitude of each pulse.
this figure, the second \( I_2 \) and/or third \( I_3 \) pulses of the incident solitons are accelerated more than the first pulse \( I_1 \) when the incident multiple-soliton goes to the sheath edge, and they are compressed there. The reflected soliton \( R_1 \) is produced at this point. As the second pulse goes towards the reflector with larger velocity, the reflected second pulse \( R_2 \) is produced near the reflector. Here, the velocity of each pulse is observed to be \( v_1 = (2.7 \sim 3.1) \times 10^5 \) cm/sec for the initial soliton, and \( v_{R1} = (0.94 \sim 1.0) \times v_1 \), and \( v_{R2} = (0.35 \sim 0.41) \times v_1 \) for reflected 1st and 2nd pulse, respectively. The point \( z=0 \) corresponds to the reflector position.

3. Discussions

An outline of the profile of the space potential in front of the reflector, which is biased deep in negative, is shown in Fig. 2(a), where \( D \) is a typical soliton width of the incident first soliton, \( \phi \) is a space potential, dotted line is a static space potential \( \phi_0(z) \) which is produced by the reflector and \( \phi_1(z) \) is illustration of the fluctuating potential caused by the ion bunch of the first pulse going into the sheath edge. From these results the space potential could be given as follows,

\[
\phi(z) = \phi_0(z) + \phi_1(z).
\]

(1)

Now, we focus our discussions on the locus of the second pulse from the point of view of the energy conservation law. By employing a particle model for the pulse for the second pulse, we can obtain,
\[ v_{12}^2 = v^2 + 2e\phi(z)/m, \]  

(2)

where, \( v_{12}, v \) and \( \phi(z) = \delta \phi \) are, respectively, the velocity of the second pulse at \( z=\infty \), that in the area \( L_1 \) and the space potential. \( m \) is an ion mass. From the experimental results on the amplitude of the first pulse, we can obtain \( |\delta \phi| = 0.26v \) as \( \delta n/n = 0.13e\delta \phi/KT_e \), with the electron temperature \( KT_e = 2eV \). Therefore, we may put \( \delta \phi = -0.26\sin(z - 1.5)\pi/D \), at \( L_1 \). The point \( z=1.5 \) corresponds to the sheath edge. From Eq. (2), the ion velocity \( v \) in the area \( L_1 \) is given as follows,

\[ \frac{v}{v_{12}} = \sqrt{1 + 0.53\sin(z-1.5)\pi/L_1}, \quad (1.5 < z < 2.0) \]  

(3)

Here, we put \( v_{12} = C_s(1 + 0.4 \times 0.11) \) at \( 1.5 < z \) with \( C_s \) the ion acoustic speed and/or \( \phi_0(z) = 0 \). In the area \( L_2 \) we can also put,

\[ \frac{v}{v_{12}} = \sqrt{1 - 0.53\sin(z-1.1)\pi/L_2}, \quad (1.1 < z < 2.0) \]  

(4)

where, \( \phi_0(z) = 0 \), and \( z = 1.1 \) is the point of \( \phi(z) = 0 \). When we put \( \phi(z) \ll |\phi_0(z)| = 50e^{-z} \) in the area \( 0 < z < 1.1 \),

\[ \frac{v}{v_{12}} = \sqrt{1 + 22.8e^{-z}}, \quad (0 < z < 1.1) \]  

(5)

should be satisfied. In Fig. 2(b), the calculated results of Eqs. (3) through (5) for the locus of the second pulse are shown with solid lines, and solid circles show the experimental results for \( \theta_1 = 30^\circ \) and \( V_b = -50 V \).

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References

1 Introduction

It is usual in studies of solitons and double layers in fluid plasmas to make the assumption of Boltzmann electrons, i.e. because of their high mobility, the electron mass is neglected\(^1\)\(^-\)\(^5\). In a recent paper\(^6\) it has, however, been suggested that electron inertia effects on ion-acoustic solitons may exceed relativistic effects.

The purpose of this study is to determine finite electron mass effects on the amplitudes and existence domains of ion-acoustic solitons. We consider a plasma consisting of cold hydrogen ions and two isothermal electron fluids at different temperatures. The electron density distributions are determined using the plasma fluid equations, thus introducing the electron mass.

Instead of using the small-amplitude KdV equation we carry out arbitrary-amplitude calculations from the fluid equations\(^1\)\(^-\)\(^5\). The existence of a soliton is deduced from the form of the Sagdeev pseudopotential\(^2\)\(^,\)\(^3\).

Our main result is that particularly in the case of rarefactive solitons, the existence domain is considerably over-estimated by the Boltzmann electron model.

2 Basic Equations

The plasma is assumed to be infinite, collisionless and unmagnetised. The normalised plasma fluid equations are then given by

\[
\frac{\partial n_j}{\partial t} + \frac{\partial (n_j u_j)}{\partial x} = 0, \tag{1}
\]

\[
\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + \frac{\gamma_j}{m_j n_j} \frac{\partial p_j}{\partial x} = \frac{Z_j e_j}{m_j} \frac{\partial \phi}{\partial x}, \tag{2}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = n_h + n_c - n_i, \tag{3}
\]

where standard normalisations have been used except for temperature which is normalised with respect to an effective electron temperature given by \(T_{\text{eff}} = T_h T_c / (n_{\text{co}} T_h + n_{\text{ho}} T_c)\). The subscript \(j = i, c, h\) refers to the ions, cold and hot electrons. Equation (2) explicitly introduces the electron mass, with the isothermal electrons having \(\gamma_{\text{c,h}} = 1\). The plasma is regarded as being undisturbed at \(|x| \to \infty\) and the following normalised boundary conditions hold there:

\[
\phi, \frac{\partial \phi}{\partial x}; u_i, u_c, u_h \to 0; n_i, n_c, n_h \to 1, n_{\text{co}}, n_{\text{ho}}; p_i, p_c, p_h \to 0, n_{\text{co}} T_c, n_{\text{ho}} T_h. \tag{4}
\]
3 Arbitrary-amplitude theory

We present an outline of this approach here. Further details can be found elsewhere\textsuperscript{1-4}. We seek time-independent stable soliton solutions and transform into a frame that is stationary with respect to the soliton using $\xi = x - \mu t$, where the Mach number $\mu$ represents the normalized soliton speed. This enables us to integrate (1) immediately, yielding

$$u_j = \mu \left(1 - \frac{n_{o_j}}{n_j}\right)$$  \hspace{1cm} (5)

for the velocities of both the electrons and the ions. With the aid of the equation of state $p_j n_j^{-\gamma} = p_{o_j} n_{o_j}^{-\gamma}$, one finds

$$p_i = 0; \quad p_{c,h} = n_{c,h} T_{c,h}.$$  \hspace{1cm} (6)

Using equations (5) and (6) in equation (2) and integrating, we obtain the ion density explicitly,

$$n_i = \frac{1}{\left(1 - \frac{2}{n_i^\gamma}\right)^{\frac{1}{\gamma}}},$$  \hspace{1cm} (7)

while the densities of the two electron species may be found by solving equations of the form

$$m_e \mu^2 \frac{n_{e,ho}^2}{n_{e,h}^2} + 2 T_{c,h} \ln \left(\frac{n_{c,h}}{n_{e,ho}}\right) = 2 \phi + m_e \mu^2.$$  \hspace{1cm} (8)

Once $n_i, n_c$ and $n_h$ have been determined, equation (3) becomes

$$\frac{d^2 \phi}{d\xi^2} = n_h + n_c - n_i = N(\phi, \mu) = -\frac{d\psi(\phi, \mu)}{d\phi}$$  \hspace{1cm} (9)

where $\psi(\phi, \mu)$ is the Sagdeev potential, defined as

$$\psi(\phi, \mu) = -\int_0^\phi N(\xi, \mu) d\xi.$$  \hspace{1cm} (10)

To obtain soliton solutions for fixed $\mu$, the following conditions must hold: $\psi(0) = \partial \psi(0)/\partial \phi = 0, \psi(\phi_0) = 0$ for some $\phi_0$ and $\psi(\phi) < 0$ for $0 < |\phi| < |\phi_0|$. Equation (9) can then be integrated with initial value $\phi(0) = \phi_0$ to yield a soliton solution.

4 Results and discussion

We have studied both compressive (positive electrostatic potential) and rarefactive (negative potential) solitons. In general we find that for compressive solitons the inclusion of finite electron mass has a smaller effect on amplitude and existence criteria than is the case for rarefactive solitons.

In all the figures below, the dashed line corresponds to the case where the electrons have a Boltzmann density distribution and the solid line to that in which the electron mass is taken into account. The former are essentially the results found by Baboolal, et al\textsuperscript{9}. The data presented here are for a temperature ratio $T_h/T_c = 30$, but they are typical of results for other values of the temperature ratio. In Figure 1 we show the Sagdeev potential for a typical rarefactive soliton ($\mu = 1.1; n_{e,ho} = 0.2$). The pseudopotential well is seen to be deeper in the
case of finite electron mass than for the Boltzmann electrons, and it cuts the x-axis at a greater negative value of electrostatic potential. Thus the corresponding soliton has a slightly larger amplitude than one would deduce from a model neglecting the electron mass, as is shown in Figure 2. Our calculations for compressive solitons showed little effect on amplitude and are not presented.

A more important consideration than the effect on soliton amplitude is the effect of finite electron mass on the existence domains of the solitons. Figure 3 shows a comparison between such domains for compressive solitons. The parameter space we consider is the soliton amplitude plotted against the normalised unperturbed cold electron density. The parameter labelling the different curves is the Mach number. As can be seen, in general the curves for the Boltzmann and finite mass electron cases lie on top of one another, which indicates, as already mentioned, that the soliton amplitude is well-represented by the 'massless' model. However, we do note that solitons for a particular Mach number exist for a larger range of cold electron density values in the Boltzmann case than when the electron mass is taken into account. The 'massless' electron model also admits soliton solutions for higher Mach numbers (e.g., \( \mu = 1.55 \)), where no solitons exist for finite electron mass.

In Figure 4 we present the existence domain for rarefactive solitons. As was seen earlier for a particular value of \( n_{\infty} \), the maximum negative potential (amplitude) of a soliton is generally found to be somewhat greater when the electron mass is considered than when it is neglected. On the other hand, the Boltzmann electron model overestimates considerably the range of \( n_{\infty} \) and \( \phi_c \) over which solitons will occur\(^3\). This result obviously has considerable importance in that, guided by the 'massless' electrons model, one may seek solitons in a part of parameter space which is totally inappropriate.

Using a temperature ratio value of 15, we sought double layers which were predicted by the Boltzmann electrons model\(^1,3\), but were unable to find them when including finite electron mass. A careful search did, however, reveal their existence for \( T_h/T_c > 20 \). Details will be presented elsewhere.

We should note that our calculations have dealt with the case of the smallest possible ion/electron mass ratio, viz. a hydrogen plasma. Clearly if one is dealing with more massive ions, the neglect of electron mass is more easily justified.

In dealing with finite electron mass in this paper, we have nevertheless assumed that because of their high mobility, both electron species are isothermal fluids. An alternative model for higher temperature ratio is one in which the more sluggish cold electrons are assumed to form an adiabatic fluid, while the hot electrons are isothermal. Results of these calculations will be presented elsewhere.

5 References

Figure 1: Typical Sagdeev potential.

Figure 2: Typical rarefactive soliton.

Figure 3: Existence domains for compressive solitons in $\phi_o - n_{co}$ plane.

Figure 4: Existence domains for rarefactive solitons in $\phi_o - n_{co}$ plane.
The theoretical studies using the fluid model show that, in a two electron temperature plasma, the ion-acoustic rarefactive soliton (IARS) exists only for certain density and temperature ratios of the hot and cold electron species /1,2/. This is in broad agreement with experimental observations /3/. However, experimental study of the propagation characteristics of the IARS shows that there are discrepancies between the theoretical predictions and experimental observations. Recently, Sayal and Sharma /4/ have studied IARS taking kinetic effects of electrons and using fluid equations for cold ions. Their results are closer to the experimental observations /3/ but some discrepancy in the variation of width with amplitude is still unexplained. Here, we try to explain this discrepancy assuming that there might be a small fraction of impurity ions in the experiment /3/. It is found that the results are improved qualitatively as well as quantitatively.

We consider a plasma consisting of two cold ions having densities $N_1$, $N_2$, masses $m_1$, $m_2$ and ionization $z_1$, $z_2$ and two electron components having different temperatures $T_c$, $T_h$ and densities $n_c$, $n_h$. We assume that electrons follow the kinetic model while the ions are governed by the fluid equations.

The density of the electrons can be given as follows /4/:

$$n_s = \alpha_s A_s (1/\gamma_s \phi) + 1/\gamma_s |\beta_s| X(\beta_s \phi)$$

(1)

where $s = c$ or $h$ corresponds to cold or hot component respectively. $\beta_s$ is an arbitrary number associated with reflected electrons. $\psi$ is amplitude of the potential $\phi$ and $-\psi \leq \phi \leq 0$ for the rarefactive soliton. $\alpha_s = n_{os}/n_o$, $\Theta_s = T_{eff}/T_s$, $\phi = \psi + \rho$ is normalized with $T_{eff}/e$, $T_{eff} = T_h T_c/(\alpha c_h + \alpha c_T)$
\[ a_s = \left( 1 + \frac{1}{\gamma |\beta_s|} \right)^{-1}. \]

\[ X(\beta_s x) = \exp(\beta_s x) \text{erf}(\gamma(\beta_s x)) \] or \[ \frac{2}{\gamma} W(\gamma(-\beta_s x)) \] for \( \beta_s \geq 0 \) respectively,

\[ W(x) = \exp(-x^2) \int_0^x \exp(t^2) \, dt, \quad l(x) = \exp(x) \left( 1 - \text{erf}(\gamma x) \right). \]

Since ions are assumed to be cold, the dynamics of the ions can be determined by fluid equations

\[ \frac{\partial N_1}{\partial t} + \frac{\partial}{\partial x} (N_1 u_1) = 0 \]
\[ \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = - \mu_1 z_1 \frac{\partial p}{\partial x} \]

where \( i = 1 \) and \( 2 \), \( \mu_1 = 1 \) and \( \mu_2 = m_1/m_2 \). Ion density \( N_i \), velocity \( u_i \), \( x \) and \( t \) are normalized by \( n_0 \), \( \gamma(T_{\text{eff}}/m_1) \), \( \lambda \text{Deff} \) and \( \omega_{pi} \) respectively.

For the steady state solution of (2) and (3), we use the transformation \( \xi = x - \mu t \) and get,

\[ N_i = M \alpha_1 / (M^2 - 2 \mu z_1 \rho)^{1/2} \]

where \( M = \sqrt{\gamma(T_{\text{eff}}/m_1)} \), \( \alpha_1 \) = normalized unperturbed density of \( i \)th ion species.

Ions and electrons densities are coupled with the normalized Poisson’s equation

\[ \frac{\partial^2 \varphi}{\partial \xi^2} = \frac{n^+ - n^-}{n_0} N_1 - z_2 N_2 \]

Using the conditions that as \( |\xi| \to \infty \), \( \varphi \) and \( \varphi' \to 0 \) we get from (1), (4) and (5)

\[ \frac{1}{2} \left( \frac{\partial \varphi}{\partial \xi} \right)^2 = - V(\varphi) \]

where

\[ V(\varphi) = M^2 (\alpha_1 - \alpha_2/\mu_2) - M ((M^2 - 2z_1 \varphi)^{1/2} + (M^2 - 2\mu_2 z_2 \varphi)^{1/2}) \]

\[ - \frac{\alpha_{1 c}}{\theta_c} t_c(\varphi) - \frac{\alpha_{2 c}}{\theta_c} t_c(\varphi) \]

\[ f_1(\varphi) = 1(\Theta_s(\psi + \varphi)) - 1(\Theta_s(\psi)) + \frac{2}{\sqrt{\gamma}} \left( 1 - 1/\beta_s \right) \Theta_s (\gamma(\psi + \varphi) - \gamma \psi) + \frac{1}{\beta} \Theta_s \gamma \beta_s \]

In order to get rarefactive solitary solution of (6), Sagdeev potential (7) must satisfy following conditions

at \( \varphi = 0 \), \( V''(0) = (z_1^2 \alpha_1 + z_2^2 \mu_2 \alpha_2)/M^2 - n_c'(0) - n_h'(0) \leq 0 \)
at ϕ = −ψ, \( V'(−ψ) = M(z_1 \alpha_1 / (M^2 + 2z_1 ψ)^{1/2} + z_2 \alpha_2 / (M^2 + 2μ_2 z_2 ψ)^{1/2}) \)

\[ - \frac{α A}{c} - \frac{α A}{h} ≤ 0 \]  

(8. b)

and \( V(−ψ) = 0 \)  

(8. c)

where \( n_s(0) = α A \theta θ_s \left( θ ψ \right) ± \sqrt{| β_s | X(θ ψ)} \) and + or − sign refer to \( β_s \) or < 0.

The nonlinear dispersion relation for IARS obtained from (7) using condition (8. c) is

\[ M^2(\alpha_1 + α_2 μ_2) - M((M^2 + 2z_1 ψ)^{1/2} + (M^2 + 2μ_2 z_2 ψ)^{1/2}) \]

\[ - \frac{α A}{c} / \frac{c}{c} (−ψ) + \frac{α A}{h} / \frac{h}{h} (−ψ) = 0 \]

(9)

Perturbation in bulk ion density \( N_1 \) at \( ϵ = 0 \) i.e. at \( ϕ = −ψ \) is

\[ \delta N_1 = α_1 / (M/(M^2 + 2z_1 ψ)^{1/2} - 1) \]

(10)

From (6) we can define the width of the soliton as follows

\[ d = \int_{−ψ+δ}^{−ψ} \frac{dϕ}{V'(−ψ)} = (2(ψ_1 - ψ)/V'(−ψ))^{1/2} + \int_{−ψ}^{−ψ/2} \frac{dϕ}{\sqrt{-2V(ϕ)}} \]

(11)

where \( ψ_1 = 0.99 ψ \).

Numerical analysis is performed with (9), (10) and (11) along with the conditions (8.a&b) for different values of charge multiplicity and concentration of impurity ions, taking suitable values of \( β_c = 1.28 \) and \( β_h = 0.1 \) as in ref./4/ and keeping \( T_h / T_c = 14 \), \( α_h / α_c = 12.6 \) as in experiment /3/.

Fig.(1) shows the variation of Mach number with the amplitude \( (δN_1) \) of the IARS. We have chosen suitable values of \( z_1, z_2 \) and \( α_2 \) and compare with the experimental values as obtained by Nishida and Nagasawa /3/ and earlier theoretical results (i.e. \( α_2 = 0 \) /4/). In fig.(2) the variation
of square of width with the inverse of the amplitude is shown for the same sets of parameters as in fig. (1). We find that the presence of a very little concentration of negative ion impurity ($\alpha_2 = 0.002$, $z_2 = -2$) improves the results qualitatively as well as quantitatively. In fig. (2) we see that in the absence of impurity ions (i.e., $\alpha_2 = 0$), the width decreases as amplitude increases up to a certain value of amplitude after that the width abruptly increases while in the presence of a small fraction of negative ion impurity ($\alpha_2 = 0.002$, $z_2 = -2$) the width decreases as amplitude increases and the graph is more nearer to the experimental values. We also see, from fig. (1) that for $\alpha_2 = 0.002$, $z_2 = -2$ the curve is seen to be in good agreement with experimental observations.

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TRANSPORT OF TIME-VARYING CURRENTS BY WHISTLER WAVES

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Time-varying currents in magnetized plasmas are of fundamental interest as electromagnetic wave phenomena and of importance in many applications such as plasma switches \cite{1}, pulsed beams \cite{2}, and electrodynamic tethers \cite{3}. The present work addresses switched and pulsed plasma currents in the parameter regime of electron MHD \cite{4}, i.e. magnetized electrons, unmagnetized ions, time scales $\omega_{ce}^{-1} < t < \omega_{ci}^{-1}$, spatial scales $r_{ce} < r < r_{ci}$. Transient currents are established between electrodes immersed into a uniform, effectively unbounded magnetoplasma. It will be shown that the currents are not only due to particle collection or emission but also induction by the insulated conductors connecting the two electrodes. The unique feature of the present experiment is the capability to map with probes the vector magnetic field in three-dimensional space and time $B(t, \mathbf{r})$, from which the total current density (conduction + displacement) is calculated $J(t, \mathbf{r}) = \nabla \times B(t, \mathbf{r})$ for arbitrary (asymmetric) field topologies \cite{5}. The major results include the observations that (i) pulsed currents are transported by whistler wave packets, (ii) the associated electromagnetic fields are essentially force-free $J \times B + \nabla \psi = 0$ such that large currents can be transported without pinching, kinking, etc.; (iii) that cross-field current closure is established by Hall currents associated with space-charge fields; and (iv) that moving dc current sources emit transient currents forming Cherenkov-like structures.

The experiments are performed in a device schematically shown in Fig. 1. Time-varying magnetic fields associated with currents from biased electrodes are measured with a magnetic probe consisting of 3 orthogonal loops movable throughout the central uniform plasma volume. From many ($> 10^5$) highly reproducible pulses the field topology $B(t, \mathbf{r})$ is assembled with the help of digital data storage and analysis.

Fig. 1: Experimental setup. The present paper deals with pulsed currents from biased electrodes.

Fig. 2 shows contours of constant axial current densities $J_z(y, z)$ at different times after switching on a positive bias to a 5cm dia. disk electrode. The current front propagates at a speed $v_z = 10^8$ cm/s which exceeds the electron thermal velocity. By modulating the current (0.5 ≤ $f$ ≤ 5MHz) and measuring the axial wavelength, a dispersion $\omega/\omega_{ce} = (k_z c/\omega_{pe})^2$ matching that of low frequency whistlers along $B_0$ has been measured /5/. The propagation speed is independent of the thermal speed of the collected particles, i.e. the same when collecting slow ions or emitting a fast electron beam /3/. The dashed contours in Fig. 2 ($J_z < 0$) are due to induced return currents providing a coaxial current closure to the chamber wall which is the negative electrode. In addition to the axial current component there are azimuthal and radial currents to be shown next.

**CONTOURS OF $J_z(y,z)$**

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Fig. 2: Current density contours $J_z(y,z)$ at different times after applying a positive bias between an electrode and the chamber wall. The current front propagates in the whistler mode. Induced return currents (dashed contours) flow coaxially back to the wall.


Fig. 3 displays magnetic field components $\mathbf{B}(t)$ produced by a current pulse rather than a current step (Fig. 2). The duration of the pulse ($\Delta t = 0.2\mu s$) is shorter than the transit time through the device such that the current pulse becomes detached from all boundaries, hence all fields and currents must close within the plasma. The axial current $I_1$ produces an azimuthal magnetic field $\mathbf{B}_a$ shown as vectors (d) and field lines (e). A radial space-charge electric field produces an electron Hall current $I_0$ which generates an axial magnetic field component (a,b). Superimposing the solenoidal and toroidal field contributions schematically yields coaxial field helices (c). Both the time-varying magnetic field $\mathbf{B}(t)$ and $I(t)$ have similar topologies. A study of the electromagnetic force density $f = I \times \mathbf{B} + \rho \mathbf{E}$ (where $\mathbf{B} = \mathbf{B}(t) + \mathbf{B}_0$) has indicated a negligible force on the conducting fluid, i.e. the electrons /8/. Large current pulses ($I = 150A$) with $B_0$ approaching $B_0$ exhibit the same field topologies as small pulses (0.05A).

In many situations, the two electrodes which feed current into the plasma are separated across $B_0$, such that plasma currents must also close across $B_0$. A typical example is an electrodynamic tether /3/, a laboratory model of which is shown in Fig. 4. The contour maps of the current density components $J_y = J_0$ and $J_x$ indicate that two field-aligned current channels propagate away from the electrodes. Since the total current must be closed ($\nabla \cdot J = \nabla \cdot (V \times \mathbf{B}) = 0$) there must be a cross-field current $I_x$. This current is induced by the insulated wire connecting the two electrodes. Fig. 4b shows magnetic field lines in a plane perpendicular to the tether wire. One can see two induced image currents $(V \times \mathbf{B})_x = \mu_0 J_x$ flowing across $B_0$ and propagating in the whistler mode along $B_0$. The total current $\int J_x dy dz$ equals the current $I_x$ in the wire, hence the whistler wave current provides for the cross-field closure between the field-aligned current channels. The wave current is

(a) Pulsed currents established by two tethered electrodes separated across $B_o$. The current closure across $B_o$ is accomplished by wave currents $I_p$ induced by the insulated tether wire as evident from the field line map $(B_x, B_y)(y, z)$ in (b).

essentially an electron Hall current due to a space charge electric field $E_y \parallel B_o$. Polarization and displacement currents are negligible in the electron MHD regime ($\delta/dt \ll \omega_c$).

When a dc current-carrying conductor such as the electrodynamic tether moves with high velocity across $B_o$, the stationary plasma carries a time-varying current due to the finite time the conductor spends in a flux tube ($\Delta t = d/v$, where $d$ is the conductor size and $v$ the spacecraft velocity). By superimposing many delayed current pulses ($\tau_{delay} = \Delta t$) emitted at displaced electrode positions ($\Delta y = d$) one can obtain the approximate plasma current distribution behind a moving dc current source $/9/$. Fig.5 presents contours of $J_y$ in the $y-z$ plane $\perp$ to the tether which moves with $v_y$ across $B_o$. Similar currents ($J_y, J_z$) flow from each end electrode as whistler "wings" which provide for the necessary current closure.

In summary, time-varying currents are associated with electromagnetic waves which, in the electron MHD regime, are low frequency whistlers. Besides establishing the basic physics through careful experiments a practical model for an electrodynamic tether in space has been investigated.
EXCITATION OF LOW FREQUENCY WHISTLERS
BY MAGNETIC LOOPS

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The physics of wave excitation from magnetic loop antennas in magnetized plasmas is of general interest in space and fusion plasmas. In the former area, large scale structures such as electrodynamic tethers and magnetic loops /1/ are useful for exciting very low frequency electromagnetic waves for communication purposes. In the fusion area antennas are used for rf heating near ion cyclotron and hybrid frequencies /2/. Fundamental questions deal with the coupling, often lumped into a single parameter, between the antenna and a plasma eigenmode, the radiation resistance /3/, and the radiation patterns /4/, nonlinear effects at high power levels /5/, sheath resonances and instabilities /6/, motional effects /7/ and noise limits for receiving antennas /8/. In the present work the excitation of low frequency whistlers (ωci << ω << ωce) by magnetic antennas is investigated in a uniform, effectively unbounded magnetoplasma. In contrast to previous studies, the complete wave magnetic field in space and time is measured. This yields a detailed picture of the three-dimensional field topology between near-zone and far-zone, and it allows the calculation of plasma currents (∇ x B = μJ) which are often ignored in wave studies. The major findings are that spatial whistler wave packets generally assume a 3-D magnetic field configuration (B_x, B_y, B_z ≠ 0) consisting of nested toroidal and solenoidal contributions. The electric field is dominated by a radial space charge field while cross field currents are mainly Hall currents. The coupling efficiency (B_wave/B_ant) increases with density. The real part of the antenna impedance is not only due to the radiation resistance but predominantly due to transit-time electron damping. The latter also causes efficient electron heating which can modify the plasma conductivity and modulate the radiation pattern /5/. Moving an antenna across B_o causes a Cherenkov-like radiation pattern, consisting of whister "wings" /7/.

The experiments are performed in a large (1m diam. x 2.5m length), magnetized (B_o = 15G) afterglow plasma with parameters given in Fig.1. Time-varying magnetic fields are detected with a magnetic probe containing three orthogonal loops movable throughout the central uniform plasma volume. From many (>10^5), highly repeatable events, the field topology B(t) is assembled with the help of digital data storage and processing.

Fig. 1 Experimental setup.

Fig. 2 displays magnetic "field lines" \((B_y, B_z)\) in the y-z plane at different times after applying a single current pulse to the magnetic loop antenna which is sketched in the top panel. The actual magnetic field lines pass at different angles through the y-z plane since there is also a \(B_x\) component (Fig. 3). One can see both the dipolar near-zone field of the loop as well as two induced image loops propagating in \(\pm z\) direction away from the antenna. The propagation speed \((v_x = 10^8 \text{ cm/s})\) is that of low frequency whistlers, \(v_g = 2v_{ph} = 2(c/\lambda_{pe})(\omega/\omega_{ce})^{1/2}\), although the single pulse introduces a spread in frequencies \((\epsilon = \Delta T_{\text{pulse}}^{-1} = 5 \text{ MHz})\), hence also in group and phase velocities. However, an advantage of the pulse excitation is the easy distinction between plasma response and imposed antenna fields, the latter vanishing after the end of the pulse (bottom two panels).

Fig. 3 shows the magnetic fields and current densities in an x-y plane at a distance \(z = 30\text{ cm}\) from a 5cm

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**Fig. 2** Field lines connecting the \((B_y, B_z)\) components in the y-z plane at different times \(t\) after applying a current pulse to the loop antenna sketched in the top panel. Induced image current loops propagate in the whistler mode away from the antenna.

**Fig. 3** Comparison of magnetic field lines and current density lines for the transverse field components in the x-y plane \(\perp B_0\). Field lines and vector fields show a toroidal topology, linked with the axial \(B_z\) and \(J_z\) components (bottom panels).

Fig. 4 Contours of the axial wave magnetic field component $B_z(y,z,t)$ showing the evolution of the wave from the near-zone into the far-zone. V-shaped contours arise from the higher propagation speed along $B_0$ than oblique to $B_0$. Small wavelets are due to dispersion of the whistler wave pulse.

Fig. 5 Three-dimensional view of the loop excited whistler wave packet. The surface indicates a constant value of the wave magnetic energy density $B^2/2\mu_0$ at one instance of time.

diam. loop antenna. From the projected field lines or the vector fields one can identify a toroidal field topology for both $B$ and $J$. The sense of rotation reverses with radius. Likewise the axial field and current density components which are shown as contour plots in the bottom panels also exhibit sign reversals as a result of wave packet dispersion.

The transition of the wave packet from the near zone into the far-zone is shown in Fig. 4. Contours of axial wave magnetic field at different times show the formation of V-shaped phase and amplitude profiles. These arise from the higher wave propagation speed along $B_0$ than oblique to $B_0$. The major negative $B_z$ distribution is the response to the antenna current rise, the subsequent positive $B_z$ maximum is induced by the antenna current drop. Several small wavelets near the wave front are created by dispersion or, alternatively, by successive induction with sign reversals according to Lenz's law. Note that the wave energy flows predominantly along $B_0$ while the phase normals are highly oblique to $B_0$. A three-dimensional display of the wave magnetic energy density is given in Fig. 5. The surface $B^2/2\mu_0 = \text{const.}$ is azimuthally quite symmetric. Note that this field distribution develops self-consistently in a uniform plasma free of boundary effects. The wave packets are neither plane waves nor column eigenmodes such as helicons.

Fig. 6 presents a physical model which explains the observed field topology in simple terms. During the antenna current rise an inductive electric field \( E_\text{L} = -\partial A / \partial t \) anti-parallel to the antenna current is imposed on the plasma. The response of the unmagnetized ions can be neglected compared with that of the electrons which undergo radial \( E_\text{L} \times B_\circ \) (adiabatic compression). A negative space charge accumulates on axis near the antenna, while a positive charge remains off axis due to the loss of electrons. The excess space charge has two consequences: (i) A radial space charge electric field \( E_\text{sc} = -\nabla \phi \) drives an azimuthal Hall current \( I_\theta = -neE_\text{sc} \times B_\circ / B_\circ^2 \) which generates the axial "induced" magnetic field \( B_\theta(t) \) (see Fig. 5). (ii) Field-aligned currents \( I_z \) are set up to neutralize the excess space charges. These generate the observed azimuthal magnetic fields \( B_{\theta 0} \) (see Fig. 4). The induced currents and fields are accelerated by \( I \times B \) forces away from the antenna. Far from the antenna near-zone they propagate at approximately constant speed as electromagnetically force-free fields, \( I \times B + neE = 0 \) \( \phi \). When the antenna current decreases in time, the sign of the induced fields, charges, and currents reverses. The space charge electric field in the whistler wave has been directly measured with an emissive dipole probe.

![Fig. 6 Physical model explaining the observed fields and currents induced by a rising current in the magnetic loop antenna.](image)

The near-zone fields are at least one order of magnitude larger than the wave magnetic fields. When the antenna current is increased, significant electron acceleration and heating is observed. As long as the plasma is collisionless \( (v_e \ll \omega \ll \omega_{ce}) \), the field topology and wave propagation are not significantly changed up to \( B_\text{wave} \approx B_\circ \). However, in an initially cold, Coulomb-collision dominated plasma the electron heating creates a channel of highly conducting plasma in which the wave propagates without damping. Such conductivity modifications produce nonlinear propagation effects similar to filamentation instabilities \( /5/ \).

In summary, a detailed study of the excitation of low frequency whistlers by magnetic antennas has been performed. The plasma anisotropy and space charge effects in the regime of electron MHD \( /10/ \) give rise to complicated fields for even symmetric antenna geometries. Nonlinear effects need further investigations.

NON-STATIONARY PROPAGATION OF NONLINEAR ION ACOUSTIC WAVES IN A PLASMA WITH NEGATIVE IONS

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I. INTRODUCTION

It is well known that small amplitude ion acoustic waves in plasmas can be described by Korteweg-deVries equation. However, the response of the plasma to disturbances is drastically modified by the presence of negative ions. In a plasma with electrons, one species of positive ions and one species of negative ions, ion acoustic waves of small amplitude and weak nonlinearity can be described, in terms of nondimensional quantities, by the equation

\[
\frac{\partial \phi}{\partial \tau} + \frac{1}{2} \left( \frac{3 \alpha_\alpha}{V^4} - 3 \frac{\beta_\alpha}{Q^2V^4} - 1 \right) \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \phi}{\partial \xi^3} = 0
\]

where \( \phi \) denotes the wave potential, normalized by \( kT/e \), \( k \) being the Boltzmann constant, and \( T \) the electron temperature. \( Q = m_\beta / m_\alpha \) is the ratio of the negative ion mass to the positive ion mass. The unperturbed densities, \( n_{\alpha_0} \) and \( n_{\beta_0} \), of positive and negative ions are normalized by the electron density, so that the quasi-neutrality condition is \( n_{\alpha_0} = n_{\beta_0} \). The time \( \tau \) is normalized by the ion plasma period \( \omega_p^{-1} \) and the coordinate \( \xi \) normalized by the Debye length \( \lambda_D \), in the frame moving with the sound velocity \( V = (n_{\alpha_0} + n_{\beta_0}/Q)^{1/2} \).

There is a critical density of the negative ions at which the coefficient of the nonlinear term of the K-dV equation (1) vanishes. Therefore, at the critical density, it is necessary to consider nonlinear terms of higher orders to get, for example, the modified K-dV equation derived by Watanabe.

In the present work the modified K-dV equation derived by Watanabe is generalized to include cylindrical and spherical geometries, and similarity analysis is applied in order to find non-stationary solutions.
2. THE MODIFIED KORTEWEG-deVRIES EQUATION

Watanabe\(^2\), considering a plasma with the negative ions at the critical density and by assuming that the ions are cold, has derived the modified K-dV equation

\[
\frac{\partial \phi}{\partial \tau} + \alpha \frac{\partial}{\partial E} \left[ \phi^2 \frac{\partial \phi}{\partial E} \right] + \frac{1}{2} \frac{\partial^3 \phi}{\partial E^3} = 0 ,
\]

(2)

where

\[
\alpha = \frac{1}{4} \left( 15 \frac{n_0}{V^6} + 15 \frac{n_{BO}}{Q^3V^6} - 1 \right).
\]

In the derivation of equation (2) Watanabe used the stretched variables \( \xi = \varepsilon^{1/2} \) \((x - Vt)\) and \( \tau = \varepsilon^{3/2} \) \(Vt\), where \( \varepsilon \) is a small parameter, \( x \) is the normalized cartesian coordinate, \( t \) is the normalized time, and \( V \) is the normalized sound velocity \( V = (n_{BO} + n_{BO}/Q)^{1/2} \).

The equation (2) has been deduced for plane geometry, considering only one cartesian coordinate, and can be generalized to include cylindrical and spherical geometries. Maciel and Sudano\(^3\) has derived a K-dV equation which holds for the three geometries. A similar derivation to that one presented by Maciel and Sudano yields, for the present case,

\[
\frac{\partial \phi}{\partial \tau} + \frac{\nu}{2} \frac{\partial \phi}{\partial \tau} + \alpha \frac{\phi^2}{2} \frac{\partial^2 \phi}{\partial \xi^2} + \frac{1}{2} \frac{\partial^3 \phi}{\partial \xi^3} = 0 ,
\]

(3)

where \( \nu = 0, 1 \) and 2, respectively, for plane, cylindrical and spherical geometries. The second term represents the geometrical effect.

By writing \( \phi_1 = \tau^{\nu/2} \), equation (3) becomes

\[
\frac{\partial \phi_1}{\partial \tau} + \alpha \frac{\nu}{2} \frac{\partial \phi_1}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 .
\]

(4)

Following the similarity method, we search for solutions of equation (4) that are invariants under a one-parameter group of transformation defined by

\[
\tau = R^{\phi_2} \tau , \quad \xi = R^{\phi_2} \xi \quad \text{and} \quad \phi_1 = R^{\phi_2} \phi_1 .
\]

(5)
where \( R, a, b \) and \( c \) are constants. The equation (4) is invariant under this group if \( b/a = 1/3 \) and \( c/a = \nu/\alpha - 1/3 \), and the group invariants are

\[
\eta = \frac{\xi}{\tau^{1/3}} \quad \text{and} \quad f(\eta) = \frac{\phi_1(\xi, \tau)}{\nu/2 - 1/3}
\]  

(6)

The equations (4) and (6) yield

\[
(\nu - \frac{1}{3})f(\eta) - \frac{1}{3} \eta \frac{df}{d\eta} + \alpha f^2(\eta) \frac{df}{d\eta} + \frac{1}{2} \frac{d^3f}{d\eta^3} = 0
\]

(7)

Let us consider the case \( \nu = 0 \) (plane geometry). In this case equation (7) reduces to

\[
\frac{d^3g}{d\zeta^3} - g^2 \frac{d}{d\zeta} - g(\zeta) = 0
\]

(8)

where \( g = 3^{1/3} 2^{1/6} \) and \( \zeta = (2/3)^{1/3} \eta \).

Assuming that \( g^2/\zeta \to 0 \) when \( \zeta \to \pm \infty \), the nonlinear term can be neglected for large values of \( |\zeta| \) and the equation (8) reduces to the linear equation

\[
\frac{d^3g_L}{d\zeta^3} - \zeta \frac{d}{d\zeta} g_L - g_L = 0
\]

(10)

which describes the asymptotic behaviour of the ion acoustic wave.

Integrating equation (10) and assuming that when \( \zeta \to \pm \infty \),

\[
\frac{d^2g_L}{d\zeta^2} = C \text{Ai}(\zeta)
\]

(12)

where \( C \) is a constant and \( \text{Ai}(\zeta) \) is the Airy function.
The asymptotic solution (12) can be used to determine the initial conditions for a numerical integration of the nonlinear equation (8). A numerical integration has been carried out where $\zeta = 2$ has been chosen as the initial point for the integration, and $C = 1$ in the asymptotic form (12). The results show that the nonlinear wave has roughly the same behaviour as the asymptotic solution but with smaller oscillation amplitude.

For the cylindrical and spherical geometries, similar derivations to that one carried out for the plane geometry yield linearized equations whose solutions diverge as $\zeta \to \infty$. However, it is possible that part of those solutions are physically meaningful because similarity analysis sometimes gives solutions which are not valid everywhere. This question requires further analysis.

3. CONCLUSION

Similarity analysis was applied to modified K-dV equations which describe small amplitude ion acoustic waves in a plasma with negative ions at the critical density. In the case of the plane geometry, it was shown that the asymptotic solution satisfies the Airy equation. Furthermore, through numerical integration, it was shown that the nonlinear equation has solutions which resembles an Airy function but with smaller amplitude of oscillation. In the case of cylindrical and spherical geometries, the present method of analysis gives linearized equations which have divergent solutions so that their physical meaning requires further discussion.

REFERENCES

RELAXATION AND SELF-ORGANIZATION OF A NONNEUTRAL PLASMA

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I. Overview
The properties of nonneutral systems have been elucidated in several elegant experiments\cite{1,2}. As plasmas, these systems have the virtue of being confinable for long times, since conservation laws place strong bounds on particle loss. Detailed experiments have explored the quasistatic evolution of a well-confined configuration near equilibrium. The transient, non-equilibrium phase during which the plasma achieves its steady-state profile is difficult to probe experimentally. However, because of the short-time scales involved this is a regime well suited to particle simulation. This paper discusses the results of a particle-simulation code of a nonneutral plasma that is confined in a slab-equivalent of a Penning trap. The early collisionless relaxation is examined, and a time-dependent steady state is observed to result. In this state, the system achieves the shape of a football, composed of a fluid-like core and a kinetic halo. When this quasi-equilibrium is externally cooled, it is found to develop spatial rings reminiscent of liquid crystals. This crystalline structure is robust and 'melts' when it is heated.

II. Geometry

This study uses a bounded, 2-1/2 D magnetized particle-simulation code\cite{3}. The magnetic field lies along z, and the plasma is spatially localized along x. The y coordinate is ignorable. The charges are confined radially by the magnetic field, and axial confinement is achieved by an external vacuum potential

$$\phi_{0}(x, z) = \phi_{0} [z^{2} - x^{2}]$$

Fig. 1 shows the initial configuration. A rectangular region of cold plasma is placed in the confining external field. The dimensions are chosen such that the self-repulsion is too weak to balance the axial compression of the electric field. The cyclotron frequency is initially eight times the plasma frequency.

III. Collisionless Relaxation

As expected, the system in Fig. 1 initially contracts and thereafter undergoes compression oscillations. A single global mode appears, with a characteristic frequency, although the system is very nonlinear and
strongly nonuniform. This oscillation frequency is about two to three times the plasma frequency, and still several times below the cyclotron frequency. After several oscillations, the system undergoes a spatial decoupling, and the edge particles (in radius) start interacting resonantly with the global mode. The edge particles are rapidly accelerated to large energies, and the system arrives at a time-dependent steady state, as seen in Fig. 2. Note that the core region relaxes to a 'football' shape, even in the absence of collisions. The state shown in Fig. 2 persists for a long time, and external damping is required to achieve a stationary equilibrium in a reasonable time-scale.

IV. Cooling and Crystallization

The time-dependent steady state previously described is subjected to external cooling. Isotropic cooling is observed to yield expansion across the magnetic field in addition to cooling. When external damping is applied only in the x and z directions, the plasma is observed to cool without expansion. Fig. 4a shows that the 'halo' region disappears and the edge particles lose energy. The core region shows the first signs of organized structure; an outer spatial ring of particles separates out from the rest of the plasma. Further cooling causes more rings to form as seen in Fig. 4b.

Due to the strong magnetic field, charges are confined to the field lines on which they were initially placed. Thus, the final configuration contains the same line-integrated density profile as it started with. However, varying the density profile does not affect the collisionless relaxation process or the final 'football' shape significantly.

When the external damping is removed the cooled system experiences a slight rise in temperature. This heating is accompanied by a weak acceleration of the edge particles, suggesting that the edge 'halo' is a universal feature in a time dependent system that attempts to match to stationary external constraints.

The self-organizing state is quite robust. When the external electric field is suddenly increased in strength by 10%, the temperature of the particles rises only slightly and the crystalline structure remains unaffected. Most of the energy goes into a global compression mode. Heating the plasma progressively destroys the organization of the particles, the outer ring being the last one to 'melt'.

Acknowledgment:

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References


Figure 1. Initial configuration of the slab Penning trap. The solid rectangle represents a nonneutral plasma, while the dashed lines represent the externally imposed electric field that produces axial confinement. Radial confinement is achieved by a magnetic field.

Figure 2. $t = 30/\omega_p$: Time dependent steady state achieved through collisionless processes. Most of the plasma behaves as a core fluid undergoing oscillations in the external field. The low density edge plasma becomes a 'halo' having a hot ring distribution that encloses (in phase space) the core particles.
Figure 3. $t = 30/\omega_p$:
Configuration space. The core region achieves the 'football' shape that is typical of the cold equilibrium. The edge 'halo' region is seen to be spatially extended in $z$. Its function is to match the time-dependent fields generated by the oscillating core plasma to the external time-independent confining field.

Figure 4. Cooling and crystallization: As the hot system in Fig. 3 is cooled by friction, the edge region is quickly lost and the plasma comes to a stationary equilibrium. Further cooling causes radial structure to appear, reminiscent of crystal formation. Initially an outer ring forms, but as cooling is continued, a sequence of rings appear towards the center.
LOW-DIMENSIONAL ATTRACTORS ARISING FROM THE INTERACTION OF DRIFT WAVES WITH LOW FREQUENCY INSTABILITIES

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1. Introduction

Nonlinear interaction of plasma waves has been the topic of many theoretical and experimental works. However, the theory of nonlinear dynamics of dissipative systems has shed new light on several plasma wave phenomena. Theroretical investigations\(^{1,2,3}\) predict low-dimensional phase space attractors for interacting drift-waves and in the interaction of drift-waves with ion-sound-waves, respectively. Experimental evidence for such attractors has been given in a magnetized double-plasma device\(^4\), where, under certain conditions, interaction between ion-sound-like waves and ion drift-waves is observed.

The present contribution gives a classification of these ion-sound-like waves and characterizes the interaction process. The phase space attractors are investigated using improved numerical methods.

2. Experiment

The KIWI-experiment is a magnetized triple plasma device which consists of two independent plasma chambers ('magnetic box', 92 cm dia. × 90 cm) and a central tube section (30 cm dia. × 200 cm) immersed in a set of 14 water-cooled magnetic field coils. A dense, quiet, and homogenous plasma \((N_e \leq 10^{11}\text{cm}^{-3}, T_e = 2\text{eV})\) diffuses from the magnetic boxes into the magnetized tube section. The plasma chambers and the tube section are separated by mesh grids with a transparency of about 50%, which can be biased for controlled injection of electrons or ions. The double-plasma mode is achieved by activating one plasma chamber and using a grid at the far end of the tube section as a plasma-loss surface. By positively biasing the grid between the plasma chamber and the tube section, a supra-thermal electron beam is injected into the plasma background. Langmuir-probes are used for density, temperature and potential diagnostics. Both radially and axially movable probes are available. Quantitative plasma potential measurements with emissive probes are performed in the usual manner\(^5\). Dynamical measurements are performed by recording time series of probe signal fluctuations, i.e., temporal changes of ion saturation current and floating potential. Power spectra of potential and density fluctuations are calculated from these high-resolution time series.

3. Experimental Results

An axial electron drift \(\propto I_e\) is well known to destabilize drift waves\(^6\). For this reason, the axial drift voltage \(U_d\) along the magnetic field has been chosen as control parameter for the dynamical state of the plasma. In the case of additional electron rich injection into the background plasma, we observe as critical phenomenon anomalous resistivity (AR) in the \(U_d(I_e)\)-characteristic, the occurrence of a low-frequency ion-sound-like instability and after the onset of AR a broad fluctuation power spectrum. In Figure 1, the transition to AR is shown. The dashed line is a sketch of the discharge-like resistivity characteristic for weak or no electron injection. From two selected corresponding power spectra, the transition from single-mode to turbulent spectra is clearly observable. From measured density profiles, the anomalous radial transport due to drift-wave turbulence can be seen as mechanism for this type of AR.

In order to understand this transition, a frequency-wavenumber spectral analysis\(^7\) and the calculation of high-ordered spectra\(^8\) has been carried out. We find a strong low-
frequency instability \((f < 3\text{kHz})\) coexistent with drift-waves. These two instabilities are clearly distinguishable by their fluctuation profile, parameter dependence and propagation properties. In case of a stable plasma, measurements of the stationary potential profile \(\Phi_p(z)\) have shown the existence of a stationary weak double layer (DL) with a normalized potential jump \(\epsilon \Phi_0/k_B T_e \approx 1\). This DL extends from the grid towards the tube section over about 300mm. For the unstable case, however, a sawtooth-shaped strong oscillation of the plasma with small wavenumbers \(k_\parallel = O(0.001\text{cm}^{-1})\) is found. It propagates with a phase velocity of \((2-3)\) times the ion-sound velocity \(C_s\). Hence, the observed low frequency oscillation can be interpreted as 'potential relaxation instability' (PRI) of the unstable DL. This phenomenon has often been described in the context of DLs and should not be confused with ion-sound waves\(^9\). In addition, for \(T_i/T_e = 1/10\), as in the KIWI-device, Landau-damping suppresses the global propagation of ion-sound waves.

Besides the fundamental frequencies \(f_D\) and \(f_F\) (drift-wave and PRI), the discrete power spectra show combination frequencies \(f_D \pm f_F\) due to their interaction (Figure 1). For more complicated situations, bispectral analysis has been applied to distinguish between self-excited modes and waves arising from nonlinear interaction. Figure 2 shows a typical bicoherence spectrum: Nonlinear coupling between the PRI, the drift wave and its higher harmonics up to order three is clearly indicated by peaks at frequency resonance points \(f_1 \pm f_2 = f_{1,2}\). The wavenumber analysis of the data, however, shows that the analogous wavenumber resonance-condition \(k_1 \pm k_2 = k_{1,2}\) is not fulfilled. Therefore, the coupling process cannot be considered as resonant three-wave interaction. It rather appears to be a nonresonant, parametric modulation of the drift-wave by low-frequency electron-pressure and density oscillations.

Using Taken's theorem\(^{10}\), the phase space of a dynamical system can be reconstructed from a single observed observable. We have calculated the dimensionality of phase space attractors for several different control parameter values \(U_d\), where we have used the correlation-integral algorithm\(^{11}\) with \(N = 15000\) vectors to ensure a small statistical error. The resulting uncertainty is estimated with roughly \(\Delta D_2 = 0.2\). The final result gives the correlation dimension \(D_2\) as a function of the drift voltage \(U_d\) as displayed in Figure 3. For comparison, the earlier results\(^4\) are shown in the same figure. The onset of AR is used as reference point for direct comparison. First, we find an excellent agreement between the \(D_2\)-values up to the onset of AR. Even the dip just before the onset of AR is reproduced. After the onset of AR, the dimensionality increases rapidly in both cases. In conclusion, the attractor in the coupling of PRI and drift-wave approaches values lower than \(D_2 = 5\) and seems to be independent from experimental boundary conditions.

To characterize the transition process more clearly, slightly different discharge condition have been chosen. A transition from monochromatic spectra with a single drift-mode to complicated broadband spectra is found, again accompanied with AR. Figure 4 shows the corresponding dimensionalities \(D_2\). In addition, we have calculated the largest Lyapunov-exponent \(\lambda\) from our experimental time series\(^{12}\). The monochromatic state appears as one-dimensional with a Lyapunov-exponent close to zero, as expected. With increased drift voltage, the PRI is excited and again, the described interaction process has a low-dimensional attractor \((3 \leq D_2 \leq 5)\) with positive Lyapunov-exponents \(\lambda\). After the onset of AR, both \(D_2\) and \(\lambda\) diverge. For characteristic values of \(U_d\), a three-dimensional reconstruction of the attractors is also displayed in Figure 4.
4. Conclusion

The KIWI experiments have shown that previously observed low-dimensional attractors in the interaction of drift-waves and ion-sound-like waves are reproducible in the very details and seem to be independent from the experimental arrangement. The ion-sound-like wave is identified as potential-relaxation instability. Its interaction process with drift-waves is a nonresonant parametric modulation and not a three-wave decay scheme.

5. Acknowledgements

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6. References


Figure 1: Transition to AR with corresponding power spectra.
Figure 2: A typical bicoherence spectrum.

Figure 3: Correlation dimension $D_2$ versus $U_d$ in comparison with [4].

Figure 4: Correlation dimension $D_2$ and largest Lyapunov-exponent versus $U_d$. 
The KdV equation for ion acoustic waves admits soliton-type of solutions. A compressive perturbation is known to evolve into a soliton due to the balance between the nonlinear steepening and dispersive effects [1–5]. The balance between non-linearity and dispersion cannot be achieved for rarefactive perturbations and these perturbations do not evolve into solitons. The present paper reports the observations on launching and evolution of large amplitude rarefactive ion acoustic waves in a linear unmagnetised plasma. Observations on the ion velocity distribution associated with these large amplitude waves are also presented. Particle trapping and modification of distribution function in the presence of rarefactive pulse is observed. There are also indications of phase space vortex formation in the wave train observed at the trailing end of the pulse.

The experiments were carried out in a linear, unmagnetised, uniform, plasma 50 cm dia and 120 cm in length. The chamber was pumped to a base pressure of \(10^{-5}\) cm Hg and operated at neutral argon pressure of \(10^{-4}\) torr. The average plasma density obtained is \(5 \times 10^{9} \text{cm}^{-3}\), \(T_{e} \sim 4.0\) eV, \(T_{i} \sim 0.2\) eV and \(C_{s} \sim 3.0 \times 10^{5}\) cm/sec. Waves were launched using a square pulse of rise time \(\sim \omega_{pi}^{-1} \sim 1 \mu\text{sec}\), applied to a circular grid of dia 20.0 cm, with a repetition rate of 1 kHz.

In the experiment rarefactive pulses of different amplitudes were launched. The pulses of initial amplitude \(\delta n/n \sim 1.2\%\) started out as symmetric pulse of width \(\sim 5.0\lambda_{D}\) close to the launcher. The pulse broadened due to dispersion and damped as it moved away from the launcher. The velocity of these pulses were found to be closer to \(2.94 \times 10^{5}\) cm/sec. This is slightly smaller than the acoustic speeds. The pulses with amplitude 6% also behaved in a similar fashion. But the pulses of \(\delta n/n \sim 7.0\%\) and more showed some additional features. These pulses also started out as symmetric pulses very close to the launcher and were dispersed and damped as they moved away from the launcher. The leading edge of the pulse becomes sharper than the trailing edge. At a distance about 25–30 \(\lambda_{D}\) the pulses start becoming asymmetric i.e. have shoulders of unequal amplitude. Beyond this the pulse splitted into two separated by a local maxima. Fig. (1) shows the traces of density variation for a wave with initial amplitude 9%. In this case the split is observed very close to the launcher itself. The pulses after splitting moved at different speeds, one moving at a speed slightly lower than the acoustic speed. A further propagation beyond 55–60 \(\lambda_{D}\) lead to a further split into three pulses. Higher initial amplitude pulses split into even more pulses. For initial \(\delta n/n \sim 9.0\%\) one of the trough was found to be moving with a speed \(\sim 0.9C_{s}\), and the other at \(\sim 1.2C_{s}\).

The splitting of the initial pulses into more than one pulses cannot be understood on the basis of fluid theory. These are phenomenon occurring at large amplitude of refractive pulses, having a negative potential structure, giving rise to the possibility of trapping of ions moving with speeds closer to the \(C_{s}\). This would lead to wave particle interaction, modifying the particle distribution function, in the phase space. In order to investigate this phenomenon, the ion energy distribution was measured at various positions in the pulse, as shown in top trace in the in fig (2), using a boxcar integrator and a three grid
amplitude neuron.

Figure 1: The density signals showing the wave propagation for initial amplitude $\delta n/n \sim 9\%$.

...retarding potential energy analyser, with a resolution better than 0.1 eV. The output of the RPA was averaged using the boxcar integrator and digitized. The collector current $I(\phi)$ as a function of the applied bias on the selector grid can be written as

$$I(\phi) = \int_{\phi_1}^{\phi_2} A e^{v f(x, v, t)} dv$$

...apart from a geometrical transparency factor. The $dI/dV$ would be a measure of $f(x, v, t)$. The was calculated digitally and is shown in fig. (2). Different curves have been taken at different time delays, as marked in the top trace in fig (2). The trace 1 to 6 in fig. (2) show a strong modification of the energy distribution of ions near an energy of 0.8 eV from the center of the distribution. These modifications are similar to those obtained in the numerical calculation of Sakanaka [8], where the trapping was observed in a beam interaction.

In fig. (1), the first pulse is the main pulse of the rarefactive wave. This pulse is a negative potential structure and its trailing edge resembles to a shock like structure. It was found that along with this symmetric pulse a low frequency ($\sim 30$ kHz $< f_{pi}$) appears in the trailing edge. These oscillations appear as a wave train in the trailing edge moving with speed slightly slower than $C_s$. Once formed these structures travel in the system to the end of the device, without any appreciable change in the width. These oscillations do not agree to the usual Airy function type of response as reported by Ikezi et al. (1973). The Airy response was seen at a frequency ($\sim \omega_{pi}^{-1}$). This is 300 kHz for the parameter of our experiment. Also Ikezi et al. (1973) had demonstrated the period of Airy type of oscillations increase with propagation distances. No change in the time period was seen in the present experiment.

In an experiment by Saxena et al. [6], a rarefactive pulse was found to fission into several pulses. In this experiment the fissioning occurred at a distance of about 65-70 $\lambda_D$ whereas in the present experiment the fissioning occurred very close to the launcher $\sim 25\lambda_D$ and for higher amplitude pulses this occurred even further closer to the launcher. The rise time of the pulses were slow in the experiment of Saxena et al. [6].

The measurement of energy distribution of ions showed strong modification near the phase velocity of waves. This indicates the formation of phase space holes due to trapping.
Figure 2: The top trace shows the time delays at which energy distributions were measured in the pulse. The traces in the bottom show the modified distributions of ions.
of ion in the potential of the wave. These measurements are similar to those of Pécseli et al. [9] in a double plasma device obtained for a shock wave formed by two beam interaction. The oscillations in the trailing edge and its propagation also indicate the trapping of particle in the phase space. The launched perturbation is at the time scale $\sim \omega_{pi}^{-1}$. Initial discontinuity in the density during the impulse leads to the formation of a sharply peaked electric field. The electric field would accelerate ions from this region and create a local depletion of ions. This situation is similar to one in numerical calculation by Hasegawa and Sato [11] for two beam instability. In their case the electric field generated by the initial discontinuity in density evolved into a shock followed by a hole in the ion phase space. These holes appeared as regions of density depletions in front of the shock very similar to the oscillations in trailing edge in our case. In the above mentioned numerical calculation, stable holes with width $\sim 100\lambda_D$ were observed. Widths observed in present experiments are $\sim 200\lambda_D$. Hasegawa and Sato [11] showed that a negative potential structure in the presence of a current in the plasma evolved into a double layer, which is known to be a BGK mode (Bernstein et al. [7]). The formation of double layer is facilitated by reflection of electrons from the wave potential. This impedes the current and leaves behind an ion rich region. In the absence of this current the region remains depleted of ions and a hole in the phase space is formed. The measurement of $f(v)$ in the main pulse shows the modification of the ion distribution and provides a strong evidence for the formation of a hole in ion phase space. This hole is asymmetric hole with shoulders of unequal amplitude. A stable hole with width $\sim 200\lambda_D$ is also observed in the trailing edge of the perturbations. The behavior of these holes are similar to those obtained in the calculations of Sakanaka [8]. In his case also Airy function type of oscillations were not observed. This agrees quite well with our measurements. However there is no sustained beam in our experiment.

In conclusion rarefactive ion acoustic waves investigated in the linear unmagnetised plasma have shown several interesting characteristics of excitation and propagation. The waves with amplitude $\sim 1.2\%$ and more show several features which do not agree with the KdV equation. The rarefactive perturbations excited by a fast step ($\omega \sim \omega_{pi}$) are found to evolve into holes in ion phase space. Both asymmetric and symmetric holes, have been observed in the presence of these large amplitude rarefactive ion acoustic waves.

References

Cascading Bifurcations to Chaos in a Current-Carrying Ion Sheath

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Recently, cascading bifurcations followed by the onset of chaos were observed in a current-carrying plasma sheath when an external periodic oscillation was applied to a fine-meshed grid which divided the plasma produced by dc discharge. In this experimental set-up, the potential at the grid is negatively biased to the extent that electrons cannot penetrate up to the grid and, therefore, the resultant sheath is an ion sheath which is detatched from the plasma by an ordinary sheath. The ion sheath on both sides of the grid forms a potential well in which the ions oscillate to induce a primary motion responding to the external oscillation.

The structure of the ion sheath can be studied based on an ion fluid:

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nv) = 0, \quad (1)
\]
\[
\frac{\partial v}{\partial t} + v \frac{\partial}{\partial x} v = \frac{e}{M} E, \quad (2)
\]
\[
\frac{\partial E}{\partial x} = 4\pi en. \quad (3)
\]

For the stationary sheath, the flux is constant:

\[
v = I_0. \quad (4)
\]
From eqs. (2) and (3), we have

$$\frac{d}{dz} \left( \frac{I_0}{n} - \frac{E^2}{8\pi M} \right) = 0,$$

which gives

$$n = \frac{n_0}{1 + (A/2)(E^2/E_0^2 - 1)},$$

where $A = (\omega_p E_0 / 4\pi e n_0 v_0)^2$. Substituting eq. (6) into the Poisson equation (3), we have an implicit expression for $E$ with respect to $x$ as

$$x - x_0 = \frac{eE_0}{M\omega_p^2} \left\{ (1 - A/2)(E/E_0 - 1) + (A/6)(E^3/E_0^3 - 1) \right\}.$$

The electric potential is expressed in terms of $E$ as

$$\phi - \phi_0 = -\frac{1}{2} \frac{eE_0^2}{M\omega_p^2} (E^2/E_0^2 - 1) \{1 + (A/4)(E^2/E_0^2 - 1)\}.$$  

For a large electric field we have, from eqs. (7) and (8),

$$\frac{E}{E_0} \approx \left( \frac{6(x - x_0)}{A \frac{eE_0}{M\omega_p^2}} \right)^{1/3},$$  

$$\phi - \phi_0 \approx -\frac{1}{2} \frac{eE_0^2}{M\omega_p^2} \frac{A}{4} \left( \frac{E}{E_0} \right)^4,$$

which are combined to give an explicit expression of the Child-Langmuir law of space-charge limited current in a plane diode:

$$\frac{\phi_0 - \phi}{\frac{eE_0^2}{M\omega_p^2}} \approx \frac{6^{4/3}}{8} \left( \frac{eE_0}{M\omega_p^2} \right)^{4/3} \left( \frac{4\pi e n_0 v_0}{\omega_p E_0} \right)^{2/3}.$$  

It is worthwhile to note that for small values of current $(4\pi e n_0 v_0 < \sqrt{2}\omega_p E_0)$ density becomes singular, as is seen from eq.(6), and the electric field and the potential are both multivalued. This reflects the fact that the Child-Langmuir ion sheath is detached from the main part of the plasma by the sheath to which warm electrons penetrate to some extent.
Now we consider the ion dynamics in the above ion sheath, which is described by the following equations of motion:

\[
\frac{dx}{dt} = v, \tag{12}
\]

\[
\frac{dv}{dt} = \frac{e}{M}(E_0 - E). \tag{13}
\]

Differentiating eq.(3) with respect to \( t \), and invoking an introduction of a damping term which ensures a well-defined attractor and \( E \) to be replaced by its asymptotic value \( E \approx -\frac{e}{\omega_p^2}J(J = 4\pi e_n v) \), we obtain

\[
\frac{d^2J}{dt^2} + \frac{J}{1 + A[\nu J + \nu^2 J^2/2]} + \nu \frac{dJ}{dt} + \Omega E_{\text{ext}} \cos(\Omega t) = 0, \tag{14}
\]

where \( \omega_p t, \nu/\omega_p \) and \( \Omega/\omega_p \) are replaced by \( t, \nu \) and \( \Omega \), respectively. Equation (14) reduces to a form of Duffing’s equation by a simple transformation when \( A \) is small and the second term can be expanded with respect to \( A \), indicating that eq.(14) exhibits rich varieties of dynamical behavior. Equation (14) is solved numerically by changing \( E_{\text{ext}} \) with \( A, \nu \) and \( \Omega \) fixed or changing \( \Omega \) with \( A, \nu \) and \( E_{\text{ext}} \) fixed. In both cases we observe the cascading bifurcations to chaos. Figure 1 shows trajectories in \( (J, \dot{J}) \) space and the Fourier spectrum of \( J(t) \) for various values of \( E_{\text{ext}} \) with \( A = 1.6, \nu = 0.25 \) and \( \Omega = 0.615 \). In this case the period-doubling bifurcations are followed by the onset of chaos as \( E_{\text{ext}} \) increases. Further increase in \( E_{\text{ext}} \) leads to period-tripling bifurcations to chaos. Then the system reverts back to period doubling bifurcations.

A similar series of bifurcations is obtained in the case when the frequency of the external oscillation is changed with the fixed amplitude \( E_{\text{ext}} = 3.0 \) for \( A = 1.6 \) and \( \nu = 0.25 \).

The above scenario of bifurcations to chaos is also applicable in those cases with different values of \( A \) and \( \nu \). Which appears first, period-doubling or period-tripling, depends on the value of the parameters \( A \) and \( \nu \). For example, when \( A \) and \( \nu \) are chosen as 1.0 and 0.1, respectively, with \( E_{\text{ext}} = 5.2 \) fixed, period-tripling appears first, and then period doubling follows as the frequency increases as is shown in Fig.2.
The bifurcation sequences revealed above agree well with the experimental results by Komori et al. Although we have not examined the mathematical structure of the chaos obtained under eq. (14), we may say that the underlying physics of the observed chaos can be explained by our theory.

Fig. 1

Fig. 2
EXPERIMENTAL MEASUREMENTS OF CASCADING BIFURCATIONS TO CHAOS IN AN ION SHEATH

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Abstract Cascading bifurcations to chaos are investigated experimentally and theoretically in a current-carrying stable ion sheath. A dc plasma current is required to produce an electron-depleted thick sheath on a grid, which obeys the Child-Langmuir law of space-charge-limited current in a diode. Bifurcation cascade and chaotic behavior are exhibited when an external periodic oscillation is applied to the grid, and is in good agreement for the first time with a theory, which describes ion dynamics in a nonlinear potential well, formed as an ion sheath on the both sides of the grid. A fractal dimension predicted by the theory is verified by the experiment.

1. Introduction

It is of current interest to investigate nonlinear physical systems which exhibit chaotic behavior. Universal characteristics of chaos have been observed in experiments conducted on a variety of nonlinear media as well as in numerical simulations. Recently, several experiments have been reported on chaotic behavior in plasma systems /1, 2/. Two routes to chaos, period doubling and intermittent chaos, are demonstrated in these experiments, and fine structures such as periodic windows are also observed. However, to our knowledge, the experimental observations of chaotic behavior in plasmas are not clearly understood and are still open to studies of their underlying physics.

The present paper gives an experimental study of cascading bifurcations to chaos in a current-carrying stable plasma, and compares it with a theory which describes our nonlinear system.

2. Experimental apparatus

The experiment is performed in a large, unmagnetized plasma device 70 cm in diameter and 120 cm in length, equipped with multidipole magnets for surface plasma confinement, as shown in Fig. 1 /2, 3/. An argon plasma produced by a dc discharge between filaments and the chamber wall is divided by a fine-meshed grid made of 0.05 mm diam stainless-steel wires spaced 0.5 mm apart. Typical parameters of the plasma designated T are \( n_0 = (0.9 - 7) \times 10^8 \text{ cm}^{-3}, T_e = 0.3 - 0.7 \text{ eV} \) and \( T_i \approx 0.1 \text{ eV} \), where \( n_0 \) is the plasma density, and controlled by changing the heater currents. At first, the density of the plasma \( D, n_D \), is chosen to be less than \( n_0 \) by about one order of
magnitudes, so that the plasma space potential $\phi_{OD}$ of the plasma D is higher than the plasma space potential $\phi_0$ of the plasma T by a few volt. The symbol $\Delta \phi$ denotes the potential difference between $\phi_{OD}$ and $\phi_0$ ($\Delta \phi = \phi_{OD} - \phi_0$). To drive a dc plasma current, a dc voltage $V_0$ is applied between the grid and a 12 cm diam target, that is, the grid and target are negatively and positively biased, respectively, as shown in Fig. 1. Plane Langmuir probes 6 mm in diameter are used to measure the plasma parameters and their fluctuations. The plasma space potentials are measured with emissive probes, and the ion temperature in the two plasmas is obtained with Faraday cups. The gas pressure $p$ is usually kept at $\sim 2 \times 10^{-4}$ Torr, and is sometimes varied in the range of $(1 - 4) \times 10^{-4}$ Torr. The time-averaged value $I_0$ and fluctuating component $I$ of the plasma current are observed from the voltage drop across the resistor $R_1$.

![Fig. 1: Schematic of experimental apparatus. $R_1 = R_2 = 50 \Omega$ and $C = 3.3 \mu F$.](image1)

![Fig. 2: Dependence of $d$ on $V_0$.](image2)

3. Experimental results and discussion

Figure 2 shows the $V_0$ dependence of the sheath thickness $d$ of the sheath $S_{GT}$ on the grid, facing the plasma T. Circles, triangles, and squares are obtained at $I_0 = 1, 1.5$ and 2 mA, respectively, and solid lines represent the curves proportional to $V_0^{3/4}$. There is good agreement between the solid lines and measured $d$'s. Thus, the potential profile in the sheath $S_{GT}$ is confirmed to be described by the Child-Langmuir law of space-charge-limited current in a plane diode: $I_0 = \left( \frac{4}{\sqrt{2}} \varepsilon_0 A / 9 d^2 \right) (e/m_i)^{1/2} V_0^{3/2}$, where $A$ is the area of the grid and $m_i$ is the ion mass. The sheath thickness $d$ obtained experimentally is found to agree with the thickness given by this equation within a factor of 3. This factor is associated with the reflection of ions from the plasma D to the plasma T by $\Delta \phi / 3$.

If an extremely electron-depleted sheath $S_{GT}$ is formed on the grid, a coherent instability, associated with an ion transit in the sheath, appears for wide parameter
regimes, as shown in the top traces of Figs. 3(a) and 3(b) /3/. An important point is that there is a threshold $\Delta \phi = \phi_{OD} - \phi_0$ to excite the instability /3/, and therefore, we can stabilize the plasma by decreasing $\Delta \phi$, that is, by increasing $n_{OD}$. Figure 3(a) indicates that $\Delta \phi$ becomes small with an increase in $n_{OD}$. In this figure, $n_{OD}$ is varied from $7 \times 10^7$ cm$^{-3}$ to $5 \times 10^8$ cm$^{-3}$, while $n_0$ is kept at $\sim 7 \times 10^8$ cm$^{-3}$. Apparently, the decrease in $\Delta \phi$ leads to the stabilization of the instability, as shown in Fig. 3(b). The ion sheath $S_{GT}$ is found to be affected little by the change of $n_{OD}$, so that there is the extremely electron-depleted ion sheath on the grid, as before, whose potential profile is described by the Child-Langmuir law. When an external periodic oscillation whose amplitude is denoted by $V_{ext}$ is applied to the grid under such a condition, a sinusoidal perturbation at the driving frequency $f_1$ can be induced in the plasma current. Nonlinear behavior is observed by gradually increasing $f_1$ from 100 kHz and keeping $V_{ext}$ at $\sim 4.7$ V. The first subharmonic appears at $f_1 = 139$ kHz, and a very clear period-doubling sequence is obtained, as shown in Figs. 4(a)-4(c). Further period doublings are hardly measured, possibly because the rapid convergence rate of the doubling sequence makes it very difficult to observe them. Increasing $f_1$ further produces chaotic behavior. This state is characterized by broadband noise in the frequency spectrum, as shown in Fig.

![Fig. 3](image1.png)  
**Fig. 3:** Effects of $n_{OD}$ on (a) the potential $\phi$ around the grid and (b) on the instability.

![Fig. 4](image2.png)  
**Fig. 4:** Frequency spectra of $I$. 

![Graph](image3.png)
4(d) /2/. The correlation dimension $\nu$ is obtained to be $1.54 \pm 0.22$ in this chaotic regime, suggesting that the number of degrees of freedom is small in our system. Further increases in $f_1$ cause period tripling [Fig. 4(e)]. Thus, it is clearly demonstrated that the bifurcation sequence leading to chaos in our system is the same as Feigenbaum's period-doubling route to chaos.

Nonlinear behavior of the oscillations is also realized by increasing $n_0$ with $f_1$ and $V_{\text{ext}}$ fixed or increasing $V_{\text{ext}}$ with $f_1$ and $n_0$ fixed. Although a set of cascading bifurcations to chaos is obtained when $p$ is changed in the range of $1 \times 10^{-4} \lesssim p \lesssim 4 \times 10^{-4}$ Torr, whether $p$ itself is relevant to the nonlinear behavior is not clear since $n_0$ is varied with the change of $p$. These observations are confirmed to be reproducible in our experiments. However, they are very sensitive to the parameters such as $n_0$, $V_{\text{ext}}$, and so forth. For example, which appears first, period doubling or period tripling, depends on these parameters. Furthermore, a mixture of period doubling and period tripling bifurcations is sometimes observed. Thus, there are many scenarios of nonlinear behavior of the oscillations, depending on the parameters.

A theory of cascading bifurcations to chaos in a current-carrying ion sheath has been developed by Kono et al. /4/. They have derived the differential equation with three dimension, which governs ion dynamics in an ion sheath potential well formed on the both sides of the grid. Bifurcations to chaos of $I$ are obtained when an external oscillating term is added to the equation. The correlation dimension in the chaotic regime is found to be $1.55 \pm 0.2$ which agrees with the experimental results $\nu = 1.54 \pm 0.22$. A rich variety of nonlinear behavior is also observed, depending on the parameters. Such a good agreement between experiment and theory indicates that the nonlinear behavior in our system is surely caused by the ion dynamics in an anharmonic potential well, that is, in the Child-Langmuir sheath.

4. Conclusions

A set of cascading bifurcations and a chaotic state in the presence of an external periodic oscillation are experimentally demonstrated in a stable plasma, and a physical model explaining these experimental results is introduced briefly. The most important point of this study is that the nonlinearity of a thick ion sheath on a grid causes a rich variety of behavior in our system. The thick ion sheath on the grid is formed by driving a dc plasma current, and described by the Child-Langmuir law of space-charge-limited current in a plane diode.

References

NONLINEAR EVOLUTION AND LOCALIZED STRUCTURE OF UNSTABLE WAVES
IN AN ELECTRON-BEAM PLASMA

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Since the pioneer work on nonlinear phenomena of Langmuir waves done by Zakharov\(^1\)), considerable progress has been made in this area\(^2\)). There have been growing interests\(^3\)-\(^7\)) in nonlinear evolution of high frequency modes in an unstable plasma. The dispersion properties of unstable waves there are quite different from those in a stable plasma. We\(^7\)) have experimentally investigated nonlinear wave phenomena of unstable electron beam waves and briefly reported the formation of a localized structure. In this paper, we will present experimental results on one dimensional nonlinear evolution of initial disturbance and generation of a soliton-like excitation, and discussed these with the theory developed by Yajima and Tanaka\(^5\)).

A target plasma is produced by a dc discharge in a so-called multi-dipole device. The working pressure of Ar gas for the discharge is adjusted at about \(1 \times 10^{-5}\) Torr to reduce collisional loss of beam electrons with neutral particles. We used a pulsed electron beam with duration \(7 \mu s\), and injected it into a target plasma along an external magnetic field (\(=80G\)), because a dc beam injection strongly changes the initial plasma parameters. Figure 1 shows the experimental setup. The beam current passing through the plasma is monitored by a collector located at the opposite side of the beam gun. Density fluctuations \(\bar{n}_e\) are picked up with plane

Fig.1 Experimental setup.
probes (Mo-disk of 3mm in diameter), and captured by a fast digitizing oscilloscope (HP54111D), whose bandwidth is up to 250MHz in the single shot mode and 500MHz in the repetitive mode. Typical conditions are as follows: plasma density $n_e=3\times10^8$ cm$^{-3}$, beam density $n_b=10^{-3}n_e$, beam velocity $u_b/u_t=5\times6$ (normalized by the thermal velocity $u_t$, beam energy=40-60eV), and wavelength $\lambda=2$cm (< beam diameter 4.5cm).

When an electron beam is injected into the plasma, the plasma becomes unstable in a frequency range lower than the plasma frequency and intense noises with $\omega_e/\omega_p \leq 0.1$ are observed along the beam path ($z>20$cm). The noises are thought to be electron beam instabilities$^{7,8}$. In order to observe evolution of unstable beam waves we repetitively inject a pulsed electron beam and then a small rf signal as an initial disturbance is applied to the control grid of the beam gun. The spatial development of rf-induced unstable waves is measured by interferometry method with a boxcar integrator.

![Fig.2 Spatial evolution of rf-induced perturbations.](image1)

![Fig.3 Dispersion relation of unstable modes in the initial stage of propagation (closed circles) and the calculation from eq.(1).](image2)

Examples are shown in Fig. 2. In the initial stage of propagation ($z \leq 14$cm), the excited waves are linearly unstable, that is, exponentially grow along the beam path, but are stabilized in the nonlinear stage and attenuated, producing a symmetric envelope. Dispersion data extracted from the linear phase are plotted with closed circles in Fig. 3. The temporal growth rate $\omega_1$ is given by a product $|k_1 v|$, where $k_1$ is the
linear growth rate of unstable beam waves and \( \nu_g \) the group velocity. Solid curves indicate the dispersion relation for an electron-beam plasma system:

\[
1 = \omega_{pe}^2 \left[ \frac{1}{(\omega^2 - \kappa^2 \nu_t^2) + (n_b/n_e)/(\omega - \kappa \nu_b)^2} \right] \quad (1)
\]

The equation has a pair of complex conjugate roots, \( \omega_r \pm j\omega_i \), for a fixed wave number \( \kappa \) less than the critical value \( \kappa_{c}^z \) (\( \omega_{pe}/\nu_b \)). One of these roots, \( \omega_r + j\omega_i \) corresponds to unstable waves. Experimental data agree very well with this unstable mode. Another root, \( \omega_r - j\omega_i \), is responsible for the attenuation region (z=20~30 cm in Fig. 2).

As the wave frequency increases from 144MHz to 152MHz (critical frequency=154MHz), the growth rate \( \kappa_1 \) decreases and the amplitude becomes small. On the other hand, the width of the wave envelope \( \Delta z \) becomes broad. Figures 4(a) and 4(b) show the amplitude vs \( \kappa_1 \) and \( (\Delta z)^{-1} \) vs \( \kappa_1 \), respectively. We found that the envelope \( A(z) \) is empirically described by

\[
A(z) \propto \kappa_1 \text{sech} [\kappa_1(z-z_0)] \quad (2)
\]

where \( z \) is the distance from the beam gun, \( z_0 \) the position of the maximum amplitude. Equation (2) shows the localized structure of unstable beam waves is characterized by the
initial growth rate $\lambda_1$. Figure 5 shows that how a wave grows and propagates, when a wave packet with half width of 100ns is generated by applying a small perturbation, where the digitizing oscilloscope is used in an average mode to improve the signal-to-noise ratio. The average drift velocity $\nu_g = 2.6 \times 10^8$ cm/sec agrees with the group velocity obtained from the data in Fig. 3. The amplitude of rf-induced wave packets, the wave packet spatially changes according to the localized structure given by eq.(2). It is also interested that the wave packet separates into fast and slow parts, and appears to produce a new packet in the down stream region.

Yajima and Tanaka$^5$ investigated nonlinear evolution of unstable electron beam waves with a synergetic approach, and derived a coupled nonlinear Schrödinger equation with an additional term of beam correction. They showed that localized modes, to be called soliton-like excitations, can exist in an electron-beam plasma. Yajima and Wadati$^6$ also showed solitons arise as the competition between instability and nonlinearity. The present observations are consistent with their results.

References

Observation of two wave induced chaos in a magnetized plasma

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Introduction

The problem of wave induced stochasticity in plasmas is being studied extensively because of its general interest in chaotic dynamics and of its possible direct consequences in space and laboratory plasmas, such as anomalous transport and non-linear heating and particle acceleration [1].

Several theoretical investigations have been performed in the recent past, based mainly on hamiltonian single particle models, which predict the transition to stochasticity in terms of the wave amplitude and spectrum [2]. The majority of these theories are non self-consistent, that is, do not take into account possible mutual interactions between wave and particles. Self-consistency characterizes actual experiments, in which non-linear wave-wave interactions, in the case of several modes in the plasma, are also naturally present.

It has been shown theoretically and experimentally [3,4], that in a magnetized plasma a wave propagating at a finite angle with respect to the B-field can generate chaos in particle orbits and, consequently, fast ion heating.

The case of two (or more) waves propagating at different phase velocities is predicted to be more efficient for the heating: the threshold amplitude for the occurrence of chaos should be lower, due to a large number of resonances in phase space.

In our experiment we investigate the interaction between ions and two propagating electrostatic modes in a Q-machine plasma.

In particular, our aim is to perform and integrate observations of the collective ion response (wave characteristics), of ion kinetic features (modification of the distribution functions, time scale for heating) and of single particle orbit modifications (phase space transport).

Experimental set-up and diagnostics apparatus

The experiments are performed on the LMP barium Q-machine [5], a uniformly magnetized plasma column characterized by ion and electron temperatures of the order of 0.2 eV and densities in the range of $10^8$-$10^{10}$ cm$^{-3}$. The maximum axial B-field is 0.3 T ($f_{ci}=30$ kHz). Sheath acceleration at the hot plate causes a supersonic ion drift $v_D=10^5$ cm/s. Low degrees of spatial and temporal variations and fluctuations can be achieved in current operation (e.g. $\delta/n/n<1\%$).

Electrostatic ion waves ($f=f_{ci}$) are launched by a capacitive antenna consisting of 4 rings placed directly around the plasma column at variable relative distances and phase.

The diagnostic system is based on the technique of Laser Induced Fluorescence (LIF) [6], which provides a direct measurement of ion distributions with good spatial and temporal resolution (the latter allows time resolved measurements of $f(v)$ and synchronous detection for $f^1(v)$).

LIF can be extended to an optical tagging method, based on the spin polarization of ground state ions [7]: sets of test-ions can thus be created and followed in their evolution in order to infer the nature of particle orbits. A schematic of the LMP machine, including the electrostatic antenna and the geometry for the LIF laser beam injection and detection systems, is shown in fig.1.
Results and discussion

The wave spectrum excited by the 4 ring antenna in the plasma at one frequency $f$ ($f_{ci}<f<2f_{ci}$) is composed in the parallel plane by two modes with two different phase velocities, and in the perpendicular plane by the two branches (forward and backward) of the ESICW dispersion relation [8]. In fig.2 we see the $k_f$ spectrum at 25 kHz ($f=1.1f_{ci}$), as directly evinced from the observed form of the first order perturbed distribution function. Two distinct peaks are clearly visible, corresponding to the two phase velocities $v_\phi 1, v_\phi 2$. ($\Delta v_\phi = 5 \times 10^4$ cm/s).

The difference $\Delta v_\phi$ is such that on the parallel ion phase space one can consider, to a first approximation, only primary resonances and neglect the multiple island structure introduced by the non zero perpendicular wavenumber (natural cyclotron resonances should appear, spaced by $2\pi f_{ci}$; it is indeed their interaction which generates chaos in the one wave case).

The hamiltonian models predict therefore a transition to a stochastic regime when the wave amplitude is such that the two resonances start to overlap. A stochasticity parameter can be introduced: $K = 2(A_{11/2} + A_{21/2})$, where $A_i = e\phi_i/m(\Delta v_\phi)^2$ is the amplitude of the mode $i$ ($i=1,2$). $K=1$ is the threshold for the transition, in the frame of the single particle theory; in macroscopic terms, $K>1$ should imply a fast ion heating.

In fig.3 we plot the parallel and perpendicular ion temperatures as functions of the excitation amplitude. We notice that a threshold value exists, above which a significant heating occurs. A calibration of the wave amplitude through the ion dielectric response [9] allows us to compare the observed threshold to the theoretical prediction for the experimental wave parameters. $K=1$ corresponds to the shaded region on the amplitude axis of the graph. We see that, inside the error bar, mainly due to the wave amplitude calibration procedure, experiment and single particle theory agree.
By pulsing the wave excitation generator, and observing the time resolved ion distribution from the time $t=0$ (when the RF is started) onwards, an accurate estimation of the heating time can be achieved. More specifically, by plotting the increase in the ion temperature (or mean square ion velocity) vs. the time, if the dependence is linear, a direct estimation of the velocity space diffusion coefficient can be obtained. This is shown in fig. 4, for the case of $25 \text{ kHz}$ and just above threshold. The resulting $v$-space diffusion coefficient is more than one order of magnitude larger than the collisional coefficient.

The mechanism responsible for the heating, therefore, is independent of collisional processes and can be attributed to a transition to chaos in particle trajectories. An additional proof of this mechanism comes from the tag measurements, which evidence an exponential separation in time of initially close ion orbits, both in velocity space and in real space.

At frequencies for which only one mode in parallel can be excited in the plasma, no heating is observed. Up to amplitudes where secular perturbations of the antenna and intrinsic nonlinearities (e.g., harmonic generation) become effective. The case of two waves is then experimentally demonstrated to be more favorable in the achievement of chaos and stochastic heating.

By further increasing the amplitude of the wave(s) well above threshold, another regime is observed: heating no longer takes place, and no more than one mode seems to be excited in the plasma. On the other hand, in the pulsed regime, by following time evolution further on after the heating is reached, temperature is observed to decrease and eventually to return to its unperturbed value (or even lower).

The two results can be interpreted as a manifestation of the feed-back action of the particles on the waves. In fact, the ion orbits undergo a transition in their topology, from regular to chaotic: the plasma oscillating fields, which are issued from collective motions of the charged particles and therefore are derived explicitly from integration of the particle trajectories, are necessarily modified. In particular, experimental observations close to the threshold for the transition to the chaotic regime seem to indicate that of the two modes only one survives. The conditions for the occurrence of chaos are then no longer satisfied, and the plasma tends to relax to its unperturbed equilibrium.

Conclusions

Stochasticity in ion dynamics originated by the interaction between particles and two electrostatic propagating plasma waves has been observed in a magnetized plasma. Optical measurements at different scales, from the single particle to the macroscopic, allowed a determination of the wave features, the kinetic ion response and the plasma heating mechanism.

Chaos appears to be limited by self-consistent effects. More results on these effects, as well as the modification of ion transport in the presence of the two waves will be discussed at the presentation.

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References

Fig. 2: $k_{\parallel}$ spectrum of the wave electric field at $f=25$ kHZ, deduced from $f' (\nu_{\parallel})$ via a 1-D Vlasov model.

Fig. 3: Parallel (top) and perpendicular (bottom) ion temperatures vs. the wave amplitude. The shaded region corresponds to $K \approx 1$.

Fig. 4: Temporal evolution of the increase of the mean square parallel ion velocity; the RF burst starts at $t=0, f=25$ kHz; exc. ampl. = 15 A.U.
Observations of Intermittent Structures in association with Low Frequency
turbulence in Toroidal Magnetized Plasma

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A plasma embedded in purely toroidal field is unstable due to free energy sources available in the form of density gradients, curvature of the magnetic field, inhomogeneity in the magnetic field, etc. The effective gravity antiparallel to density gradients favors R-T type of instabilities in these regions. The density gradients itself can provide energy for drift type of oscillations in the plasma. In the nonlinear limits these instabilities are known to give rise to vortex structures in the plasma flow (Shukla, 1984, 1989 for a review on this topic). In the present paper we report the study of resulting structures from a turbulent state. A study of plasma turbulence under above mentioned conditions has been made. The general behavior of plasma turbulence in the device have been reported earlier by Bora (1989) and Prasad et al. (1992). The turbulence was interpreted to be due to R-T and drift waves, and a modification of these due to velocity shear.

The experiment was carried out in a toroidal device called BETA (inset in figure (1))(PPP-report (1984)), major Radius 45.0 cm, minor radius 15.0 cm, having a plasma column of diameter 18.0 cm, terminated by a limiter in the form of a poloidal annular ring. The plasma is produced by electron impact ionization of Argon gas at a fill pressure of \(10^{-4}\) Torr, base pressure \(10^{-6}\) Torr. The present experiments were performed at 1 toroidal magnetic field of 1 kGauss, plasma density \(n_p \sim 10^{11}\) cm\(^{-3}\) and \(T_e \sim 5.0\) eV.

Figure (1) shows the radial profiles of plasma density and potential. Both density and potential peak near the center and fall as we go radially outward or inward. The typical density variation scale length \((1/n)(dn/dz)^{-1} \sim 6\) cm. The curvature on the magnetic field provides an effective \(g \sim C_f^2/R\). This \(g\) and \(\nabla n\) are antiparallel in the radially outer locations, making this region unstable to R-T instability. Owing to these density gradients and the potential gradients the plasma density and potential fluctuate and the plasma is in a turbulent state. This turbulence has been reported earlier by Prasad et al. (1992). The oscillation on plasma density and potential are of frequencies \(\omega \leq \omega_{ci}\).

In the present paper we report the study of intermittent structures in the plasma stream lines of flow. The evidence of presence of such structures comes from the probability distribution functions of the fluctuation of potential and density. The PDF calculated at \(r = +8.0\) cm reveal that the distribution is non-Gaussian. The value of skewness parameter is non-zero. Curtosis also deviates from a Gaussian value. For negative potentials PDF falls slowly and then exists a sharp cutoff for the fluctuation beyond \(\phi/KT_e \sim 60\%\). The deviation from non Gaussian PDF is more in the central region of plasma. The plasma close to limiter ring has PDF very close to the Gaussian statistics. Figure (2) shows the calculated PDF at different \(r & z\) in the poloidal cross-section. The non-Gaussian PDF is a signature of coherent states in the turbulence as discussed by (She, 1990, Kraichnan, 1990)). The statistics of small scale fluctuations in turbulence differ from Gaussian by having large probabilities for events with higher amplitudes. This contributes to the higher order moments of the PDF e.g. Skewness and curtois (She, 1990). Under such situations secondary mechanism of energy cascade from larger wavelengths to shorter
wavelengths have been suggested by Kuznetsov et al. (1991). The bursty nature of distribution are suggestive of coherent structures in the system. Kraichnan (1990), Benzi & Biferale et al. (1991) also suggest that the higher order moments are related to strong intermittent burst in the energy transfer to small scale.

In order to study the coherent structure we measured the plasma floating potential on a grid of 15x12 using a set of movable Langmuir probes. A fixed probe at \( r = +8.0 \text{ cm} \) was used as a reference probe. The auto correlations at each point was calculated. The cross correlation between reference probes and the movable probes were also calculated using FFT based techniques. On the basis of the correlation times at different positions it was found that the fluctuations have different behavior in the regions of the plasma radially. Typical correlation time varied between 150 \( \mu \text{s} \) to 250 \( \mu \text{s} \). The contours of equal cross correlations \( < \phi(t)\phi(t+\tau) > \) plotted in a plasma perpendicular to the magnetic field reveals the formation of a closed eddy in the inner central region of plasma between \( r = -5.0 \) to \( r = 5.0 \text{ cms} \). There were no closed structures found in the radial outer location in the plasma. The cross correlation are insensitive to the sign of actual potential and hence can not represent the stream lines of flows. A more detailed information about these closed structures can be obtained by the techniques of conditional averaging. This technique is widely used in fluid dynamics (Blackwelder, 1977; Adrian, 1975) for the study of two dimensional structures. This technique has also been used by Huld et al. (1988, 1990, 1991), Pécseli and Trulsen (1989) in the investigation of plasma eddies resulting from flute type of instabilities.

A time resolved contour plots of these conditionally averaged potential are obtained. Figure (3) shows the contours obtained with a condition \( \phi_1 = -1.5\phi_{min} \). Different frames are taken at a time interval of 25 \( \mu \text{sec} \). The contours at time \( t=0.0 \text{ secs} \), show the presence of a large (\( \sim 6.0 \text{cm} \) radially \( \sim L_n \)) closed structure in the central region of plasma. These structures have negative polarity. There is also seen a positive structure in the inner side of the plasma cross section vertically upward. As time progresses the negative structures in the central region start shrinking in size and positive structures appear. At further delays the entire central region is occupied by large positive structure. The negative structure appear in the top left corner. This takes about 100\( \mu \text{sec} \). At further delays the bigger structure elongate in a direction perpendicular to the density gradient and break into smaller structures. These contours have been plotted for several conditions as
Figure 2: Calculated Probability distributions of $\phi$. Also shown is a Gaussian with the calculated standard deviation.

Figure 3: Contours of conditionally averaged potentials with a condition $\phi = -1.5 \text{ volts } \phi_{\text{rms}}$. See text for details.
mentioned earlier, the basic observation remains the same.

In conclusion we have studied two dimensional features of low frequency turbulence in a toroidal magnetized plasma. The fluctuations associated with the turbulence exhibit non Gaussian statistics in certain region of plasma. Short lived coherent structures have been observed in presence of turbulence, using the technique of conditional analysis on the potential signal. This analysis shows that there exists large structure in the inner and central region of plasma. Initially we see an structure having radial dimension as large as the density scale length. This at a later time develops into elongated closed eddy with the elongation in a direction perpendicular to the density gradient. At still later times the structure breaks into smaller structures. These smaller eddies are found to merge with the bigger ones of similar sign, at still longer times to form again a bigger structure. It is also observed that initially the central region was filled with negative polarity structures which disappears from this region at further time delays. These structures reappear again in a different inner top region of the cross section. The central region gets filled up with positive structures. This gives further signature of intermittency associated with non Gaussian statistics.

References
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A variety of instabilities can be excited by cross-field drifts in magnetized plasmas. The present work describes a sound-like cross-field instability near the lower-hybrid frequency ($f \approx f_{hy} = 100$ kHz) in a large plasma column (1m diam. x 2.5m length) immersed in a weak magnetic field ($B_0 = 15$G) /1/. The magnetized electrons perform a diamagnetic drift $\mathbf{v}_d = \mathbf{V}_p \times \mathbf{B}_0/\mathbf{neB}_0^2$ relative to the essentially stationary, unmagnetized ions ($v_n > \omega_e$). When the relative drift exceeds the sound speed, $v_n > c_s = (kT_e/m_e)^{1/2}$, unstable waves grow across $\mathbf{B}_0$ /2/. The experimental investigation includes the $\omega-k$ spectrum, saturation mechanisms, finite $\beta$ effects, particle acceleration and transport.

Figure 1 summarizes the experimental arrangement. A pulsed dc discharge is created with parameters as indicated. Plasma and wave properties are diagnosed with Langmuir probes, directional velocity analyzer, electric and magnetic field probes. The latter are useful in this high-beta plasma [$\beta = n_k T_e / (B_0^2/2\mu_0) = 0.5$] because the density fluctuations of the predominantly electrostatic instability are coupled to magnetic fluctuations. The net electron drift is obtained from $\mathbf{V} \times \mathbf{B}/\mathbf{p}_e = n \mathbf{v}_e$. The measurement technique includes conditional averaging /3/ using digital oscilloscopes. Here, a reference probe provides a conditional trigger while a movable probe signal is ensemble-averaged to obtain coherence in time and space.

The strongest fluctuations are observed off-axis where the density gradients are largest. Fig.2a shows the typical density fluctuations vs. time during the discharge pulse while Fig.2b displays its frequency spectrum.

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Fig. 2  (a) Time-dependence of the density during a discharge pulse. Large amplitude fluctuations exist off axis where the pressure gradient maximizes. (b) Frequency spectrum of the fluctuations showing a peak near the lower hybrid frequency, $f_{\text{LM}} = 120$ kHz, and high frequency tail due to wave steepening of $\delta n$, shown on upper trace.

The peak in the spectrum is below the lower hybrid frequency. The higher frequency components are phase coherent and caused by wave steepening as visible in the temporal waveform of the density fluctuations.

By moving a probe axially, radially and azimuthally it is found that the fluctuations consist of waves growing and propagating initially in the azimuthal direction along the electron diamagnetic drift $(k_x, k_y << k_z)$. The phase velocity matches the sound speed $(c_s = 2.5 \times 10^3 \text{ cm/s} = \sqrt{\gamma})$. The plasma column exhibits no radial electric fields $(E \times B_t = 0)$ but a small radial electron temperature gradient $(\partial T_e / \partial r < 0)$. Due to the

Fig. 3  Conditionally averaged density fluctuations and typical wave front. (a) $<\delta n(r)>$ at different azimuthal positions showing a constant velocity component. (b) $<\delta n(r)>$ at different radial positions showing a decreasing radial velocity component. (c) Transverse $r - \theta$ plane with a curved phase front constructed from the measured velocity components $v_{\phi} = \omega / k = (v_x^2 + v_y^2)^{1/2}$. The curvature is due to wave refraction by temperature gradients, $v_{\theta} = C_1 = (kT_e/m)^{1/2}$. 
temperature dependence of the sound speed the growing waves begin to refract radially outward /4/. Typical conditional averages of density fluctuations in azimuthal and radial direction are shown in Fig.3a,b from which the curved phase front of Fig.3c has been constructed. Due to the large size of the plasma column (average circumference 2m), the waves refract radially outward and decay rather than forming azimuthal eigenmodes as, e.g., drift waves do in small columns. As the waves refract from the azimuthal into the radial direction they stabilize since the drift no longer exceeds the azimuthal phase velocity component ($v_d < c/\cos\theta$). Thus, refraction provides a saturation mechanism for the instability.

A second saturation process is nonlinear wave steepening which is observed here for wave propagation across $B_0$ while earlier studies of sound wave steepening were done for $B=0$ /5/. Fig.4 shows a typical waveform of a large amplitude density fluctuation ($\delta n/n \geq 25\%$). The steepened wave front has a temporal and spatial scale of $-2\%$ of the wave period and wavelength, respectively. The shock thickness ($\Delta s = 1.2$ mm) is between the Debye length ($\lambda_D = 0.02$ mm) and the electron Larmor radius ($r_{\text{Larmor}} = 4$ mm). The latter condition implies that finite Larmor radius effects modify the otherwise adiabatic electron dynamics. The spikey shock electric field can accelerate the electrons directly across $B_0$ which leads to wave damping and saturation.

In addition to density fluctuations the perturbations in other physical parameters have been measured. It is found that wave density and potential are positively correlated although the Boltzmann relation is not quantitatively satisfied ($\delta n/n > e\delta \rho/\varepsilon T_e$), possibly due to resistivity along $B_0$. Axial magnetic field fluctuations are observed to be anti-correlated with density perturbations. However, the theoretical relation predicted by

![WAVE STEEPENING](image)

Fig.4 Wave steepening of large amplitude sound wave propagating across $B_0$. Expanded view indicates rise time $t_r = 0.5\mu s \ll$ wave period $t_p = 20\mu s$. The corresponding shock thickness, $\Delta s = c t_r = 1.2$ mm is smaller than the electron Larmor radius, $r_{\text{Larmor}} = 4$ mm.

Fig. 5 Comparison of density and magnetic fluctuations showing that $\delta n$ and $\delta B_z$ are anti-correlated. The time derivative of the magnetic fluctuations is proportional to the radial current density

$$\delta J_r = \nabla \times \delta B_z / \mu_0 = -\delta B_z / (\mu_0 v_p).$$

The current is caused by electron diamagnetic drifts opposed by Hall drifts $\delta E_x \times B_0 / B^2_0$.

Pressure balance is quantitatively not fulfilled, $|\delta B_z / B_0| < (\delta n/n)/\sqrt{2}(1-\delta)$. The reason is that electron diamagnetic drifts set up by the wave $\delta v_x = kT_e \nabla n \times B_0 / n e B_0^2$ are partly canceled by electron $\delta E_x \times B_0 / B^2_0$ drifts. If the Boltzmann relation were satisfied $\delta E_x = (kT_e / c) \nabla \delta n / n$ both drifts would exactly cancel ($\delta B_z = 0$). From $\nabla \times \delta B = \mu_0 \delta J$ the net perturbed current density $\delta J = ne \delta v_x$ has been quantitatively obtained ($\delta J_{\text{net}} = 100 \text{ mA/cm}^2$). The radial electron drift in the wave ($\delta v_e = 8 \times 10^5 \text{ cm/s}$) is as large as the azimuthal electron drift due to the background pressure gradient. Thus, the wave causes significant radial electron transport.

It is often assumed that large amplitude lower hybrid waves cause parallel electron acceleration and heating. Using a Langmuir probe the rapidly swept ($\Delta t \approx 3 \mu s$) current-voltage characteristics have been traced out at a wave maximum ($\delta \phi_{\text{max}}$) and minimum ($\delta \phi_{\text{min}}$). The temperature is found to be constant to within $\sim 5\%$. Since the electron pressure gradient provides the free energy for driving the instability, ion heating rather than electron heating might be expected. Similar time-resolved ion velocity analyzer traces show small bulk ion heating ($\Delta T / T_i \approx 25\%$). Although the wave potential energy exceeds the ion kinetic energy ($e \delta \phi = 0.6 \text{ eV} > kT_i = 0.4 \text{ eV}$) the short interaction time between waves and particles ($v_{ni} > v_{ai}$) prevents significant ion acceleration. However, a bulk ion drift ($v_n / c_i = 5\%$) in the direction of wave propagation has been observed. Steepened ion acoustic waves can cause a bulk ion flow /6/.

In summary, the basic properties of a pressure-gradient driven instability in a weakly magnetized plasma column have been investigated. Although the instability grows to large amplitudes ($\delta n/n \geq 50\%$) it results in little particle acceleration and heating. Numerous situations in laboratory and space plasmas exist where this instability may occur.

EXPERIMENTAL STUDIES OF ANOMALOUS POTENTIAL DROPS DUE TO ION DENSITY INHOMOGENEITIES

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Double layers form in laboratory plasmas as the response to an applied or induced voltage drop, but the significant mechanisms in different processes for double layer formation are still unclear. In a plasma maintained by local ionization of the background gas due to electron impacts, a double layer forms from an electrode sheath when a sufficiently large electron current is drawn to an electrode. The significant mechanism for the formation of the double layer is in this case the generation of positive ions in the electrode sheath [1]. Ionization phenomena also lead the voltage drop over steady double layers to relatively small values which are related to the ionization potential of the background gas /2/. In contrast to this, double layers with very large voltage drops have been observed in the "collisionless" plasmas that can be produced in Q-machines and differentially pumped triple plasma machines /3,4/. In this paper we report experiments showing the dynamic plasma potential response when a step voltage drop is applied along the plasma column in a differentially pumped triple plasma machine. The time resolution in the potential measurements is 0.5 μs which is close to the electron transit time along the plasma column. Potential changes caused by essentially the electron motion can therefore be detected. Similar investigations have been made in the axially homogeneous plasma column of a Q-machine, operated "double-ended" in the "electron rich" regime /5/. In this case, most of the potential drop first concentrates in a narrow region at the low potential end of the plasma column. On the time scale of the ion motion this potential drop begins to propagate into the plasma as a double layer. Here we shall show that a quite different plasma potential response may be obtained when the initial ion density is inhomogeneous and has an "ion density cavity", that is, a local minimum with a width of many Debye lengths. For a sufficiently low minimum density, the potential drop becomes distributed over the cavity during the first few electron transit times, and the profile steepens slowly to a double layer on the time scale of the ion motion. Similar results have also been obtained in numerical simulations /6,7/ and been predicted by a theoretical model /7/. An ion density cavity may also strongly modify the non-linear evolution of plasma waves /8/. The differentially pumped triple plasma device (Fig. 1) has a central chamber with a plasma column which is maintained by a source on each side. There plasmas are produced by discharges between filaments and the source chamber walls. The plasma column is radially confined by an axial magnetic field (5 to 50 mT). The neutral atom (argon) density in the central chamber is kept much lower than in the sources where a certain minimum neutral atom density is required to get quiescent discharges. Circular apertures, which connect the
sources and the central chamber, define the radius of the plasma column (3 cm). Potential drops exist at the apertures. They reflect some of the source electrons back to the source and accelerate source ions into the central chamber, similar to the "electron rich" boundary conditions at the plates in the Q-machine experiment. At the apertures, the density drops from the value $N_5$ in the sources to the value $N_1$ in the plasma column. For identical source parameters, a density minimum, $N_1-\Delta N$, can be produced halfway along the plasma column which then forms a symmetric cavity with a width comparable to the length of the column (Fig. 2). Asymmetric cavities can also be produced by having different source densities. The cavity is due to radial ion losses, and the depth of the cavity can be varied ($0.1 < \Delta N/N_1 < 0.7$) by varying the magnetic field and the potential of the end plates of the central chamber. $N_1$ is close to $10^{15}$ m$^{-3}$ and $k_BT_e$ typically 10 eV. A step voltage (rise time 20 ns) was applied periodically between the plasma sources, and plasma potentials were sampled with emissive probes. The good time resolution was obtained by reducing the distributed probe capacitance to ground, and it was checked by direct measurements of rise and fall times. Fig. 3 shows the plasma potential response for an asymmetric ion density cavity with a minimum at $z=20$ cm.

Fig 2. Initial axial density profile at the symmetry axis. Probe currents at plasma potential give a measure of the ion densities. The machine was operated symmetrically and only the variation between 3 and 30 cm is shown (length of plasma column 60 cm). The relative cavity depth, $\Delta N/N_1$, can be varied ($0.1 < \Delta N/N_1 < 0.7$). Close to the low potential aperture ($z=0$), the density increases sharply to the density $N_5$ in the source.
\[ \Delta N/N_1 = 0.7 \text{ and } N_s/N_1 = 10. \] Here \( N_s \) and \( N_1 \) are the densities on the low potential side. The corresponding values on the high potential side were slightly higher. Radial potential profiles are given at \( t=1\mu s \) and \( t=100 \mu s \) for different axial positions. At \( t=1 \mu s \) (about two electron transit times) the profile varies linearly over most of the length of the plasma column, and the electron emission from the low potential source is also limited by a potential minimum, situated at about \( z=7 \) cm (not shown in the diagrams). During the subsequent ion motion, the axial profile steepens, and a double layer has formed between \( z=48 \) cm and \( z=52 \) cm after about 100 \( \mu s \). Similar profiles were obtained for applied potential drops up to 400 V. Fig. 4 (left) shows potential profiles along the symmetry axis when again \( \Delta N/N_1 = 0.7 \) and

\[ \Delta N/N_1 = 0.7, \quad N_s/N_1 = 10. \] The potential drop is first \( (t=1\mu s) \) supported by an almost linearly varying potential along the magnetic field. (right) The initial density profile kept as flat as possible \( (\Delta N/N_1 < 0.1) \) and the source density is lower \( (N_s/N_1 = 3) \). Now most of the applied potential drop is first supported by a narrow region near the low potential aperture, and the potential along the plasma column is approximately constant. Then the potential drop propagates into the plasma on the time scale of the ion motion.
$N_2/N_1=10$. Fig. 4 (right) shows profiles when the initial profile was kept as flat as possible ($\Delta N/N_1<0.1$) and $N_2/N_1$ was reduced to 3. Now a quite different response is obtained. The potential is approximately constant along the plasma column during the first few microseconds, and most of the applied potential drop is concentrated to a narrow region close to the low potential source. Then this drop begins to propagate into the plasma as a double layer. This is the same type of response as observed in the Q-machine experiment.

We have shown that there are two different types of responses to a step voltage drop depending on the initial ion density distribution. These do not only differ during the first microsecond, but the subsequent evolutions on the time scale of the ion motion are also different. The ion density cavity can support very large potential drops for several electron transit times. In contrast to a double layer, where the Debye length is the spatial scaling constant, the cavity potential drop is extended over the cavity width. The cavity potential drop evolves slowly into a double layer on the time scale of the ion motion.

The results obtained are consistent with an analytical model and a particle-in-cell simulation /7/. Also these demonstrate the existence of two different states, similar to those observed experimentally. The natural ion to electron mass ratio is used in the simulation which covers about three electron transit times corresponding to about 1 $\mu$s in the experiment. Cavity potential drops are obtained when the applied voltage drop falls below a critical voltage drop which depends on only $N_0/(N_1-\Delta N)$. Here $N_0$ is the density of the injected electrons at the boundaries, and $N_0 > N_1$ in electron rich injection. For larger voltage drops applied, the potential drop instead concentrates in a "cathode sheath".

Ion density cavities may form due to nonlinear ion waves or due to the ponderomotive force from high frequency wave packets. Further investigations are required to decide whether such cavities may modify the voltage supporting capability of the plasma and introduce a new type of anomalous resistivity.

References
ION RESONANCE CONES AND INTRINSIC DIFFRACTION

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Introduction:

The radiation of a point source antenna immersed in a hot magnetized plasma has been studied during the past twenty years and is often related to the resonance cone phenomenon (see the C.M.A. diagram). Namely, the strong anisotropy of the medium for the propagation of the electrostatic waves is the reason for the existence of a narrow angular radiation of the antenna. In the low frequency range below the ion cyclotron frequency, the ion resonance cones have been first introduced by H.H. Kuehl: the reason for the existence of this phenomenon is the specific shape of the dispersion surface exhibiting a region of low damped wave-vectors with a mean group velocity dependent on the frequency value.

This phenomenon has seldom been observed experimentally in not very suitable plasma conditions, very close to the antenna and in a very limited plasma column. We are presenting here a numerical bidimensional study of the radiated potential near the point source antenna below the ion cyclotron frequency and the corresponding experimental records. The results are interpreted in terms of intrinsic diffraction in relation with a model previously introduced in the case of the electron modes.

Calculation of the radiated potential:

The dispersion relation of the electrostatic modes in the hot magnetized plasma expresses as:

$$\varepsilon(\Omega, K_\perp, K_\parallel) = 1 + \frac{\Omega_2^2}{\Omega_2^2 - K^2 \left[ 1 + \sum_{-\infty}^{+\infty} I_n(n_c) e^{\lambda_n z L(z_n)} + \frac{T_e}{\Omega_2^2} - \frac{1}{K^2} \left[ 1 + \sum_{-\infty}^{+\infty} I_n(n_c) e^{\lambda_n z L(z_n)} \right] \right]$$

where $\Omega = \omega/\omega_{ci}$, $K = k c_s/\omega_{ci}$, $\lambda_0 = K_{\perp}^2 m/M$, $\lambda_1 = K_{\parallel}^2 T_1/T_0$ and
\[ \zeta_{n} = \frac{(\Omega + n M/m)}{k_z \sqrt{2M/m}} \quad \zeta_{n} = \frac{(\Omega + n M/m)}{k_z \sqrt{2M/m}} \]

The corresponding dispersion branches are related to the pure ion Berstein mode when the wave vector is exactly perpendicular to the B field, to the neutralized ion Berstein mode when the wave vector departs from the perpendicular direction and to the ion cyclotron mode, i.e., the ion acoustic mode. The radiated potential is calculated following Kuehl's method [11]:

\[ \Phi(r) = \sum_{m=1}^{+\infty} \frac{H_0^{(1)}(k_{l,l}) \exp(ik_{l,l}r)}{8\pi\epsilon_0} \int_{-\infty}^{+\infty} \frac{d\kappa_{\|}}{D_m(k_{\|})} \]

where \( D'(k_z) = \frac{\partial D}{\partial k_z^2} \) and \( H_0^{(1)} \) is the Hankel function.

**Numerical results:**

Keeping only the least damped root of the dispersion equation, the domain of integration is truncated once the damping coefficient becomes large and there is no longer a significant contribution to the integral.

The typical results obtained with Helium and \( \omega_{pi}/\omega_{ci}=5 \), \( \omega/\omega_{ci}=0.7 \) and \( T_e/T_i=20 \) are displayed on figure 1. It is in fact necessary to choose a low mass gas in order to obtain large values of the normalized distance to the antenna due to a small ion gyroradius. The map shows the potential calculated in front of the point source antenna up to a distance 50 cm in both radial and axial directions. The resonance cone is easily seen along the dashed line and moreover an axial potential modulation is observed in the axial direction. By varying the plasma parameters, this modulation has been shown to be very dependent on the ion temperature: the modulation length is continuously decreasing with increasing the \( T_e/T_i \) ratio.

This phenomenon has been related to the particular shape of the dispersion surface. Figure 2 displays the dispersion curve \( k_{\perp}(k_z) \) for \( \omega/\omega_{ci}=0.3, 0.5, 0.7 \). The main point is that the damping (dashed line) is increasing very rapidly at some particular value of \( K_z \). This behaviour is similar to the case of the electron modes, where this sharp cut-off of the non-damped wave vectors has led to the intrinsic diffraction model [3]: in this model the radiated potential is shown to be modulated in
the same way as the optical intensity behind a screen with sharp edges. We show that this model can be applied to the ion waves radiated by a point source antenna below the cyclotron frequency, due to the existence of this rapid increase at some particular $K_2$ value.

Moreover, the sensitivity of the modulation length to the ion temperature can lead to the possible use of this phenomenon for the measurement of the $T_e/T_i$ ratio in a magnetized plasma.

**Experimental results and discussion:**

The theoretical results have been compared to the records obtained in a magnetized double plasma device, in a helium plasma with a B-field lower than 0.12 Tesla /4/. The diameter of the plasma column is 26 cm. The plasma is created by electric discharge in each end chamber. The emitting antenna is fed by a signal generator at the level +40 dBm. The potential radiated around the point source antenna is recorded by a HF probe located on a X-Y table. The signal is recorded by a spectrum analyzer and the results are stored on a PC for further processing.

Fig. 3 displays the map of the potential obtained at the frequency 100 KHz with $f/f_{ci}=0.7$, $n_e=10^{10}$ cm$^{-3}$ and $T_e=3$ eV. A potential modulation along the direction of the B-field is easily seen, although the cone structure is not evident here. The value of the modulation length has been found consistent with the expected temperature ratio $T_e/T_i=60$.

These preliminary results show that the measurement of the temperature ratio by using this axial modulation of the potential radiated by a point source antenna below the ion cyclotron frequency is easier than the estimation of this ratio by recording the angle and width of the ion resonance cone. In fact, only an axial displacement of the probe is needed. Moreover, only a rather simple experimental set-up and standard equipment is necessary.

**REFERENCES:**

FIG 1

FIG. 2

FIG 3
REFLECTION OF A DIVERGENT RESONANCE CONE NEAR THE PLASMA LAYER IN A MAGNETIZED INHOMOGENEOUS PLASMA

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I - INTRODUCTION

The potential radiated by a small antenna in a magnetized plasma exhibits very anisotropic features for two frequency ranges related to the lower and higher hybrid frequencies, $\omega_{\text{lh}}$ and $\omega_{\text{uh}}$. For the lower ($\omega_{\text{lh}} < \omega < \min(\omega_p, \omega_c)$) and upper ($\max(\omega_p, \omega_c) < \omega < \omega_{\text{uh}}$) branches, the energy flows along resonance cones, having their summit on the antenna and their axis in the magnetic field direction.

The theoretical description of electrostatic resonance cones for a warm infinite homogeneous plasma has been given first by Kuehl/1/. Temperature effects produce a secondary structure of cones of smaller amplitude.

The basic properties of resonance cones have been verified in several experiments/2,3/, namely the cone angle as a function of the frequency, plasma density and magnetic field, and the structure associated to temperature effects. However, whereas the potential amplitude has been accurately measured, there are very few phase measurements. On the other hand, due to the finite size of a real experiment, the resonance cone emitted by a point-source can undergo a reflection at the surface where the frequency equals the local plasma frequency. Only one paper/4/ deals with this problem.

We report in this paper experimental results on both amplitude and phase of the potential. We find that, for a broad range of parameters, the phase variation across a single resonance cone is $2\pi$, due to bounded plasma effects. We also study the reflection of a resonance cone at the cut-off plasma layer and give an estimate of the phase jump at the reflection.

II - EXPERIMENTAL SET-UP

Experiments have been done in a multipolar plasma device operating in a magnetic field/5/. The maximum magnetic field strength is 800 gauss. The magnetized plasma column is 1.4 m long and 26 cm in diameter. The plasma density goes up to $10^{11}$ cm$^{-3}$ and the electron temperature is 3 eV.
Figure 1 gives the overall view of the experimental set-up. The emitting point-source antenna is located at the center of the plasma column. A HF detection antenna is movable along two perpendicular directions by means of step-motors. The detected signal is sent to a spectrum analyser, that records the potential amplitude, and to a digital oscilloscope, triggered by a reference signal, from which the phase evolution can be reconstructed. The data are stored and processed by a personal computer.

III - EXPERIMENTAL RESULTS

Divergent resonance cone

We present here results for a homogeneous plasma with the following parameters: wave frequency $f=340$ MHz, electron plasma frequency $f_{pe}=1.5$ GHz and electron gyrofrequency $f_{ce}=675$ MHz. Figure 2 shows 3D maps for the potential evolution. The usual evolution of the amplitude is shown on Figure 2a, where the principal peak of the resonance cone as well as the secondary
thermal structure can be seen. Figure 2b displays for the first time a similar map for the phase evolution. There is a phase jump across the main peak, followed by a plateau in the inside region. Further analysis shows that equipotential curves are also equiphase curves, in agreement with the fact that the phase velocity is perpendicular to the group velocity.

The phase jump across the resonance cone is better shown on Figure 3, that displays the radial dependence of the potential. The asymptotic value of the phase is $2\pi$. This value is found whenever the plasma density is high enough (so that there are no electromagnetic corrections) if the perturbations due to adjacent cones are negligible, so that the cone can be considered "isolated". As a numerical computation does not predict this behavior, the phase value of $2\pi$ is probably due to bounded plasma effects: the phase variation can be perturbed by the boundaries much more easily than the amplitude.

Reflection at the plasma layer

We next study the resonance cone propagation in a linear density profile. The parameters are now: $f = 697$ MHz, $f_{peo} = 1.2$ MHz and $f_{ce} = 1.1$ GHz. Figure 4a displays the evolution of the amplitude of the incident and reflected resonance cones versus the axial distance $z$ for several radial distances $r$. The
incident cone and its thermal structure are much better defined than the reflected cone. This comes from the fact that the resonance cone is made of a broadband spectrum of wavenumbers that is strongly perturbed by the reflection process. Figure 4b shows the related phase evolution. The phase jumps associated to the incident and reflected cones are clearly seen. The total phase variation across the incident cone is again $2\pi$. A small periodic phase variation related to the thermal peaks occurs, in good agreement with numerical calculations. To determine the phase shift produced by the reflection process itself, the phase difference between the incident and reflected peaks has been measured for various radial distances and is displayed on Figure 5.

This difference varies almost linearly with the radius $r$. There is obviously a phase shift introduced by the reflection; due to inhomogeneity effects and the uncertainty on the plasma layer position, only an upper bound of the order of $\pi$ can be set. Theoretical arguments predict a value of $\pi/2$.

IV - CONCLUSIONS

The amplitude and phase of the potential radiated by a point-source antenna in a magnetized plasma have been measured simultaneously for various plasma conditions. Accurate phase measurements have been performed on the resonance cone diverging from the antenna as well as on the resonance cone reflected at the plasma layer. Phase variations associated to the thermal structure have been measured. The phase shift due to the reflexion process has been observed.

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PRODUCTION OF A HIGH DENSITY PLASMA
BY HELICON WAVES

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Abstract A highly-ionized plasma with central density $\sim 3 \times 10^{13} \text{ cm}^{-3}$ is produced by radio-frequency excitation of helicon waves, and Landau damping is experimentally demonstrated to cause efficient transfer of wave energy to electrons that subsequently suffer inelastic collisions. Nonthermal beamlike electron tails, which are considered to be closely related to the plasma production, are also observed when a small-diameter stainless-steel tube is used as a vacuum chamber.

1. Introduction

Recently, Boswell has presented a new plasma source which uses inductively coupled radio frequency power to generate a high-density plasma at low pressure, and has shown that wave properties are consistent with those expected of helicon waves /1/. In a theoretical analysis, Chen has suggested that the rate of energy absorption by wave damping may be due to Landau damping of the helicon wave which has an electric field component parallel to the magnetic field /2/.

In the present paper detailed measurements are performed of the helicon wave and the efficient plasma production. Spatial variations of damping waves are measured by interferometry to obtain damping rates and wavelengths in the plasma along the magnetic field. The damping rate of the helicon wave is consequently confirmed to have the characteristics of Landau damping although the collisional damping is not negligible in our experiments. The nonthermal beamlike electron tail is also studied to clarify its effect on the plasma production.

2. Experimental apparatus

A schematic of the experimental apparatus is shown in Fig. 1 /3/. The argon plasma produced with a helical antenna is confined in a uniform magnetic field $B_0$ of up to 3 kG. The antenna is located outside the plasma, that is, on the Pyrex tube with an inner radius $a$ of 2.5 cm and a length of 50 cm, which is connected to the end of the stainless-steel vacuum chamber of 120 cm in length and 16 cm in diameter or that of 170 cm in length and 46 cm in diameter. The stainless-steel vacuum chamber is electrically grounded.

The antenna consists of two copper ribbons of 2.5 cm in width, which are wound
around the Pyrex tube and have half of a winding, in order to excite an \( m = 1 \) mode, as shown in Fig. 1(b). The length of the antenna is chosen to be 25 cm. The rf power at 7 MHz, \( P_{rf} \), is supplied from an oscillator-amplifier system and is varied up to 2 kW. To minimize damage to the rf circuit, the rf supply is pulsed at 83.3 Hz with a 16.7% duty cycle.

Measurements of \( n_e \) and \( T_e \) are performed with a Langmuir probe calibrated against a microwave interferometer. The plasma density \( n_e \) is measured at \( t = 1.5 \) msec after the oscillator is turned on at \( t = 0 \) msec with a boxcar integrator with a gate width of 0.1 msec, while \( T_e \) is obtained in the afterglow plasma because \( T_e \) of 4-6 eV at \( t = 1.5 \) msec is considered to be affected by the rf wave. The electron temperature \( T_e \) thus obtained is found to be always 3-4 eV, and to depend little on such parameters as \( B_0 \), \( P_{rf} \) and neutral pressure \( p \). The wavelength of the helicon wave, \( 2\pi/k \), is measured by interferometry with magnetic probes, which are located inside the stainless-steel vacuum chamber, and are movable along the \( z \) axis. The magnetic probes consist of single layer solenoidal coils of 4 mm in diameter and 6 mm in length with 50 windings.

![Fig. 1: Schematic of experimental apparatus.](image1)

![Fig. 2: Dependence of radial \( n_e \) profile on \( B_0 \).](image2)

3. Experimental results

The radial \( n_e \) profile is studied by changing \( B_0 \) at \( P_{rf} = 1.4 \) kW, and the result is shown in Fig. 2. When \( B_0 \) is smaller than \( \sim 0.45 \) kG, the \( n_e \) profile around \( r = 0 \) cm has a concave shape and the distance between two ridges is almost equal to the inner diameter of the Pyrex tube. By increasing \( B_0 \), the \( n_e \) profile is varied drastically.
and becomes peaked at \( r = 0 \) cm. This dependence of the radial \( n_e \) profile on \( B_0 \) was explained by taking account of the confinement of the plasma \( /2, 3/ \). Basically, the heating and ionization will be localized near the walls, and so is the energy deposition. This causes the concave \( n_e \) profile with the ridges at the inner-wall position of the Pyrex tube. By increasing \( B_0 \), the energy confinement, which is determined by the end plate sheaths and the intensity of \( B_0 \), was considered to be better on the axis, leading to the appearance of a dense core.

When the small-diameter stainless-steel tube is used as a vacuum chamber, nonthermal beamlike electron tails \( /4/ \) are observed at the same time that the radial \( n_e \) profile is peaked, as shown in Fig. 3. Figure 4 shows that the beam is restricted radially, is the most dense, and there is no tail for \( r > 1.5 \) cm, indicating that the beam position agrees well with the peak position of the radial \( n_e \) profile. The electron energy distribution function shows that there is a bump in the range of 50-70 eV, where the cross section for the electron-impact ionization is very large. Therefore, in addition to the improved confinement of the plasma, the electron tails are considered to produce the peaked radial \( n_e \) profile, although the mechanism responsible for the nonthermal distribution tail is not yet known. If the large-diameter vacuum chamber is used, the nonthermal beamlike electron tail is not observed at a distance of \( \sim 50 \) cm from the antenna and the half-width of the radial \( n_e \) profile is found to be much larger than that obtained in the small-diameter vacuum chamber. The reason why the electron tail disappears is not clear at this stage, but considered to be that since there is a large amount of neutral argon atoms between the dense core and the chamber wall, compared with those in the small-diameter chamber, the electron tail is exhausted for ionizing neutral atoms which are fed radially.

The plasma density \( n_e \) is found to be proportional to \( p \) in the region of \( p \lesssim 1 \times 10^{-3} \) Torr, and to saturate in the high-\( p \) region when \( B_0 = 1 \) kG and \( P_{rf} = 1.5 \) kW. The
maximum \( n_e \) is \( \sim 3 \times 10^{13} \text{ cm}^{-3} \). Since the density \( n_0 \) of neutral atoms at \( p = 1 \times 10^{-3} \) Torr is \( \sim 2.6 \times 10^{13} \text{ cm}^{-3} \) at the room temperature of 300 K, it is confirmed that neutral atoms are highly or completely ionized in the range of \( p \lesssim 1 \times 10^{-3} \) Torr. In the high-\( p \) region, it is considered that there is not enough rf power for all neutral atoms to be ionized.

The phase and amplitude of the helicon wave as a function of axial position \( z \) are obtained by interferometry with the magnetic probe. The wavelength is not determined by the antenna length, and is varied automatically to provide the best coupling or to satisfy the dispersion relation which is given by \( \omega/k = 3.83B_0/ae_{\nu_0}n_e \). Indeed, there is relatively good agreement between the experimentally and theoretically obtained dispersion relations. An important point is that \( \zeta(= \omega/kv_{th}, \text{where } v_{th} \text{ is the electron thermal velocity}) \) approaches \( \sqrt{2} \) as \( n_e \) is increased with \( B_0 \) fixed, in other words, \( n_e \) has a maximum at \( \zeta \sim \sqrt{2} \) where the Landau damping rate maximizes. The variation of \( B_0 \) makes it possible to change \( n_e \) with keeping \( \zeta \) at \( \sim \sqrt{2} \), as known from the dispersion relation. Thus, the \( n_e \) dependence of damping rate \( \text{Im}(k)/\text{Re}(k) \) is obtained with \( \zeta \sim \sqrt{2} \). There is a tendency for \( \text{Im}(k)/\text{Re}(k) \) to decrease with an increase in \( n_e \). This is characteristic of Landau damping, and can not be explained by the collisional damping. However, Landau damping is smaller than the collisional damping in the range of \( n_e \gtrsim 7.2 \times 10^{12} \text{ cm}^{-3} \), so that Landau damping and the collisional damping occur at the same time in our \( n_e \) range. If the Landau damping rate plus the collisional damping rate is used as a theoretical damping rate, good agreement is found between the experimentally and theoretically obtained damping rates.

4. Conclusions

Experiments reported here demonstrate the excitation of a helicon wave in a cylindrical magnetoplasma and the production of a highly-ionized plasma. It is shown that the density has a maximum at \( \zeta \sim \sqrt{2} \) where the Landau damping rate maximizes, and that both Landau damping and collisional damping occur at the same time in our density range. Nonthermal beamlike electron tails observed in the small-diameter vacuum chamber are considered to play an important role in producing a peaked radial density profile.

References

DEVELOPMENT OF SPIRAL STRUCTURES ON A ROTATING PLASMA COLUMN WITH LARGE ION-GYRORADIUS

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Surface structuring of an expanding plasma in a magnetic field has been vitally investigated in space/1/ and laboratory/2,3/ experiments as well as in analytical/4/ and numerical/5/ calculations, as one of self-organizing processes in continuous media. In the AMPTE magnetotail barium release experiment/1/, the photoionized barium plasma generated a diamagnetic cavity by exclusion of the ambient geomagnetic field and later developed a large-scale coherent structure on the surface. In another laboratory experiment/2/, an aluminum target was irradiated by laser in a strong magnetic field (1.0 Tesla). The expanding plasma gave rise to a robust Rayleigh-Taylor-like instability where the plasma bulk was blocked by the magnetic field but the flute tips continued free-streaming at sub-Alfvénic speeds, often accompanying field-aligned striations. These observations have common features such as: (1) an effective gravity directed outward; (2) unmagnetized plasma ions; (3) the rapid instability growth.

The present paper reports an investigation on the temporal evolution of a Rayleigh-Taylor-like instability which has occurred in a rotating plasma with a large ion gyroradius $r_i\gg a$ and a large rotation frequency $\omega \gg \Omega_i$ during propagation along a uniform magnetic field, where $a$ is the plasma core radius and $\Omega_i$ is the ion gyrofrequency. In particular, the development of a coherent pattern of 'multiple spiral filament' has been observed clearly on the plasma boundary with use of the end-on framing photography and the witness plate diagnostics.

The arrangement of the experiment is shown in Fig. 1. A rotating plasma of Cu/Zn mixture is
ejected from a coaxial plasma gun and propagates in a Pyrex vacuum chamber (10 cm in diam. x 60 cm long) where a uniform magnetic field \( B_z \) of up to 3.2 kG is applied. Vacuum is better than \( 4 \times 10^4 \) Pa before experiment. The plasma is produced from brass (Cu 60% / Zn 40%) of the inner cathode of the gun through intense cross-field vacuum discharge while it is energized by a 40 \( \mu \)F capacitor bank. The current waveform is approximated to a half sine with a peak of 13 kA and a quarter period of 8 \( \mu \)s when a charging voltage \( V_c = 9 \) kV and \( B_z = 2.5 \) kG. Axial and azimuthal velocities, \( v_z \) and \( v_\theta \), result from Lorentz forces \( F_z = j_z B_\theta \) and \( F_\theta = j_\theta B_z \) (order of \( 10^3 \) N/m\(^3\)) due to the radial current density \( j_r \), where \( B_\theta \) is the azimuthal magnetic field produced by \( j_\theta \).

Two diagnostic methods were used to monitor plasma behaviors: A fast framing camera (2 \( \times \) 105 frames/s) investigated the temporal evolution in the plasma profile from both the end and side of the plasma gun. A witness plate (black Polaroid type 47 film) recorded a time-integrated plasma profile when it was exposed to the plasma flow between \( z = 1 \) and \( z = 15 \) cm and the plasma flow was energetic enough to blow off the film coating. Other diagnostics are: a fiber-optic emission spectroscopy coupled to an intensified OMA system to measure the ion species, \( T_i \) and \( \psi_i \); an ion time-of-flight detector to measure \( v_\perp \); electrostatic probes to measure \( \psi \), and the plasma potential.

Typical plasma parameters for \( I = 13 \) kA and \( B_z = 2.5 \) kG are given in Table I. From the spectroscopic analysis, it was found that the plasma consists mainly of Cu and Zn although it contains smaller amounts of H, C and O. Although mass difference between Cu and Zn is negligible (~3 %), they are predicted to have singly and doubly charged ions from \( T_i = 9 \) eV. This means that the plasma includes two groups of ions having a factor of two different \( \rho_i \) and \( \Omega_i \). We suppose that singly-charged ions with greater \( \rho_i / a \) and \( \omega / \Omega \) will be predominant for the instability growth.

A rotation frequency \( \omega \) was evaluated mainly from end-on framing photographs which indicated a constant angular rotation of irregularities on the plasma boundary for each frame time. A radial profile in \( \omega \) was determined from space-resolved, spectroscopic doppler shift of CIII 464.7 nm and confirmed the rigid rotation of the plasma core. It is noted from Table I that the plasma has sub-Alfvenic and supersonic speeds; \( v_y(\rho = 2 \text{ cm}) = v_z = 2 \times 10^4 \text{ m/s} = v_A = 0.7 c \) and \( \omega / \Omega_i > 1 \). We assume that the convective and radial pressure gradient are sub-exponential and sub-Alfvenic, supersonic, and that the plasma is relativistic. Since the conventional MHD model is restricted to the low frequency plasma response assuming \( \omega / \Omega_i < 1 \) and \( \rho_i / a < 1 \), it can not be applied to strongly nonlinear, nonequilibrium plasmas here.

<table>
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<td>\begin{tabular}{c} \text{Plasma core radius ( a )} \end{tabular}</td>
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</tbody>
</table>
Figure 2 shows $\omega$ as a function of a variable $B_z/I$. The frequency $\omega$ increases linearly with $B_z/I$ at large $B_z/I$. This relation has been explained from a simplified model/6/. The reason for $\omega=0$ at $B_z/I=10$ may be that the plasma diamagnetism cancels $B_z$ and as a result, the driving force $F_B=|B_z|$. Results of witness plate measurements are shown in Fig. 3. Each corresponds to a single exposure at $I=18\,kA$ and $B_z=2.5kG$. Immediately after the ejection from the gun barrel, plasma particles are transported outward due to the strong centrifugal force; the outer boundary expands to a radius of $\approx 2\,cm$ from its initial value of $1.15\,cm$ and at the same time, a cavity is formed inside; the expansion appears to reach equilibrium at a downstream distance of $z=4-6\,cm$; the plasma finally has an annular profile with an outer diameter of $\approx 4\,cm$ and an inner diameter of $\approx 2.5\,cm$ (Fig. 4). We also recognize that high-order density modulations (like spikes or bubbles) appear on the annular column at $z=5-6\,cm$ and often lead to the elliptical deformation of the whole column further downstream.

Figure 5 shows end-on framing photographs taken for (a) $B_z/I=17\,(13\,kA/1.5\,kG)$ and (b) $B_z/I=43\,(11\,kA/3.2\,kG)$, where $\omega$ is (a)$0.6\times10^6\,rad/s$ and (b)$2.2\times10^6\,rad/s$ and the plasma density is of the order...
Fig 5: End-on framing photographs taken for (a) $B_z/I=17(13 \text{ kA}/2.5 \text{ kG})$ and (b) $B_z/I=43(11 \text{ kA}/3.2 \text{ kG})$. The number shows the sequence of frames.

of $10^{15} \text{ cm}^{-3}$. From the scaling relation $n_\pi \propto I^{2/3}$, the plasma density for (b) is estimated to be 70% of (a). At the beginning, 8-10 pieces of thin filaments appear to grow radially in a time quite shorter than the rotation period (Fig. 5(a)). The number of filament decreases with time through the nonlinear interaction which includes coalescence or merging of filaments, to final 2-3 pieces of broader spirals. This scenario is more evident in Fig. 5(b) where $\omega$ is increased by a factor of 3.7 and the centrifugal force by a factor of 13; strong spiral filaments are clearly observed; the interaction among filaments appears to be rather turbulent; in the later stage, a low density halo is formed around the plasma core since plasma particles are transported outward through filaments; A sheared rotation should occur between the halo and plasma core; this drives a secondary instability (probably K-H instability) generating fairly symmetrical, 2 or 3 pieces of spiral arms. This transition from short to long wavelengths of instability waves has been observed also in the laser plasma experiment/3/ and in the nonlinear plasma simulation/5/. It may be regarded as the inverse cascade process in the nonlinear evolution of instability.

In summary, the growth of the Rayleigh-Taylor like instability and the subsequent structuring have been investigated for rotating plasmas with large ion-gyroradius with the framing photography. The development of spiral filaments and their shift to longer wavelengths (lower modes) through the mode coupling have been identified as typical nonlinear phenomena. A future investigation with controlled $\omega$ and $n_i$ will be essential to make detailed comparisons with theories and to understand the physical picture of instability.

WAVES DRIVEN BY STRONG TRANSVERSE POTENTIAL STRUCTURES*

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The mode characteristics of waves being driven by magnetic-field-aligned current and waves driven by strong transverse potential structures are compared. Experiments are performed in a Q-machine plasma column (3-cm radius and 90-cm length) with the following parameters: \( n = 10^8 \, \text{cm}^{-3}, B = 1.5 \, \text{kG}, m_i/m_e = 7.15 \times 10^4, T_i = T_e = 0.2 \, \text{eV} \). Experiments are aimed at producing strong shear in azimuthal electron velocity inside a narrow (diameter = 10 ion gyroradii) channel of axial electron current. The presence of a velocity shear layer is predicted to affect the electrostatic ion-cyclotron waves that can be excited by current drawn to the biased electrode. In this paper, measured mode characteristics of the ion-cyclotron fluctuations as modified by adjustable velocity shear are presented.

Ganguli, Lee, and Palmadesso developed a kinetic theory to investigate the electrostatic oscillations in a magnetized plasma containing a transverse, d.c. electric field. In the fluid limit and for perpendicular propagation, they recover the well known Kelvin-Helmholtz instability which explicitly depends on the gradient of the shear, i.e., the second derivative of the transverse flow. It is characterized by long wavelength \( (k_L L < 1; L \) is the characteristic length associated with the velocity shear) and low frequency \( (f << f_i) \). In the short-wavelength limit, the general nonlocal dispersion condition yields a new branch of oscillation which is sustained by the inhomogeneity in the energy density of the waves. The inhomogeneous energy-density driven (IEDD) instability is generally a short-wavelength \( (k_L L > 1) \) fluctuation with a broadband frequency spectrum. Depending on ambient parameters, the frequency of this instability may fall around the ion gyrofrequency and can be mistaken for the current-driven electrostatic ion cyclotron (CDEIC) instability, especially when magnetic-field-aligned current is also present.

Unlike the CDEIC instability, which grows by inverse Landau damping onto the electrons that are resonant with the longitudinal component of the wave phase velocity, the IEDD instability requires only that the Doppler shift in frequency due to the \( E \times B/B^2 \) rotation velocity \( v_E \) makes \( \omega - k v_E < 0 \), a nonresonant effect. The nonlinear spectra of the two instabilities are very distinct as demonstrated by Nishikawa et al. While the spectrum of the CDEIC instability is coherent around a single frequency, the spectrum of the IEDD instability is spiky and broadband. In the general case where a combination of both transverse, d.c. electric field and magnetic-field-aligned current are present, the growth rate is larger than a linear combination of the two sources of free energy would indicate. The interaction of this new branch of oscillation with magnetic-field-aligned current is being used to explain the short-wavelength broadband turbulence observed in the ionosphere, magnetosphere, and the jump-region of the bow shock, and it is upon this interaction that the investigation reported here is focused.

The radial structure of the electric potential in the plasma is controlled using a disk electrode placed on the cylindrical axis of the plasma, 60 cm from the plasma-source end. The disk is made of two coplanar, concentric circular
segments that are heated to prevent surface contamination. The applied voltages on the outer annular segment and on the inner button segment are independently controlled with precision power supplies. The potential structure in the plasma is inferred from the voltage of an electrically floating Langmuir probe. Fig. 1 shows two examples from a specific sequence of radial potential profiles. The reference case of this sequence for which all end electrodes are electrically floating is included.

The features of the profiles that are relevant to plasma stability are located on magnetic field lines that map to the 2-cm-diameter segmented disk electrode. A positive potential within this region, with respect to the reference profile, induces a magnetic-field-aligned electron current. Between regions of different potential, an electric field transverse to the magnetic field exists. Potential structures representing the cases of (a) large field-aligned current and (b) large transverse, localized electric field are shown in Fig. 1. This categorization identifies one of these two aspects as that having the predominant effect on plasma stability.

For the case of large magnetic-field-aligned current (FAC), waves are observed within the region $r \leq 1$ cm (the current channel region) that are identified with the current-driven electrostatic ion cyclotron (CDEIC) instability. The mode frequency of these waves is slightly larger than the ion cyclotron frequency for potassium, $f = 1.1 f_{ci}$. The spectral feature associated with the mode is narrow (2-3 kHz) as shown by the thick trace at the top in Fig. 2a.

![Fig. 1: Measured floating potential across the diameter of plasma column, 2 cm in front of segmented disk electrode. Profiles are displaced vertically for clarity. Annulus and button voltages are a) 5.5 V and 5.5 V, b) 10.5 V and 0.5 V, c) floating and floating, respectively. Hashmarks on vertical axis are 0.5 V increments.](image-url)
For the case of large transverse, localized electric field (TLE), waves are observed that are distinctly different from the FAC-driven waves. Although the TLE-driven mode frequency is in the neighborhood of the FAC-driven mode frequency, the spectral feature associated with the TLE-driven mode is broader (~10 kHz) and spikier, as shown by the thick trace near the bottom in Fig. 2a.

Fig. 2: Sequence of spectra from fluctuations in current collected by annulus segment of disk electrode. Spectra are displaced vertically for clarity. Thick-line traces are referred to in the text. Annulus voltage is 9.5 V and \((V_a - V_b)\) is identified by a number next to each trace. a) Spectra are normalized to the same maximum amplitude, b) spectra are left unnormalized.
Two additional features that distinguish the two modes can be recognized in the normalized spectra in Fig. 2a (normalizing emphasizes mode frequency) and in the unnormalized spectra in Fig. 2b (not normalizing emphasizes relative mode amplitude). First, a shift in TLE-driven mode frequency is evident in traces (4) through (10) in Fig. 2a, for which the applied voltage difference between the two circular segments of the disk electrode changes in one-volt increments from 4 V to 10 V. Second, a sensitive dependence of the TLE-driven mode amplitude on the transverse electric field strength is apparent in traces (4) through (10) in Fig. 2b. The TLE-driven mode power can be significantly larger than the FAC-driven mode power. Since the channel-averaged field-aligned current decreases as the bias on either segment is decreased, Fig. 2 implies that this mode is observed for cases in which the field-aligned current is subcritical for exciting CDEIC waves. This is consistent with a report /6/ of a reduced critical electron drift velocity in the presence of this transverse, localized, d.c. electric field.

One further distinction between the two modes is that the FAC-driven waves are detected from the fluctuations in the current collected by both the annulus and button segments, however the TLE-driven waves are detected only on the annulus. This difference suggests that the TLE-driven waves are present at radii where the transverse electric field is large and are absent at radii where the transverse electric field is small.

The observed dependence of the TLE-driven mode characteristics (spectral shape, mode frequency, mode amplitude, and radial mode structure) on the transverse localized electric field conclusively eliminates the current-driven electrostatic ion cyclotron instability as the mechanism responsible for this mode. On the basis of the results presented here the inhomogeneous energy-density driven instability /4/ is believed to be responsible. Van Niekerk et al. /7/ recently reported on azimuthally propagating TLE-driven waves, which they attribute to the same instability, with a frequency 15% higher than CDEIC waves. A complete characterization of this TLE-driven mode is presently underway.

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A direct analogy exists between the dynamics of a 2-D inviscid, incompressible fluid, and a magnetized pure electron plasma, with the fluid vorticity corresponding to the plasma density. Thus the U.C. Berkeley electron trap is a powerful tool for controlling and measuring the "vorticity" of an evolving fluid system. Here we study the evolution of a hollow ring of vorticity. Through the Kelvin-Helmholtz instability of fluid mechanics, and the equivalent diocotron instability of electron beam theory, a symmetric ring develops into individual vortices. A sufficiently asymmetric ring, however, undergoes a "winding" instability, resulting in a spiral shape. We show various images detailing the stages of the dynamics for various initial ring parameters.

Figure 1 contains a simplified side view of our experimental geometry. A magnetic field along the axis of the trap insures radial confinement of the plasma. Axial confinement is obtained by applying negative voltages to the two end cylinders. $E \times B$ drifts due to the applied magnetic field and the plasma's own electric field govern the plasma dynamics. These drifts are slow compared to the end to end electron bounce motion; therefore the behavior is largely two-dimensional.

Grounding the right end cylinder forces the plasma to stream out along the magnetic field lines, producing a visible image on the phosphor screen. Imaging destroys the plasma; however, time sequenced images can be produced because the plasmas are very reproducible. To obtain the asymmetric annular plasmas used in this experiment, we first form a quiescent electron plasma (length=4 cm, temperature=2.0 eV, density=$1.0 \times 10^8$ cm$^{-3}$) and then move it a known distance off center using feedback from a wall probe. Finally, we eject the central plasma core by briefly reducing the voltage on one end cylinder.

Figure 2 shows the time history of a typical symmetric hollow plasma. First, well developed vortices form and then the vortices merge until only one central vortex remains. The initial number of vortices (the mode number) is inversely proportional to the ratio of the thickness of the ring to the ring radius. In Fig 3 we show the results for a large number of ring thicknesses. In Fig. 4 we plot the calculated and measured mode number vs. this ratio. The results are in good agreement with theory for small mode numbers but are not in agreement for large mode numbers. We have also measured the instability growth rate and find that it is also far from the theoretically predicted rate for thin rings. We suspect that these discrepancies are due to finite length effects.

For some asymmetric rings the dynamics are entirely different. As shown in Fig. 5, asymmetric rings develop into spirals. In this case the dynamics
can be divided into an "active" and a "passive" phase. In the active phase, superposition leads to a model of a section of the asymmetric ring as two vortices of opposite sign. Such a vortex pair will travel in the direction perpendicular to the line connecting the pair centers—in this case towards the device center. After the density clump gets to the center, the interaction is largely passive, with the remaining plasma spiraling around the center because of the sheared velocity fields. The number of spiral turns increases linearly with time. As shown in Fig. 6, the plasma at radius r advances in angle roughly as \( \frac{d\theta}{dt} \propto \frac{t}{r} \). This regular dependence implies that a mean field approximation can be used to estimate the velocity shear. The winding process ends when the plasma annular Kelvin-Helmholtz instabilities are suppressed by adverse shear.

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![Fig. 1 Experimental schematic.](image-url)
Fig. 2 Hollow plasma time sequence.

Fig. 3 Instability for varying initial ring thicknesses.

Fig. 4 Measured (dots) and calculated mode numbers.
Fig. 5 Asymmetric dynamics.

Fig. 6
Dynamics of Non-Stationary Dipole Vortices in the Hasegawa-Mima Equation.

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Abstract: The dynamics of tilted dipole vortices in the Hasegawa-Mima equation is studied. A recent theory is compared with numerical simulations and found to describe the short time behaviour of dipole vortices well. In the long time limit the dipoles are found to either disintegrate or relax toward a steady dipole vortex counter propagating relative to the linear drift waves. This relaxation is a consequence of nonviscous enstrophy loss by the dipole vortex.

The Hasegawa-Mima equation can be derived for nonlinear drift waves in a low-$\beta$ plasma with a density gradient. In non-dimensional variables the equation is

$$\frac{D}{Dt} \Gamma = 0 , \quad \Gamma = \nabla^2 \phi - \phi + \beta y ,$$

where $\Gamma$ is the potential vorticity which is conserved for each fluid element, $\phi$ represents the electrostatic fluctuations, $[\cdot]$ the Jacobian and $\beta$ is proportional to the density gradient directed along $-y$. The maximum phase velocity of the linear drift waves, propagating in the electron diamagnetic drift direction, is $-\beta$. It is well known that Eq.(1) conserves mass, energy and enstrophy.

Eq.(1) has a dipole vortex solution /1/, which is exact and stationary if co- or counter-propagating relative to the linear waves. It is known that if a counter-propagating dipole vortex is initially rotated slightly, it will execute regular oscillations around the steady state magnetic flux surface /2/, i.e. the positive $x$-axis (cf. Fig.2). This behaviour may be understood as a consequence of the Lagrangian conservation of potential vorticity. Recently Nycander and Isichenko /3/ derived an approximate equation of motion for the velocity $\vec{U}$ of dipoles propagating in arbitrary directions

$$\frac{d\vec{U}}{dt} = \vec{U} \times \frac{\epsilon y \beta U S}{P_d} , \quad \vec{P}_d = \int (\phi - \nabla^2 \phi) r dx dy$$

where $S = \pi a^2$ is the area of the dipole inside the separatrix, $y$ is the $y$-coordinate of the centre of the dipole vortex and $P_d = |\vec{P}_d|$ is the magnitude of the dipole moment, with $\vec{r} = 0$ in the centre of the dipole vortex. Equation (2) is obtained by noting that the radius of curvature of the trajectory is equal to the ratio between the dipole moment and the monopole moment of the dipole vortex, and also that the monopole moment varies linearly with the $y$-coordinate of the vortex. The latter is a consequence of the conservation of potential vorticity inside the separatrix. It is assumed that the internal structure of the dipole remains unchanged, implying that $S$ and $P_d$ are constant, and that the radius of curvature of the trajectory is much larger than the diameter of the dipole. This is justified if $\beta/U \ll 1$. The theory doesn't take into account loss of enstrophy.

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Equation (2) may be integrated, and the oscillation frequency, \( \omega \), obtained

\[
\omega = \frac{U}{2} \left( \frac{\beta S}{P_d} \right)^{1/2} \frac{1}{K(k)},
\]

where \( K \) is a complete elliptic integral of the first kind with the argument \( k = |\sin(\alpha/2)| \), \( \alpha \) is the maximum angle between the trajectory and the \( x \)-axis. \( \alpha = 0 \) for an exact dipole vortex counter-propagating relative to the linear waves.

The maximum displacement, \( y_t \), along the density gradient is found to be

\[
y_t = \pm \left( \frac{P_d}{\beta S} \right)^{1/2} 2\sin(\alpha/2),
\]

and the displacement, \( L_w \), along a magnetic flux surface during one full oscillation, here referred to as the oscillation length, is

\[
L_w = 4 \left( \frac{P_d}{\beta S} \right)^{1/2} \left[ 2E(k, \pi/2) - K(k) \right],
\]

where \( E \) is an elliptic integral of the second kind.

---

**Fig.1** a) Oscillation frequency. b) Displacement along the density gradient. c) Displacement along magnetic flux surface. d) Blow up of 1a.
These expressions have been evaluated by numerical simulations. The numerical scheme is based on a pseudospectral approximation in a periodic domain and a 3rd order Adam-Bashforth predictor-corrector as time integration method. Zero-padding is used as de-aliasing. All simulations are done with a resolution of $128^2$ modes. As initial conditions we used the exact solution $f_{1/2}$ tilted at an angle $\alpha$ with propagation velocity $U = 0.1$ and size of separatrix $a = 1.0$. Unless otherwise stated $\beta = 0.05$. A special ‘cutting’-technique has been applied to avoid effects caused by the periodicity of the domain $/4/$. 

To test Eqs.(3)-(5), the dipole behaviour was simulated for different values of $\beta$ and $\alpha$. The three characteristic parameters $\omega$, $y_l$, $L_w$ were determined at the first maximum of the oscillation, and the estimated parameters were then found by extrapolation. The reason for using only a quarter of a period is that the amplitude of the oscillations changes significantly already during one or a few periods because of processes not included in the theory.

In Fig. 1a, the simulated oscillation frequency is compared with Eq.(3). We observe good agreement between theory and simulations for $\beta/U \leq 1$ except for $\alpha \leq 30^\circ$. The maximum displacement along the density gradient is plotted and compared with the value from Eq.(4) in Fig.1b, and comparison between simulations and the oscillation length from Eq.(5) is shown in Fig. 1c. In the limit $\beta/U \ll 1$, the figures show good correspondence between simulations and theory even for a very large initial angle. However, there is some unexpected deviation for small angles. In order to investigate this, a detailed study of this region has been performed. The result is shown in Fig. 1d. One may speculate that the deviation is caused by some internal transient process. It has been observed, that the internal reorganizing process has a stronger effect when $\alpha$ is small, causing the functional dependency inside the separatrix to become weakly non-linear after a quarter of an oscillation. This idea is supported by the fact that calculating the oscillation frequency of the dipole after several oscillations yields increased agreement with the theory as marked in Fig.1d with ‘x’.

In the region $\beta/U > 1$, the qualitative dependence of the oscillation frequency, $\omega$, as a function of the angle is correct, but the exact value is seen to be too low. This may be explained by energy loss causing the dipole vortex to decrease its propagation velocity. The good agreement for $y_l$ and $L_w$ supports this assumption.

Some of the dipoles disintegrate shortly after a quarter of a period, as marked by large circles in Fig.1. The large deviation for $\beta/U > 1$ and large initial propagation angle are most likely connected to the processes causing the disintegration.

As has been shown, the above theory seems to agree well within the given limits. However, the nonviscous loss of energy and enstrophy by the vortex to the surrounding plasma have a strong impact on the long time behaviour of the dipole, implying that the theory is only valid for the first few oscillations. After these the dipole vortex has changed significantly, and the theory fails to describe the trajectory of the oscillating dipole based on the initial conditions.

Simulations have been done with parameter scans in $\beta$ and $\alpha$ in order to investigate the long time evolution of counter-propagating dipoles launched at the angle $\alpha$. Figure 2a shows trajectories for dipoles at different initial angles. The most interesting feature is that the trajectory of the dipole after a sufficiently long time seems independent of the initial conditions, except for a phase shift.

For a large initial angle the trajectory is approximately sinusoidal, as one might expect. However, for a smaller launch angles the initial part of the trajectory is rather irregular, indicating that the dipole in this phase is affected by the internal reorganization process.

Simulations of dipoles with a very large initial launch angle are also shown in Fig. 2b. We observe that if the angle is too large, the dipole soon disintegrates. Thus there seems to be only two possible outcomes of a dipole vortex initially propagating at some angle. Either the dipole ends up as a steady counter-propagating dipole or it disintegrates.
The relaxation of the trajectories seen in Fig. 2a can be explained as follows. If the vortex loses enstrophy, the amplitude $\phi - \nabla^2 \phi$ must decrease in magnitude. For $\alpha$ being positive this implies, due to conservation of potential vorticity, that the cyclonic part must move a little down the gradient and the anticyclonic part a little up the gradient. This induces a clockwise torque around the centre of the dipole vortex and as a result we observe that the propagation angle decreases. Similar arguments may be adopted to explain why the co-propagating dipoles swing around and finally become a counter-propagating dipole vortex. The enstrophy loss also causes the two dipole halves to be pulled slightly apart implying a small decrease in the propagation velocity. This corresponds well with the speculations used in Fig. 1 to explain the deviation between the Nycander-Isichenko theory and the simulations for $\beta/U \gg 1$.

![Fig. 2](image)

**Fig. 2** a) Long time behaviour for small launch angles.

b) Long time behaviour for large launch angles.

Concluding Remarks

It has been shown that the theory of Nycander and Isichenko /3/ describes the short time behaviour of the oscillating dipole vortex quite well. However, since the dynamics of the non-stationary dipole is strongly altered by loss of energy and enstrophy, the theory fails to describe the long time dynamics of the dipole vortex. It has also been shown, that in part of the parameter space the short time vortex behaviour is strongly affected by initial transient effects, caused by an internal reorganizing process.

In the long time limit dipoles launched at some finite angle are found to either end up as steady dipoles counter-propagating the linear drift waves or disintegrate shortly after the initial period. Thus, the steady counter-propagating dipole vortex may be considered as an attracting solution. Our explanation of this is that the counter-propagation state is the minimum enstrophy state of the dipole vortex. The long time behaviour can be interpreted as a relaxation toward this state, caused by enstrophy loss. However, no definite quantitative confirmation of this has been obtained.

References

Localized vortices in $\eta_i$ - modes

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For a wide variety of nonlinear wave equations necessary conditions for the existence of localized, stationary structures can be found by applying a simple procedure, involving two steps /1/: First the linear dispersion relation is obtained and the regions of the phase velocity of linear waves found. Secondly, assuming that localized solutions exist, their velocities are determined by using integral relations. The obtained velocity takes the form of a "center of mass velocity". If this velocity falls outside the regions of phase velocities for linear waves then nonlinear localized vortices may exist. Otherwise, the structure will couple to the linear waves and gradually disperse. Applying this method we have shown that monopole vortex solutions exist for drift waves driven by the ion temperature gradient in a magnetized plasma, the so-called $\eta_i$-modes. Numerical solutions show that such vortices are steadily propagating and stable and they generally emerge from localized initial conditions. Our study is motivated by recent high resolution simulations of $\eta_i$-turbulence /2,3/, where it was observed that coherent vortices developed spontaneously. These had a dominating influence on the evolution of the turbulence, and the associated anomalous transport was found to be significantly reduced as compared with the predictions from quasilinear theory /2/.

The analysis is based on a simplified two-dimensional model of $\eta_i$-modes /3,4/:

$$\partial_t(1-\nabla^2)\phi - [\phi, \nabla^2 \phi] + (1-2\varepsilon_n + \alpha \nabla^2) \partial_y \phi - 2\varepsilon_n \partial_y p = 0$$

$$\partial_t p + [\phi, p] + \alpha \partial_y \phi = 0,$$

where $\varepsilon_n = L_n/R$, $\alpha = (1+\eta_i)/\tau$, $\eta_i = L_n/L_T$, $\tau = T_e/T_i$, $L_n^{-1} = \partial_x n_0/n_0$, $n_0$ is the background density, $L_T^{-1} = \partial_x T_e/T_e$, $T_e$ and $T_i$ are electron and ion temperatures, respectively, while $R$ accounts for the curvature of the magnetic field. The time is normalized with $c_s/L_n$ and the spatial coordinates are normalized with $\rho_s = (c_s/L_n)$, where $c_s = (T_e/M_i)^{1/2}$, and $\Omega_i$ is the ion cyclotron frequency. The electrostatic potential, $\phi$, and the ion pressure perturbation, $p$, are normalized with $(T_e/\rho_s)(n_0/L_n)$ and $p_{0i}(\rho_s/L_n)$, respectively. Here $p_{0i}$ is the zero-order electron pressure. Finally, $[f, g] = f\partial_y g - f\partial_y g$ denotes the Poisson bracket. In Eqs. 1-2 we have omitted the effects of magnetic shear and thus the parallel dynamics. Curvature effects are retained and the instability is reproduced by the equations for a certain critical value of $\eta_i$.

The regions of linear wave propagation are found by deriving the dispersion relation in the local approximation. Linearizing and Fourier transforming Eqs. 1-2 we obtain

$$\omega^2(1+k^2) - \omega k_y (1 - 2\varepsilon_n - \alpha k^2 + 2k^2 \alpha \varepsilon_n) = 0.$$  

(3)

By examining Eq. 3 we find that the condition for instability, i.e. $\text{Im} \omega > 0$, is $\alpha > 0$. To obtain the localization condition we solve the dispersion relation for $k^2$ in terms of $u \equiv \omega/k_y$:

$$k^2 = \frac{u^2 - (1 - 2\varepsilon_n)u + 2\varepsilon_n \alpha}{u(u + \alpha)}.$$  

(4)

The condition for the existence of propagating linear waves is $k^2 > 0$, provided the phase velocity $u$ is real. For $\alpha < 0$ linear wave propagation is possible in a restricted regime of

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u-values /4,5/, and localized vortex solutions may be possible both for positive and negative u-values. It has been shown that dipole vortices exist in these regions /4,6/. To see if monopole vortices are also possible we derive the center of mass velocity $V$ for any localized solution. This is done by multiplying Eq. 1 with $y$ and integrating over the $(z, y)$-space:

$$V = \frac{d}{dt} \langle y \phi \rangle = 1 - 2\varepsilon_n - 2\varepsilon_n \langle y \rangle$$

(5)

where we have introduced the notation $\langle f \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f \, dx \, dy$. Thus the velocity of localized coherent structures depends on the ratio $\langle p \rangle/\langle \phi \rangle$. It should be stressed that the local ratio between $p$ and $\phi$ can only be considered a free parameter inside the separatrix, separating trapped fluid from free fluid. Outside the separatrix this ratio will be determined by the incoming fluid and the upstream boundary condition at infinity, which gives $p/\phi = a/V$. Clearly $V$ can take any value by properly choosing $p$ and $\phi$, thus localized solutions exist.

Explicit monopole solutions can be constructed /5/ by a perturbation analysis /1/. We have investigated the stability of the monopole solutions numerically. The set of equations (1-2) is solved in a two dimensional domain with periodic boundary conditions. We have employed a fully de-aliased spectral method /7/. To facilitate the numerical solutions we equipped Eqs. 1-2 with dissipative terms: $-D_1 \nabla^4 \phi$ accounting for cross-field dissipation in Eq. 1 and $D_2 \nabla^2 p$ accounting for thermal conduction in Eq. 2. The initial value for $\phi$ is obtained by solving the modified Poisson equation $(1 - \nabla^2) \phi = S$, where the source function is a Gaussian $S = S_0 \exp[-(x^2 + y^2)/\sigma^2]$. This ensures that the zeroth-order solution is stable in the isotropic case. The initial value for $p$ is equal to $Q_0 \phi$. We have performed runs with $\varepsilon_n = 0.25$ and $\sigma_0 = 1$ and various values of $a$ and the initial amplitude ratio $Q_0$.

Figure 1 shows a typical example of a steadily propagating, stable monopole vortex for the case where the velocity is outside the region of linear waves and $a < 0$. After an initial transient period with internal rearrangements to set up the proper solution, the vortex is seen to propagate with a constant speed in the $y$-direction somewhat below $V_0$, where $V_0 \equiv 1 - 2\varepsilon_n - 2\varepsilon_n Q_0$. (See also Fig. 2). The vortex is very slowly decaying due to the dissipative terms introduced. In all cases we found that the speed of the vortex is below $|V_0|$. (As was discussed in connection with Eq. 5 $Q_0$ can only determine the ratio $p/\phi$ inside the separatrix, while this ratio outside the separatrix will be determined self-consistently by the outer fluid. Thus, $V_0$ cannot be expected to give the final velocity exactly. The discrepancy is inversely proportional to the amplitude of the vortex). From Fig. 2 it is observed that although the vortex propagates mainly in the $y$-direction, i.e. perpendicularly to the gradients there is also a weak propagation along the gradient. In the present case the propagation occurs up the density-gradient, but due to the symmetry of Eqs. 1-2 vortices with opposite signs of $p$ and $\phi$ will propagate down the gradient. It is thus evident that the vortices will transport trapped fluid much more effectively perpendicularly to the gradients (along the magnetic flux surfaces) than along the gradients (across the magnetic flux surfaces), and the presence of long-lived vortices in the turbulence may actually reduce the cross field transport. Results like those shown in Figs. 1 indicate that the monopole vortex is an attractor, at least for circularly symmetric initial conditions. Evolutions similar to Fig. 1 were also observed for moderately positive $\varepsilon$-values when $|V_0|$ was sufficiently large. In the strongly unstable regimes the flow was dominated by the linear instability of the wake field.

For values of $V_0$ resulting in velocities inside the region of linear wave propagation a propagating monopole vortex was still observed to form from the initial conditions. This structure was found to radiate linear waves and gradually disperse as shown in Fig. 3. Nevertheless it was sufficiently long-lived to propagate (with a velocity below $V_0$) several
Figure 1: The evolution of a localized monopole vortex. Both the spatial structure of \( p \) and \( \phi \) are shown. Positive values are shown with full lines, negative ones with dashed lines. Parameters are \( Q_0 = -1.67, \alpha = -0.2, \varepsilon_n = 0.25, D_1 = 0.01, D_2 = 0.02 \) and \( V_0 = 1.3 \). The calculation was performed spectrally with a resolution of 128 x 128.

Figure 2: The \( x \)- and \( y \)-position of the maximum of \( p \) (full line) and the minimum of \( \phi \) (dashed line).
times its own width along the $y$-direction before it completely dispersed. Also for this case we observed a slight propagation in the $x$-direction.

The evolution of the single vortices as observed in the numerical studies is seen to be well described by the phenomenological theory outlined in the introduction /1/. We have also investigated the interaction among two vortices /5/. Due to the large freedom in choosing the velocity of the vortices an interaction may be realized in a number of different ways. For the head-on collision we found that after a strong interaction two vortices propagating away from each other emerge. In addition some fragments appear that may finally form a new small vortex. This interaction is markedly different from what is observed for the vortex interaction in one field models as for instance the Hasegawa–Mima model for drift waves /8/. There vortices of different sign will pair to form a dipole vortex and like-signed vortices will coalesce to form a larger vortex /8/.

In conclusion, we have revealed the existence of monopolar vortical structures in $\eta_\pm$-modes. In the stable regime these vortices are steadily propagating and stable and may even be attractors for a class of initial conditions. Thus we imagine that they will appear in the saturated stage of the $\eta_\pm$-mode driven turbulence. They may then have strong influence on the dynamics of the turbulence and on the associated transport in particular, as indicated in numerical simulations /2/.

References

BREAKING OF WAVES INTO WAVE PACKETS OBSERVED IN A BEAM-PLASMA SYSTEM

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Breaking of slow ion-beam modes into wave packets in an ion-beam-plasma system is experimentally studied. When pump waves of the mode with a frequency around 200 kHz have amplitudes larger than a critical one, the wave breaking into many packets is observed to start at an axial position ($x \sim 9$ cm) and the packet amplitudes to rapidly grow with increasing $x$. Here, resultant wave packets keep their forms for a long enough time and are reproducible. Further, the damping of packets or the creation of new packets during interaction between them is observable.

INTRODUCTION

Breaking of waves into many packets in plasmas may be caused by various kinds of causes. In particular, the modulational instability\(^1\) is one of the most important causes among them and attracts interest of many workers in the field of plasma physics. Ikezi et al.\(^2\), studying the possibility of the instability in ion acoustic waves, showed experimentally that the ion acoustic waves could not be modationally unstable. However, Honzawa et al.\(^3\) found experimentally that the \textit{fast} ion-beam modes in a beam-plasma system were self-modulated above a threshold and this phenomenon could be explained by the modulational instability. But, in the above experiment\(^3\) it was very difficult to have a useful information on the evolving process of individual wave packets or the interaction between packets. This is because the waveforms of resultant wave packets were always restless and irregular.
In this paper, we report experimental results on the breaking of the slow ion-beam modes into many packets. Here, the waveforms of packets are considerably settled and reproducible. Thus, the results obtained are expected to give a useful information on the interaction between packets.

EXPERIMENTAL METHODS AND RESULTS

Experiments were carried out using a double plasma (DP) device. In the device two argon plasmas were independently produced by dc discharges. Applying a dc voltage $V_B$ to the 'driver' plasma chamber with respect to the grounded 'target' plasma chamber, we could inject a steady-state ion-beam into the target plasma and formed an ion-beam-plasma system there. In this case, the average energy of an injected ion-beam was usually in a range of 10-17 eV. Thus, the beam velocity $v_b$ adopted here was estimated to be several times larger than the ion acoustic one $C_s$, so that the system used was regarded to be linearly stable.

In order to excite waves in such a beam-plasma system, an rf voltage $V_{rf}$ with a frequency around 200 kHz was superimposed to the driver plasma chamber and thereby the intensity of the injected ion-beam was modulated. As a result, we could excite waves in the beam-plasma system. In such a case, generally we can have some different kinds of modes in the system. However, under our present experimental condition only the slow ion-beam mode was observed at large axial distances ($x > 4$ cm). This fact was confirmed from the observed dispersion relation of excited waves and the dependence of their velocities on the beam velocity $v_b$, the latter of which is shown in Fig.1.

When the rf voltage $V_{rf}$ was increased, we could observe first the rapid increase in amplitude of the excited wave and then the occurrence of the wave breaking into packets above a critical $V_{rf}$. Typical examples of the
wave signals, observed at a fixed position \((x = 10.8 \text{ cm})\) and at various values of \(V_{\text{rf}}\), are shown in Fig. 2. In each pair of traces in this figure, the upper trace corresponds to the applied rf signal, which is externally amplitude-modulated at a low frequency of about 700 Hz, and the lower to the observed wave signal. We can see from these that one or more humps are formed on the wave envelope at higher \(V_{\text{rf}}\) than a critical one and the number of humps increases with increasing \(V_{\text{rf}}\). The result reveals that there is a threshold in \(V_{\text{rf}}\) or wave amplitude for the wave breaking to occur.

Further, the evolution of the wave envelope during propagation was also studied. Typical evolving forms of the envelopes are shown in Fig. 3. Here, the frequency of the carrier wave, which is smoothed out and is not visible in Fig. 3, is about 210 kHz. In this figure the top trace corresponds to the applied rf signal and the lower traces to the evolving wave envelopes observed at various \(x\). These indicate that the observed carrier wave rapidly grows in amplitude at an early stage \((x < 9 \text{ cm})\) and then breaks into many wave packets at larger \(x\). Moreover, we can see the evolving features of individual wave packets from such pictures. Based on similar data to Fig. 3, the changing wave amplitudes are plotted in Fig. 4 as a function of \(x\) for three different values of \(V_B\), which correspond to three different \(V_B\).

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1. For example, V.I. Karpman, Non-Linear Waves in Dispersive Media (Pergamon Press, Oxford, 1975) p106, etc.
[Fig. 1] WAVE VELOCITY $U_w (x10^5$ cm/s) vs. ION BEAM VELOCITY $V_b (x10^5$ cm/s)

[Fig. 2] Applied RF signal

[Fig. 3] RF signal observed at $X$ (cm)

[Fig. 4] Wave amplitude vs. DISTANCE $X$ (cm)
EXPERIMENTAL MEASUREMENT OF THE DEFLECTION OF AN ELECTRON Beam DUE TO THE PONDEROMOTIVE FORCE OF A HF WAVE IN A WAVEGUIDE

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INTRODUCTION

A large-amplitude high-frequency wave induces on charged particles an average (low-frequency) effect due to the so-called ponderomotive force. Much theoretical work has been devoted to the determination of the ponderomotive force in a magnetic field/1/. In particular, single particle/2/ and fluid/3/ methods have been used to derive the expression of the ponderomotive force and the effects that it produces on an ensemble of particles such a plasma or a particle beam. So far, only one experiment has been done by Vaclavik et al./4/ to check the predictions of theoretical models; the perturbation of an ion beam by an azimuthal electric field was measured; the wave frequency was of the order of the ion cyclotron frequency. The conclusion was that the experimental results were in accordance with the predictions of the fluid theory. However, the electric field structure at such low frequencies is more or less well defined, and important parameters such as the Debye length were kept constant. We present in the following the results of an experiment on an electron beam. Due to the higher frequency, a rectangular waveguide can be used, thus allowing for an accurate knowledge of the wave field. Furthermore, the Debye length and other physical quantities are varied over a large range, which allows to check more carefully the theories. Our results confirm the conclusions of Vaclavik et al./4/. In particular, it is shown that the ponderomotive beam deviation exhibits a rather strong increase when the Debye length becomes smaller than the beam radius, that is when the fluid effects become significant.

THEORY

In the fluid description, in the steady state case, the non-linear
force acting on the electron beam can be written as

\[ F_p = -V\Phi + B_0 \times \nabla \times M \]  \hspace{1cm} (1)

where the ponderomotive potential \( \Phi = \frac{e_0}{4} (\delta_{jk} - \epsilon_{jk}) E_j^* E_k \), the RF induced magnetization \( M = \frac{e_0}{4} \partial_{\omega} \epsilon_{jk} E_j^* E_k \) and \( \epsilon_{jk} \) is the dielectric tensor \(/1,4/\). In the case of a perpendicular force induced by an azimuthal electric field \( E_0(r) \), it is easy to show from the momentum conservation equation that the electron beam experiences an azimuthal deviation with a velocity \( V_\theta \) given by

\[ m_e n_e V_\theta = \frac{e_0}{4} \frac{\omega_0^2 - \omega^2 + \omega_\infty^2}{(\omega^2 - \omega_\infty^2)^2} \partial_r |E_0|^2. \]  \hspace{1cm} (2)

Since the perpendicular components of the dielectric tensor involve the quantity \( (\omega^2 - \omega_\infty^2)^{-1} \), the magnetization part of the perpendicular ponderomotive force becomes dominant in the vicinity of the cyclotron frequency. Hence the beam deviation is always in the same direction, regardless of the sign of \( \omega - \omega_\infty \).

The RF induced magnetization occurs only if collective effects come into play. In other words, the density of the electron beam must be high enough so that the Debye length is smaller than the beam diameter. From Eq.(1), it is easy to show that the RF induced magnetization current can play a dominant role only in the case of the perpendicular ponderomotive force.

**EXPERIMENTAL RESULTS and DISCUSSION**

The experimental set-up is reported on Figure 1. The whole system lies in a vacuum vessel (pressure 10^-6 torr) immersed in an axial magnetic field up to 800 gauss along the z-direction (see figure 1 for the coordinates). Electron are emitted by a thin tantalum foil heated at 2300 K and accelerated by positively biased electrodes. The beam then goes through a rectangular waveguide cavity. The cavity is excited by a generator with adjustable frequency and output power. The mode in the cavity has a frequency of 1.75 GHz, with a TE_{10} polarization,
corresponding to an electric field $E_y(x)$. This reproduces the situation of the theoretical model, with $F_p$ perpendicular to the magnetic field.

and the expected deflection in the $y$-direction (for small deflection angles, $v_y \approx v_0$ to second order). The electron current escaping the cavity is detected by an electrostatic probe, made of a wire, 3 cm long and .1 mm in diameter, parallel to the $x$-direction and movable in the $y$-direction. The beam has a density up to a few $10^6$ cm$^{-3}$, its diameter is roughly 1 cm, its energy of the order of 50 eV.

Fig 1: Experimental set-up

\[ d(\text{mm}) \]

\begin{tabular}{c|c|c|c|c|c|c}
\hline
1.0 & 0.8 & 0.6 & 0.4 & 0.2 & 0.0 \\
\hline
1.6 & 1.7 & 1.8 & 1.9 & 2.0 & \\
\hline
\end{tabular}

error bars

Fig 2: Beam deviation versus $f_{ce}$

\[ d(\text{mm}) \]

\begin{tabular}{c|c|c|c|c|c|c}
\hline
1.0 & 0.8 & 0.6 & 0.4 & 0.2 & 0.0 \\
\hline
1 & 2 & 3 & 4 & 5 & \\
\hline
\end{tabular}

Fig 3: Beam deviation versus $r/\lambda_{De}$
Figure 2 shows the deviation amplitude of the beam as a function of the electron cyclotron frequency $f_{ce}$. In this figure, the width of the dotted region accounts for the magnetic field inhomogeneity in the cavity. The figure clearly shows that the deflection always occurs in the same direction, regardless of the sign of $(\omega^2 - \omega_b^2)^{-1}$, as predicted by the fluid theory. Figure 3 shows the influence of the Debye length $\lambda_D$ on the deviation. The beam temperature necessary to evaluate $\lambda_D$ is obtained by using the relationship between the beam energy $E_b$ and its initial energy dispersion $\Delta E$, $T_b/\Delta E = \Delta E/2E_b /5/$. For $r/\lambda_D < 3$, no deflection is measured, whereas there is a fairly constant deflection for $r/\lambda_D > 3$. This threshold illustrates the essential role of collective effects in the beam, in agreement with the fluid theory.

In conclusion, we have directly measured the deflection of an electron beam under the action of the perpendicular ponderomotive force. The evolution of the deflection with $(\omega^2 - \omega_b^2)$ agrees with the results of Vaclavik et al./4/ and stresses the importance of the ponderomotive magnetization current. Furthermore, we have explicitly shown that there is a density threshold for the occurrence of a significant deflection. The totality of these results confirms the existence of fluid effects in the action of the perpendicular ponderomotive force.

REFERENCES


GENERATION OF ION CYCLOTRON WAVE IN AN ION BEAM BY A PERIODIC MAGNETIC FIELD

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Basic investigation of charged particle beams in a periodic structure is important for the study of stability of plasmas and excitation of electro-magnetic waves such as in the Ubitron and the free electron lasers /1-3/. In previous experiments, an ion beam injected axially along a multiple magnetic mirror field was observed to cause the spatial cyclotron resonance at a resonance condition /4/. This paper presents an observation of strong excitation of electrostatic ion cyclotron waves near this resonance condition in the multiple mirror field. It is considered that the waves are not destabilized by a linear interaction between spiral ion beams and plasma /5,6/, but are caused by the explosive instability /7/ due to a periodic magnetic field.

When an ion beam with parallel velocity, $V = V_{b}$, is injected along a straight magnetic field, $B = B_0 z$, the dispersion relation of electrostatic waves in the fluid theory is shown in Fig.1, where the parameters are selected from the experimental condition ($T_e = T_i = 0.009 T_b$, $T_e$, electron temperature, $T_i$, ion temperature, $V_{b}$, injection beam energy, $n_b = 0.2 n_i$, $n_b$, beam density, $n_i$, ion density, $k_z = n_i r_b$, $k_z$, wave number in perpendicular direction, $r_b$, beam radius). In addition, a new branch ($\omega = 0$, $k_z = 2\pi/L_c$) can be introduced when the magnetic field is modulated with a period length, $L_c$, which is regarded as a stationary wave with zero frequency. Since there are negative-energy waves and positive-energy waves in the dispersion relation, the explosive instability is expected to occur. In Fig.1, the ion cyclotron mode, $m_1 (\omega = \omega_1, k_z = k_1)$ and the slow beam mode, $m_2 (\omega = \omega_2, k_z = k_2)$ and the periodic structure of the magnetic field, $m_3 (\omega = 0, k_z = 2\pi/L_c)$ can be coupled, resulting in a simultaneous growing of the former two waves. The coupling conditions are

$$\omega_1 = \omega_2 \approx \omega_{ci}, \quad \text{(1)}$$
$$k_1 \approx 0, \quad k_2 \approx 2\pi/L_c, \quad \text{(2)}$$

where the perpendicular wave numbers for $\omega_{ci}$ and $\omega_{ci}$ and slow beam mode, $m_2 (\omega = \omega_{ci}, k_z \approx 2\pi/L_c)$ and the periodic magnetic field, $m_3 (\omega = 0, k_z = 2\pi/L_c)$ satisfy the coupling condition indicated by three allows.

Fig.1: Dispersion relation for ion beam-plasma system.

The electrostatic ion cyclotron mode, $m_1 (\omega = \omega_{ci}$, $k_z \approx 0)$ and slow beam mode, $m_2 (\omega = \omega_{ci}, k_z \approx 2\pi/L_c)$ and the periodic magnetic field, $m_3 (\omega = 0, k_z = 2\pi/L_c)$ satisfy the coupling condition indicated by three allows.
the ion cyclotron mode and the slow beam mode are assumed to be the same.

The experiment is carried out at Tohoku university using a Double-Plasma (DP) type Q-machine /8/ as shown schematically in Fig.2(a). Two plasmas produced by the contact ionization of potassium atoms at hot plates of 35mm in diameter, are confined by a uniform magnetic field, $B_0=0.1-0.4$ Tesla. Since plasmas are separated by a grid biased negatively, cylindrical ion beams are continuously injected into the target plasma along the magnetic field when a positive bias voltage is applied to the hot plate of the driver plasma (HPD) as shown in Fig.2(b). The column of the target plasma is about 3m long. The target plasma has density $n=10^7-10^9$ cm$^{-3}$, $T_e=T_i=0.2$ eV. The beam density, $n_b=10^7$ cm$^{-3}$, the beam energy, $\psi_b=5-100$eV, the beam radius, $r_b=1.1$cm and the total beam current, $I_b\leq100\mu$A. The background pressure is about $1.3\times10^{-4}$ Pa and collisionless condition is satisfied.

A periodic magnetic structure (multiple mirror field) is produced by setting iron rings with same intervals as shown in Fig. 2(c). Usually, the number of the period, $N=6$, the period length, $L_c=10$cm, the mirror ratio, $R_m=1.3$ at $B_0=0.27$ Tesla. By changing the shape of iron rings, the parameters of the periodic magnetic field can be changed. The ion beam current and fluctuation of ion current are measured by a grid of Faraday-cup analyzer with 3mm in diameter and a small asterlike probe.

![Fig.2](image)

Fig.2: (a) schematic of experimental setup, (b) potential distribution along the axis and (c) measured magnetic field, $B_z$ along the axis at $r=0$.

![Fig.3](image)

Fig.3: (a) fluctuation spectrum of ion saturation current measured 20cm downstream from the last mirror point. The beam energy is selected to maximize the amplitude. (b) the peak frequency and injection beam velocities in (a) as a function of the magnetic field.
Figure 3(a) shows the frequency spectrum of ion saturation current measured by an asterlike probe at 20cm downstream from the last mirror point of the multiple mirror field. For each magnetic field, the injection beam energy is selected to maximize the instability. Figure 3(b) gives the injection beam energy to maximize the instability and its peak frequency as a function of the magnetic field. The frequency of the instability is almost same as the ion cyclotron frequency, and the optimum condition of the instability generation is $V_b \approx \omega_c L_c / 2\pi$, which satisfy the coupling conditions. These results are in good agreement with the theoretical prediction.

The growth of the instability along the beam flow through the periodic magnetic field is measured by a movable analyzer. Figure 4 shows the axial variation of ion current $j_b$, its fluctuation amplitude, $\tilde{j}_b$, and the ratio $\tilde{j}_b/j_b$ at the axis, where the injection beam energy, $\Psi_b=23\text{eV}$, $B_0=0.13\text{Tesla}$, and the frequency, $f=116\text{kHz}$ $=1.08\omega_c/2\pi$. The ion beam current density is modulated in the periodic field by the magnetic focusing effect of the spatial cyclotron resonance (4). On the other hand, the fluctuation with the ion cyclotron frequency is excited in the periodic magnetic field, growing along the magnetic field with strong amplitude modulation.

![Figure 4: Axial variation of ion saturation current $j_b$ at the axis, fluctuation amplitude of ion current $\tilde{j}_b$ with frequency, $f=116\text{kHz}$, and the proportion of the fluctuation, $\tilde{j}_b/j_b$ (initial beam energy, $\Psi_b=23\text{eV}$ and $B_0=0.27\text{Tesla}$).](image1)

![Figure 5: (a) Axial variation of the fluctuation amplitude of the saturation current and corresponding phase variation along the axis at a fixed time at $t=1.0\text{cm}$. Closed circles and the open circles correspond to the phase variation of the top and bottom of the wave form, respectively. (b) Amplitude and phase variation of a model wave, $F=\cos(\omega t)+\cos(\omega t-k_z)$. Solid lines and broken lines correspond to the top and bottom of the wave form, respectively.](image2)
The wave gradually damps after the exit of the periodic field. From this measurement, it is confirmed that the electrostatic wave with the ion cyclotron frequency is excited by the periodic structure of the magnetic field at the coupling condition. The measurement of radial profile of the fluctuation amplitude proves the wave observed not to be an ion cyclotron drift wave. According to the measurement of the amplitude of the wave vs. the injection beam energy, the condition of the maximum growth of the wave is different from the condition that the ion gyration energy becomes maximum. This result comes to a conclusion that the instability is not linearly excited by an ion beam of high perpendicular energy. On the other hand, the strong modulation of the wave amplitude observed, which is independent of the modulation of the beam current density, suggests that two waves with same frequency and different wave numbers are simultaneously excited. To get further characteristics, the axial variation of the wave phase at a fixed time is measured by using a boxcar integrator, as shown in Fig.5(a). The closed circles and open circles correspond to the top and the bottom of the excited wave form (180° different) at a fixed time. The phase change is not linear and has a periodic pattern. In order to explain the measured result, the spatial variation of amplitude and phase at a fixed time is calculated for superposed two-waves, \( F = \cos(\omega t) + \cos(\omega t, z) \), as presented in Fig.5(b). The amplitude has strong spatial modulation and the phase has a periodic jump, both of which are in good agreement with the experimental result shown in Fig.5(a). Thus, it is considered that the two waves are simultaneously excited and grow in the periodic magnetic field. The frequency of the two waves is about ion cyclotron frequency and the wave numbers in the axial direction are about \( 0 \) and \( 2\pi/L_c \). Our experimental results consist with the theoretical prediction of the explosive instability by the periodic magnetic field.

In conclusion, the electrostatic ion cyclotron wave and the slow beam mode are simultaneously excited at the coupling condition in an ion beam passing through the periodic magnetic field. We believe that this is the first observation of the electrostatic explosive instability induced by the periodic magnetic field. This mechanism of the coupling is same as that of the excitation of the Raman type free electron laser and various kinds of periodic structures in a coordinate system moving with the beam will give rise to this kind of instability.

References

NON-LINEAR ION ACCELERATION IN ELECTROSTATIC WAVE FIELDS

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1. Introduction
The interaction of plasma ions with large amplitude electrostatic (ES) waves propagating perpendicularly to the background magnetic field shows a variety of amplitude dependent features which are important in the physics of plasma heating and ion acceleration in rf-fields. This is a result of the interesting properties of the single particle ion dynamics in the wave field. At a high enough field strength, stochastic ion heating takes place with the exact mechanism depending on the \( \omega/\Omega \) ratio. These processes have been invoked as possible ways of absorbing wave power in lower hybrid and ion Bernstein wave (IBW) heating of hot fusion plasmas [1,2]. Experimentally, stochastic heating has recently been observed in a single mode ES wave [3] and found to be a probable candidate for fast heating of cold ions, caused by drift waves [4]. In addition to these stochastic mechanisms, even at lower field strengths the ions are susceptible to strong acceleration on a slow time scale, which may indicate hot ion generation in modulated fields.

2. On Ions in Electrostatic Wave Fields
In rf-heating of fusion plasmas in the ion cyclotron range of frequencies some or all of the power is coupled to electrostatic modes. The latter prevails in the case of IBW-heating, while in the fast wave heating case parasitic coupling may lead to a fraction of the total power being in the ES branch [5]. The launched power propagates through a low density and temperature edge plasma in the vicinity of the antenna. In tokamaks, if we assume a typical edge density of \( 10^{18} \text{ m}^{-3} \), a temperature of 50 eV and a parasitic coupling of a few per cent of the total fast wave power flux of 1 MW/m², the corresponding ES wave field strength is of the order of 1000 V/cm. This estimate can be derived from a 1-D definition for the kinetic energy flux \( \dot{E} = \sigma E_x dE_x / dx \) [6], where \( \sigma \) is the thermal correction to the dielectric tensor element \( \epsilon_{xx} \) second order in Larmor radius and \( E_x \) is the ES wave field in the radial, or perpendicular to the magnetic field, direction.

Given high enough field strengths, the dynamics of ions with kinetic energies typical of such edge plasmas can be analysed numerically by integrating the single particle equation of motion

\[
m_i \frac{d}{dt} \vec{v} = Z_i e ( \vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r}) )
\]

Here \( \vec{v} \) is the ion velocity, \( m_i \) is its mass, \( Z_i e \) is the charge and \( \vec{E} \) and \( \vec{B} \) are the ES
wave electric field and the background magnetic field, respectively. We formulate the problem by taking the magnetic field to lie in the z-direction and to be constant in space and restricting the electric field to the x-direction. The neglect of a possible parallel electric field component is of no real concern as far as $k_z \ll k_\perp$ holds for the parallel and perpendicular wavevector components. This condition is usually valid in the case of ES waves arising due to parasitic coupling in FW experiments and in most IBW experiments, as well.

Most studies on the wave–particle dynamics have assumed a single mode interacting with the ions. We allow in the following the field to consist of several modes with different wavenumbers $k_\perp$, and the electric field is thus written as $E = \sum E_i \cos (k_{\perp,i} x - \omega t + \theta_i)$, where for the relative phases $\theta_i = 0$ is used for mode–locked cases while random ones are chosen for $\theta_i$ in the case of fully dephased modes. $\omega$ is the angular frequency, common for all the modes. For simplicity, equal amplitudes $E_i = E$ are taken in the following.

3. Results

We concentrate in the following on the development of the ion kinetic energy as a function of time. The parameters of the calculations have been chosen to be appropriate for ES modes in the IBW heating or created through parasitic coupling in fast wave heating. In such experiments, the creation of fast ions at the plasma boundary is of great interest because of the sputtering of the wall structures. Accordingly, we take $\omega/\Omega = O(1)$, $B = 2.34$ T and study the behaviour of D$^+$ ions. The initial ion kinetic energy is 50 eV.

Fig. 1 shows a case, in which a single deuteron is accelerated in an electrostatic field with $|E| = 1000$ V/cm and $\omega/\Omega = 1.88$. The ion orbit has been integrated numerically using an accurate adaptive ODE solver. Adiabatic oscillations of the particle energy characteristic of a trapped particle with a conservative Hamiltonian [7] are clearly seen in the Figure. The present field level is well below the stochastic limit, above which irreversible heating takes place. Nevertheless, the maximum ion energy is much larger in this example than the initial one.

To study the dependence of the ion energy on the electric field we consider an ensemble of 100 ions distributed randomly at $t = 0$ in wave phase and direction of initial velocity. By increasing the field strength we see in Fig. 2 how after reaching a threshold in $|E|$ the original behaviour of oscillations around the original kinetic energy changes to those with a long period depicted in Fig. 1. With still higher fields, the maximum ion energy approaches the value of about 5 keV corresponding to the separatrix condition $J_{n=2}(k_{\perp,\perp}/\Omega) = 0$ [2]. Because of averaging, the mean energy of the ensemble approaches a steady–state value depending on $|E|$. Unlike the field strength, the $\omega$ to $\Omega$ ratio has only a qualitative effect, as can be concluded on the basis of Fig. 3, where similar calculations are shown for a few values of this ratio. These demonstrate a dependence of the threshold seen in Fig. 2 on the $\omega$ to $\Omega$ ratio, while at higher field levels the picture of long term oscillations holds still. The change in the ion dynamics due to a multimode spectrum is unexpectedly large, as we show in Fig. 4. The modes have been taken to be such that $k_{\perp,i}$ is distributed equally around $k_\perp = 800$ with the spread $\Delta k$
being 10. The amplitudes of the modes are normalized so that the total energy in the spectrum remains approximately the same in every case, i.e. that contained in a single 1000 V/cm, k_L = 800 mode. The mode phases have been chosen at random and the particle kinetic energies have been distributed according to a Maxwellian with T = 50 eV. The distribution of the original wave energy contained in a single mode to several of equal strength does not affect the general picture of long term energy oscillations, although the dynamics is more rapid. On the other hand, the structure of the phase space becomes in the multimode cases such that the accessible range of kinetic energy to the ions becomes wider.

4. Discussion

In this paper we have considered the case of electrostatic wave interacting with low energy ions in a magnetized plasma. The numerical examples shown demonstrate that an ion population with a low enough temperature is rapidly accelerated to high energies assuming an undepleted electrostatic wave. From the point of view of heating of fusion plasmas, such acceleration is of interest as a localized source of hot ions. As the ions move with a velocity v_z along the magnetic field lines the field amplitude varies strongly along their paths. The characteristic length of the localized field in front of the antenna has a measure of the antenna width w, which gives an autocorrelation time t_c = w/v_z for the wave–particle interaction. If t_c is at least of the order of the characteristic period of the slow adiabatic oscillations in the field, ions with high perpendicular energy can escape the field structure. This may result in plasma heating or sputtering of limiter and walls structures connected by the field lines. For w = 20 cm and v_z = 10^6 m/s, we have t_c = 2 × 10^6 s, which is of the same order as the oscillation periods in Figs. 1 to 4. This suggests a heating mechanism in front of present ICRF antennas and can be related to the observed problems in IBW heating experiments. One question not answered by our numerical calculations is how a large amplitude ES wave propagates in low density, cold plasmas. For this purpose, a self-consistent particle simulation is needed.

References
Figure 1a. The behaviour of a single deuteron kinetic energy as a function of time with $|E| = 1000$ V/cm, $\omega/\Omega = 1.88$ and $k_1 = 800$ 1/m.

Figure 2. The average energy of 100 deuterons as a function of time. $T_0 = 50$ eV, other parameters as in Fig. 1. Solid line: 600 V/cm, dotted line: 700 V/cm, dashed: 800 V/cm, chain: 1000 V/cm.

Figure 3. The energy time histories for a single mode case with $|E| = 1000$ V/cm for some values of the $\omega/\Omega$ ratio. Solid line: 1.8, dotted line: 1.88, dashed: 1.98, chain-dashed: 2.1.

Figure 4. The energy of a Maxwellian ion population vs. time. Initial energy 50 eV, 1 mode case: solid curve, 11 modes: dotted and chain: dashed.
Electron Acceleration by $v_p \times B$ Mechanism in Vacuum

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Abstract

A new type electron linear accelerator, based on $v_p \times B$ acceleration mechanism observed originally in plasma, has been demonstrated in vacuum. In this scheme a static magnetic field is applied vertically to the wave propagating direction and the particles are accelerated along the wave front at constant phase with respect to the wave. The linac consists of electron gun, magnetron for a RF source, slow wave structure and a pair of coil for an applied magnetic field. Energy gain of 12.8 keV is observed at an initial energy of 66 keV when an external magnetic field of 2.5 G is applied. The energy gain is 30% higher than that of a conventional linac. The output energy increment is proportional to the square of the applied magnetic field $B$. The comparisons between the experimental and theoretical values are given.

Recent interests of charged particle accelerators are focused on generating a much higher acceleration gradient. In the last decade some of new acceleration schemes using a longitudinal electric field of the wave, including a plasma wake field accelerator (PWFA), a plasma beat wave accelerator (PBWA) and a $v_p \times B$ scheme (or a surfatron), have been proposed. Some of the experimental results based on the above mentioned scheme have been reported to give the higher acceleration gradient. The new schemes using plasmas can be expected to generate much higher electric field, but it is also easy to predict some difficulties of controlling plasma parameters. The $v_p \times B$ acceleration phenomena have been observed in the experiments of microwave-plasma interaction. In this scheme a static magnetic field is applied vertically to the wave propagating direction and the particles are accelerated along the wave front at constant phase with respect to the wave, and the acceleration continues until the trapping condition breaks.

In the $v_p \times B$ system a charged particle is trapped in an electromagnetic wave (TM wave) with a phase velocity $v_p$ propagating in the $z$ direction immersed in a static magnetic field $B$ in the $x$ direction (Fig. 1). The Lorentz force $F_y = qv_z B_0$ accelerates the particle in the $y$ direction, and the resulting velocity $v_y$ produces the Lorentz force in the $-z$ direction. If the wave electric field satisfies the following condition, the charged particle is trapped in the electric field in phase space,

$$E_m > \gamma_p B_0 v_y,$$

where $E_m, \gamma_p, v_p$ are the maximum electric field of the wave, the relativistic factor defined by $\gamma_p = (1 - v_p^2/c^2)^{-0.5}$ and the velocity component in the $y$ direction, respectively. The particle is continuously accelerated until the condition breaks.

Energy gain in steady state for a unit length is the same as that originally obtained on the surfatron,

$$\frac{d\gamma}{dy} = \gamma_p \frac{q P_{\perp}}{c^2},$$
\[
d\gamma \frac{dz}{dz} = \gamma \frac{\omega_c^2 z}{c^2} \left( 1 + \frac{\omega_c^2 z^2}{c^2} \right)^{1/2}
\]

where \( \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \) and \( \omega_c \) is the electron cyclotron frequency.

Figure 2 is a schematic diagram showing an electron beam gun, a slow wave structure and an energy analyzer. The injected electron is initially accelerated by a high voltage power supply, with maximum energy and current of 100 keV and 1 mA, respectively.

A hair-pin type cathode is made of a tungsten filament which is originally made for an electron microscope. The accelerated electrons through the accelerator are detected by an electron energy analyzer.

The pulsed electromagnetic wave is generated by a magnetron which is triggered by the external timer with a typical pulse width of 5 \( \mu \)s in repetition at 10 Hz. We used two different power sources, one has a maximum power of 2.5 kW and the other has 10 kW, the emitted frequencies of them were measured by comparing with the standard oscillator to give 2.465 GHz and 2.459 GHz, respectively. Although both of the original specifications are 2.45 GHz, the frequencies were upper-shifted due to too much of the applied cathode voltages so as to obtain higher output power. The generated microwave is converted into TM wave through the slow wave structure, and is absorbed by a non-reflection dummy load.

The slow wave structure consists of parallel fin electrodes with 48 cm long in the wave propagation direction. Its propagation mode of the microwave is designed to be \( 2\pi/3 \) mode and the phase velocity is 0.464c (corresponding energy of 66 keV) at the RF frequency of 2.45 GHz. The cross section of the structure is rectangular. In general the circular type acceleration cavity is popularly used for the linear accelerator to generate the maximum longitudinal electric field at the center of the cavity. In the \( v_p \times B \) scheme, however, the electron orbit is parabolically bent by the external magnetic field, in other words, the electron is accelerated in two dimensional. Although the maximum electric field strength is weaker in the present case, we can choose the structure with no obstacles in the \( y \) direction. This is convenient to demonstrate the \( v_p \times B \) acceleration principal. When it works in a relativistic regime, the particle orbit is strictly linear and the circular type slow wave structure could be employed so as to obtain a strongest field.

An example of experimental results with rf power of 2.5 kW is shown in Fig. 3. The horizontal and vertical axes indicate the incident electron energy \( E \) and the energy increment \( \Delta E \), respectively. Symbols \( \Box, \Delta, \circ \) and \( \triangle \) stand, respectively, for the value of the external magnetic field of 0, 2 and 3 Gauss. When \( B_0 = 0 \) G, the accelerator operates as the conventional linac. Therefore one can easily compare the \( v_p \times B \) accelerator with the conventional one in a same machine. The slow wave structure strongly resonates with the microwave around \( E_0 = 48 \) and 57 keV. Data show that the energy increment of the \( v_p \times B \) scheme is larger than that of the conventional one at every incident electron energy. When the incident beam energy is 48 keV, for example, the observed electron energy is 56.0 keV (\( \Delta E = 8.0 \) keV) with a static magnetic field of 3 G, while in the conventional accelerator the energy reaches 52.0 keV corresponding \( \Delta E = 4.0 \) keV. The expected energy increment is calculated from Eqs. (2) and (3) for the case of \( B_0 = 3 \) G at \( E_0 = 48 \) and 56 keV. The results are shown by the solid circles in Fig. 3. The disagreement between the experiment and the calculated value is explained as follow; The Eqs. (2) and (3) are valid only for the
steady state of the acceleration only. In the experiment, however, the electrons are not in the steady state but oscillate around the equilibrium point in the phase space because the acceleration time is not sufficient to reach the steady state. Sufficient acceleration with condition (\(\omega t \gg 1\); in the present machine \(t > 20\) nsec is required) is assumed in Eq. (2).

Figure 4 shows the energy increment as a function of the static magnetic field intensity. Up to 2 G, the energy increment \(\Delta \varepsilon\) is proportional to \(B^2\), i.e., \(\Delta \varepsilon - \Delta \varepsilon\ (B_0 = 0\) G \(\propto B^2\), while over 2 G, \(\Delta \varepsilon\) decreases sharply. This dependence can easily be explained by the Lorentz force acceleration in the \(y\) direction. After the value of \(B\) exceeds 2 G, the electron cannot be trapped by the wave potential anymore. These results mean that the trapping condition is no longer maintained. The underlying physics producing a critical magnetic field \(B = 2\) G is explained using Eqs. (2) and (3).

A new electron linear accelerator based on the \(v_p \times B\) acceleration scheme has been demonstrated to work successfully in a vacuum system. The experimental results show that proposed scheme accelerates the electron more effectively than the conventional linear accelerator. The measured data are in good agreement with the theoretical results containing weak relativistic effects.

References

Figure 1.
Schematic diagram of $v_b \times B$ acceleration scheme

Figure 2.
Experimental setup

Figure 3.
Energy increment $\Delta \varepsilon$ vs incident energy

Figure 4.
Energy increment $\Delta \varepsilon$ vs the static magnetic field
Plasma Wave Wakefield Excited by Short Pulse Microwave

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Abstract

The plasma wakefield can be excited by using short, high-power microwave pulse for demonstrating a laser wakefield accelerator principle (LWFA). Although the conditions of the experiments are different from those of the LWFA, the plasma wave is excited, non-resonantly by a short microwave pulse. Close to the resonant density for the resonance absorption of the electromagnetic wave, new nonlinear phenomena, tentatively interpreted as the electron wave soliton, have been observed. The excitation of the plasma wave is due to the ponderomotive force \( \text{grad}E^2 \) of the short microwave pulse.

Several electron acceleration schemes using plasma have been proposed.\(^1\)–\(^3\) They have an advantage of getting high field gradient for electron acceleration. Specifically, the \( V_p \times B \) acceleration scheme has high acceleration rate with better efficiency compared with other schemes. This, however, needs strong wave fields for achieving its best qualities. Laser wakefield accelerator (LWFA) is one of the plasma-based acceleration schemes, and employs a short, high-power, single frequency laser pulse.\(^4\) The LWFA has some advantages. For example, it can get long acceleration distance, or it is not necessary to adjust the plasma density very precisely to designed certain value. LWFA can work as an accelerator as it stands. Moreover, it could be used for a driver wave for the \( V_p \times B \) accelerator, which could achieve much higher energy than the original LWFA scheme.

In this paper, first experimental results by a short microwave pulse, and some explanations for the excitation mechanism of the plasma wave are presented. This is essentially the same method of the LWFA.

The ponderomotive force produced by the microwave acting on the plasma is given by

\[
\vec{F}_{\text{pond}} = e \vec{v} \phi_L(r, z, t)
\]

where, \( \phi_L = -m_e c^2 a_L^2/(2|e|) \) is ponderomotive potential and \( a_L = |e| a_L/(m_e c^2) \) is the normalized amplitude of vector potential radiation field. This field moves with the wave group velocity. The radial ponderomotive force expels electrons radially outward, while the front (back) of the laser pulse exerts a forward (backward) force on the electron. As the plasma electrons move around the microwave pulse, the plasma waves can be generated.

Experiments are performed in a cylindrical stainless steel chamber with multi-dipole magnets on the outside of the wall, as shown in Fig.1. To demonstrate the excitation of the wakefield, a uniform plasma density and no reflection of the microwave are required. For this purpose, the other end of the chamber from the antenna is covered with acrylic resin plate, through which the wave comes out of the chamber. The argon plasma is produced by pulse discharge (discharge duration of 3 msec and 10 Hz repetition). Typical plasma
parameters are a maximum plasma density \( n_0 = 2 \times 10^{11} \text{cm}^{-3} \), an electron temperature \( T = 2 - 3 \text{ eV} \), which are obtained by Langmuir probe method. The p-polarized microwave with a frequency 2.86 GHz (corresponding critical plasma-density is \( n_c = 1 \times 10^{11} \text{cm}^{-3} \), a maximum power of 10 kW and typical pulse width 200 nsec is irradiated through a high gain horn antenna installed with metal plate lens in front of it. The field strength in the plasma are measured by a crystal detector or a spectrum analyzer, through a tiny plate probe (1 mm x 1 mm) or a cylindrical probe (0.1 mm dia. x 1 mm).

Typical waveforms are shown in Fig. 2, where bottom trace is the signal of the incident microwave, and top one is the plasma density fluctuation detected through probe which is biased to \( V = -20 \text{ V} \) at \( Z = 37.6 \text{ cm} \) from the antenna edge. This position is roughly 5 mm down the resonance absorption layer. The scale of the horizontal axis is 200 ns/div. There are several peaks in the plasma wave. When the pulse width of the incident microwave becomes larger, these peaks go out. This instability has a threshold. They seem to be wakefield, because it is nonlinearly excited after the incident microwave power shut off.

Figure 3 shows the spectrum observed close to the resonance absorption layer with and without plasma. We can see lower frequency wave which corresponds to Fig. 2. Figures 4(a) and (b) show spectra of the plasma wave observed around 1 GHz at \( Z = 14 \text{ cm} \) and 15 cm respectively. As clearly seen, the plasma wave can also be excited with frequencies of about 0.99 and 1.02 GHz. When the detecting point moves up to higher density layer \( (Z = 15 \text{ cm}) \), both frequencies become higher. If a plasma frequency is 1 GHz, the corresponding plasma density is \( 1.2 \times 10^{10} \text{cm}^{-3} \). This value is in reasonable agreement with \( n_0 = 1.3 \times 10^{10} \text{cm}^{-3} \) which is measured by Langmuir probe method. The propagation velocities \( (Z \text{-direction}) \) of the nonlinear waves are also measured to show the velocity of \( v = 2 - 3 \times 10^7 \text{cm/s} \). The larger peak has higher velocity. Since the electron thermal velocity is estimated to be \( v_{th} = 1 \times 10^8 \text{cm/s} \), the velocity of this nonlinear wave is slightly slower than \( v_{th} \). The present pulses propagate out of the resonance layer and also from the layer with highest intensity of the electric field. When a resonance layer exists within a chamber, a familiar Airy function type standing wave structure has been observed. In this case, the plasma waves can be excited, typically, at two layers, one at the resonance layer and the other at the first, highest peak position near the cut-off layer as an example is shown in Fig. 5. Here, the wave can be observed through the interferometer method with the reference frequency of \( f_r = 2.86 \text{ GHz} \), so that other frequencies cannot be detected. The waves are excited at both sides, where the ponderomotive force, which is proportional to grad \( E^2 \), is maximum.

To explain above experimental results, the original theory for LWFA should be modified. The following conditions are required; the electron temperature \( T_e \) is non-zero, and the group velocity of the driving wave is zero, because the short pulse, electromagnetic wave forms a standing wave structure not to propagate. Still there exist areas of high gradient of \( E^2 \). Therefore, even if the microwave pulse does not pass through the plasma, the ponderomotive force due to high gradient of \( E^2 \) can exist: Therefore, it is possible to consider that the ponderomotive force acts on electrons, and the plasma wave can be excited. Under these assumptions, the excited plasma wave is expected to propagate with the velocity less than thermal velocity \( v_{th} \). As mentioned above, the observed propagation velocity is slightly slower than the thermal velocity in reasonable agreement with the
Theoretical prediction. Near the resonance layer, we have several possibilities for exciting solitons, 1) envelope soliton of electron plasma wave, 2) ion acoustic soliton, and 3) decay instability for exciting ion acoustic wave. The decay instability has finite growth time of several microseconds such as already shown in the previous works, with longer pulse duration, and we could neglect this phenomenon at present. The ion plasma frequency is estimated as 10–12 MHz, and 2nd possibility could survive. The first possibility is also expected because the cavity formation time is also order of ion plasma wave time scale. Anyhow, multiple "soliton" such as observed in the present experiments with short pulse duration have never been observed so far, and we need more theoretical works for interpretation.

In conclusion, the plasma wave (wakefield) was non-resonantly excited by short microwave pulse. The method of excitation was slightly different from that of the original LWFA theory. However, the key mechanism of the excitation can be explained by the theory with slight modification. New "soliton" phenomena have been observed near resonance absorption layer.

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References

Fig.1. Schematic of experimental apparatus.
Fig. 2. Typical wave forms observed near resonance layer.

(a) Electron density perturbations

(b) Incident microwave pulse

Fig. 3. Typical example of the spectrum observed with and without plasma near the resonance layer.

(a)

(b)

Fig. 4. Example of the spectrum observed in the underdense layer at (a) \( Z = 14 \text{ cm} \) and (b) \( Z = 15 \text{ cm} \).

Fig. 5. Typical example of the electric field \( E^2 \) and the plasma wave pattern.
Direct Measurement of Velocity Space Transport
in a Fully Ionized Plasma

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We report the direct measurement of velocity space transport coefficients in the direction of the confining magnetic field, which may be interpreted as the Fokker-Planck coefficients. The measurements were made using the techniques of laser induced fluorescence [LIF] and optical tagging, which allows ions to be followed in real and in phase space. In a plasma with nearly thermal fluctuations only, the results agree well with test-particle calculations made by N. Rostoker. In the presence of larger amplitude drift waves, however, there is a pronounced enhancement of the diffusion coefficient at a velocity less than thermal speed. The measurements reported here are unique in that they deal with the bulk of the velocity distribution, where most particles reside, allowing direct comparison with theoretical calculations such as the Fokker-Planck coefficients. A schematic of the experiment, which was performed in the U.C.Irvine Q-Machine, is shown in Fig. 1.

The barium plasma column length was 1.3m and its diameter was 5 cm. Two laser beams were used: one, the tagging beam, pumped the ground state transition at 4934Å and filled a metastable state, and the other, at 5854Å, de-excited the metastable transition and thus identified the tagged particles. The two beams were superimposed and directed axially through the collector end of the plasma column. The usual solid cold end plate was replaced by a fine mesh grid that allowed the laser beams to pass through with minimal attenuation.

The photon emission from the tagged particles was picked up by lens systems which directed the the optical signal to photomultipliers [PMT], one for each laser beam. The output of the lasers were pulsed by means of acousto-optical modulators. The pulsing sequences are shown in the figure. The pulses were 10msec. long with a basic repetition rate of 769Hz [1.3msec period].

Selecting a laser wavelength, or frequency, corresponds to selecting a plasma ion velocity. Sweeping the laser frequency amounts to scanning the plasma ion velocity, which allows the determination of the ion velocity distribution, from which the ion temperature and other information can be determined. To take the measurements the laser output was pulsed as described above except that the pump, or tagging laser was pulsed at half the repetition rate of the search laser. This allowed alternate pulses of the search laser to be used to subtract out background light. The first measurement was
made using the sequence labelled "I" in the Fig.1, which indicates that except for alternate pulses both the tagging beam, which was set at a particular frequency, or velocity, and the search beam were on simultaneously. The frequency of the search beam was then swept. Since the sweep rate was so much slower than the pulsing rate, the velocity distributions obtained were, essentially, continuous distributions of the tagged particles averaged over 10msec intervals. Ten microseconds is long enough for significant diffusion in velocity space to have taken place, but much shorter than the time required for the tagged particles to equilibrate with the background. Hence this is, essentially, a diffusion problem in which a source consisting of a single [d-function] velocity continuously feeds a diffusing population. The search beam picks up what has happened to the particles that were tagged at that particular velocity over the interval of 10msec. Thus as the laser scans over its frequency range, the result from the search beam is a distribution that shows how many particles that were originally at a specific velocity have diffused to other velocities over an interval of up to 10msec.

In principal this should yield enough information to enable the calculation of a velocity-space diffusion coefficient. However, it turns out to be simpler and easier to understand if we use this as an initial distribution, since it includes all the vagaries of pumping the metastable states etc., and then comparing it to a distribution taken at a later time with the tagging beam turned off. This was done with the pulsing sequence labelled "II" in Fig.1. Here the tagging beam is turned on for 10msec, and at the end of the pulse, as it is turned off, the search beam is turned on for 10msec. Now the problem presented is that of an initial tagged distribution that is the result of a d-function stimulus after diffusion during a time that we take to be 10msec. The moments of each distribution were calculated for many conditions and then compared with the calculation of the Fokker-Planck coefficients as explained below.

The initial distribution of tagged particles can be represented to a good approximation, by

$$ f_T(v,t=0) = \frac{n_T}{\sqrt{2\pi \sigma_T^2}} \exp\left(\frac{(v-v_T)^2}{2\sigma_T^2}\right) $$

(1)

where \( T \) refers to the tagged particle, \( \sigma_T \) is the standard deviation of the initial tagged particle distribution. The time evolution of eq.(1) is given by

$$ \frac{\partial f_T(v)}{\partial t} + V_v \cdot \vec{\nabla} f_T + \Gamma = 0 $$

(2)

The experimental results can be compared with the three-dimensional theory for ion-ion collisions in a fully ionized plasma given by the Fokker-Planck equation

$$ \frac{\partial f}{\partial t} \bigg|_{\text{coll}} = \frac{\partial}{\partial v} \left( \delta v \right) f(v,t) + \frac{1}{2} \sum_{i,j} \frac{\partial^2}{\partial v_i \partial v_j} \left[ \langle \delta v_i \delta v_j \rangle f(v,t) \right] $$

(3)
The first and second terms are interpreted as the convection and diffusion terms respectively. When \( \langle \frac{\delta v}{\delta t} \rangle \) and \( \langle \frac{\delta v_j}{\delta t} \rangle \) are known the future of the distribution is known to second order in \( dv \).

Calculation of the Fokker-Planck coefficients for this case was done by N. Rostoker\(^3\). The term for parallel convection is given by

\[
\langle \frac{\delta v_z}{\delta t} \rangle = C_z = \frac{2 \pi e^4 n \ln \Lambda}{m_i^2} \frac{1}{v_{th}^2} \frac{\partial}{\partial x} \text{erf}(x)
\]

(7)

The diffusion of test ions in a quiet plasma along the magnetic field is given by

\[
\langle \frac{\delta v_z \delta v_z}{\delta t} \rangle = D_{vv_z} = \frac{8 \pi n e^4 \ln \Lambda}{4 \sqrt{2} v_{\text{th}}^2} \frac{1}{x} \frac{\partial}{\partial x} \text{erf}(x)
\]

(8)

where \( v_{\text{th}} \) represents both parallel and perpendicular contributions to the thermal energy, \( x = v/(\sqrt{2} v_{\text{th}}) \), and \( z \) represents the coordinate axis aligned with the magnetic field. The results are shown in Figs. 2-3, all shown in plasma reference frame coordinates. The measurements were taken near the axis of the plasma column so as to avoid complications arising from drift waves and other phenomena associated with the steep density gradients at the edges. The techniques for determining ion density and temperature are described in ref. [1]. The data for the velocity space diffusion coefficient, \( D_{vv_z} \), as a function of plasma density in a "quiet" plasma for a typical value of \( v_z \) show good agreement with a linear variation with density. Additional measurements of the diffusion coefficient taken with different values of \( v_z \) yielded similar results, indicating good agreement indicating classical diffusion theory.

Fig. 2 shows a comparison of the variation of \( D_{vv_z} \) versus the tagged velocity, \( v_T \), normalized to the thermal velocity, \( v_{\text{th}} \). The open circles were taken in the quiet plasma condition and show good agreement with Rostoker's theory. The solid squares show data taken when the plasma density profile was altered to have a sufficiently large gradient to destabilize drift waves with \( \text{dn}/n \sim 6.0\% \). The dramatic departure from theory is quite apparent, the data indicating a peak in the axial diffusion.
Coefficient at a velocity about $-0.7 v_{th,z}$. At velocities higher than [plasma frame] zero, agreement with classical predictions seems to be restored. Figure 3 shows a comparison of the convective coefficient, $C_{vz}$, with the normalized velocity $v/v_{th,z}$. Within experimental error the mean value agrees with theory in the quiet case. However, the errors bars are too large to draw firm conclusions about agreement with classical theory insofar as the variation with velocity is concerned. When fluctuations are present, the scatter of points does not allow any further conclusions to be drawn. The relatively large error bars are due to the fact that the convective coefficient, being linear in $v$, is very susceptible to the relative drift of the tagging and search lasers, while the diffusion coefficient, being dependent on the square of the velocity, is not as sensitive.

![Figure 3](image)

**FIG. 3** Plot of the longitudinal velocity convection coefficient as a function of the normalized tagged ion velocity for a quiet plasma (open circles) and one in which drift waves are present (solid squares). The solid line is the theory for a quiet plasma, Eq. (2).

Measurements agree well with classical predictions. Second, when fluctuations significantly greater than thermal are present, classical collisional theory is insufficient to predict results. Something very definite happens when fluctuations are present. We are actively pursuing an explanation at this writing. The dependence of the perpendicular diffusion coefficient on the magnitude of density fluctuations has already been discussed by one of the authors.6

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PHASE CONJUGATION VIA BRILLOUIN ENHANCED FOUR-WAVE MIXING IN A PLASMA

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INTRODUCTION:

Detailed studies of phase conjugation via Brillouin enhanced four-wave mixing (BEFWM) of high power microwaves in a large, well diagnosed unmagnetized hydrogen plasma have been performed. Both the transient and steady state behaviour are shown to agree well with the predictions of two-fluid plasma theory for low power, low density operation. Strong suppression of the BEFWM interaction is observed at an electron density \( n_e = n_e/4 \), indicative of the two plasmon decay instability. High power, high density operation is seen to deviate strongly from simple theoretical predictions; evidence of the onset of ponderomotive whole-beam self-focusing is given.

BEFWM THEORY:

BEFWM, in its simplest form, can be modeled as a pair of simultaneous three-wave mixing processes. In the first process, a strong pump wave \( E_2 \) of frequency \( \omega_2 \) mixes with a weak signal wave \( E_s \) of frequency \( \omega_s = \omega_0 + \omega \) to generate a density "grating" in the plasma. In the second process, another strong pump wave \( E_1 \) of frequency \( \omega_1 \) (antiparallel to the first) scatters off this grating to generate a phase conjugate wave \( E_c \) of frequency \( \omega_c = \omega - \omega_1 \). As the pump intensities are increased, however, this simple model breaks down as the grating formed by \( E_1 \) and \( E_c \) begins to interfere (either constructively or destructively) with the primary grating formed by \( E_2 \) and \( E_s \).

As per Williams et al., simple two-fluid theory suffices to calculate the low frequency plasma response to the beating of two transverse electromagnetic waves. In the limit of small \( m/M \), the quasineutral steady-state density response is found to be

\[
\frac{\tilde{n}}{n_0} = \frac{(m/2M)k^2v_{i1}v_{i2}}{(\omega - k\cdot U_i)(\omega - k\cdot \bar{U}_i - iv_i) - k^2c_s^2}
\]

where \( c_s = [(\gamma e T_e + \gamma i T_i)/M]^{1/2} \) is the ion acoustic speed, \( U_i \) is the ion drift velocity, \( v_i \) is the ion damping rate, and \( v_i = eE_j/m\omega_i \) is the first order response to the external field \( E_j \). The density response reaches a maximum when \( (\omega - k\cdot \bar{U}_i)^2 = k^2c_s^2 \), at which

\[
\frac{\tilde{n}_{\text{res}}}{n_0} = i\frac{m}{2M} \frac{k_c}{v_i} \frac{v_{i1}v_{i2}}{c_s^2} = i\beta (E_1 \cdot E_2), \quad \text{where} \quad \beta = \frac{m}{2M} \frac{k_c}{v_i} \left( \frac{e}{m\omega_{i0}c_s} \right)^2
\]

The quasineutral pulse (step) temporal density response, at resonance, is found to be

\[
\frac{\tilde{n}_{\text{res}}}{n_0} = \beta (E_1 \cdot E_2) (1 - e^{-t/\tau_1}) \sin(\omega_{\text{res}} t)
\]
Returning to BEFWM, we consider the case where all four waves are polarized parallel to each other (but perpendicular to the interaction plane), and where grating $\tilde{n}_1$, formed by $E_s$ and $E_2$, is resonant. Note that this implies that grating $\tilde{n}_2$, formed by $E_s$ and $E_1$, is also resonant. The density gratings $\tilde{n}_1$ and $\tilde{n}_2$ at resonance are:

$$\frac{\tilde{n}_1}{n_0} = \beta E_2 E_2 \sin(kr - \omega t)$$

$$\frac{\tilde{n}_2}{n_0} = \beta E_1 E_2 \sin[(k + \Delta k)r - \omega t]$$

where $\Delta k$ is the wavenumber mismatch between the waves. Making the usual slowly-varying approximation for the fields (expressed in terms of their rms values) we obtain:

$$\frac{\partial E_1}{\partial r} = \frac{g}{2} E_c [E_2 E_s \exp(-i\Delta kr) + E_1 E_c]$$

$$\frac{\partial E_2}{\partial r} = \frac{g}{2} E_c [E_2 E_s + E_1 E_c \exp(i\Delta kr)]$$

$$\frac{\partial E_s}{\partial r} = \frac{g}{2} E_2 [E_2 E_s + E_1 E_c \exp(i\Delta kr)]$$

$$\frac{\partial E_c}{\partial r} = \frac{g}{2} E_1 [E_2 E_s \exp(-i\Delta kr) + E_1 E_c]$$

with boundary conditions of $E_1(L) = E_{10}$, $E_2(0) = E_{20}$, $E_s(L) = E_{s0}$, and $E_c(0) = 0$. This set of coupled equations is expressed here in the form utilized by Scott$^2$ to explain BEFWM in general, nonlinear media except that the SBS gain coefficient $g$ is now

$$g = \frac{\beta \alpha p_0}{k_c c^2} = \frac{1}{4\pi} \frac{n_0 k c_s r_c}{n_e v_1 M c_s^2}$$

Equation (5) can be solved analytically assuming undepleted pump beams. Under low gain conditions and a short interaction length limit ($\Delta k L \ll 1$), the steady state response is simply $E_c = \frac{1}{2} g E_1 E_2 E_s \omega L$, which is identical to that of the primary grating formed by $E_s$ and $E_2$, yielding

$$E_c(t) = \frac{1}{2} g E_1 E_2 E_s \omega L \left[ 1 - e^{-r_1/\omega} \right]$$

**EXPERIMENTAL RESULTS:**

The experimental studies are performed in a cylindrical chamber$^3$ in which unmagnetized, low density $H_3$ plasmas are formed. Within the chamber are two large microwave horns which launch antiparallel 3.24 GHz pump waves, and a smaller horn which transmits (collects) the signal (conjugate) wave. Due to the $\approx 38^\circ$ tilt angle between the pump and signal waves, both a large-$k$ (formed by $E_s$ and $E_2$) and a small-$k$ (formed by $E_2$ and $E_s$) grating are created in the plasma. The conjugate wave thus results from the scattering of $E_2$ off the large-$k$ grating, and of $E_1$ off the small-$k$ grating.
Figure 1 displays the large-\( k \) ion wave (grating) amplitude, obtained by a 38 GHz scattering system aligned at a 19° tilt angle, and the conjugate wave amplitude as a function of the difference frequency \( \Delta f \) between the signal and pump waves. The solid lines represent least squares fits using Eq. (1), summed over 28°–48° for the scattering data and 32°–76° for the conjugate wave data to account for the range of angles contained within the signal beam. The jump in the fitted responses at \( \Delta f = 0 \) arises from the nonstationary nature of the gratings, as ion waves observed at one location (and tilt angle) were actually formed at another location (and tilt angle) within the plasma.

The scattering data, taken at time \( t = 25 \) μsec after the onset of \( E_s \) and \( E_p \), are found to be consistent with \( c_s = 1.0 \times 10^6 \) cm/sec and \( v_i = 220 \) kHz. The conjugate data, taken at \( t = 14 \) μsec under similar conditions, are consistent with \( c_s = 1.1 \times 10^6 \) cm/sec and \( v_i = 290 \) kHz. Provided in Fig. 2 are sample time-histories of the scattering and conjugate wave signals, taken from the same data sets as Fig. 1. The data demonstrates excellent agreement with Eqs. (3) and (7), as represented by the solid lines in Fig. 2.

Figures 3 and 4 present the scattering signal and conjugate wave power dependence on input pump wave power, at two different times into the high power microwave pulse. The signals show the expected linear dependence with pump power up to a power level of \( \approx 5 \) kW. Figure 5 presents the conjugate wave density dependence with \( P_1 = 10 \) kW, \( P_2 = 7.5 \) kW and \( P_3 = 1.2 \) kW. The conjugate wave amplitude is seen to deviate from linearity near 0.25\( n_e \), disrupted possibly by the large amplitude ion fluctuations often observed in the presence of the \( 2\omega_{pe} \) instability\(^4\), and again at higher densities. The nonlinear effects, exhibited at high pump powers and high plasma densities, are consistent with the onset of ponderomotive whole-beam self-focusing. Although the power levels used here are low, they are sufficient to disrupt the BEFWM interaction by dephasing the wavefronts of the electromagnetic waves as plasma is pushed out of the BEFWM interaction region.
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REFERENCES:
High Frequency Modes of a Plasma Column Loaded in a Sheath helix

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This paper investigates the high frequency ($\omega > \omega_p$, $\omega_e$), slow wave modes ($V_{ph...} < c$) of a magnetized plasma column loaded inside a sheath helix. The system geometry is shown in Fig.1. All relevant quantities are illustrated in the figure. Consider small amplitude waves (varying as $\exp(-i(\omega t-kz+m\phi))$; $m$: an integer) propagating in the system. This problem has been treated in [1], where the general dispersion relation (DR), was derived in connection with the Alfvén wave modes of the system.

The high frequency slow wave solutions of the DR derived in [1], are presented here for the $m=0, \pm 1$ modes ($\omega > \omega_p$). The values of the different parameters are chosen as: helix radius ($r_h$) = 7.35 cm and 3.5 cm; helix pitch ($p$) = 2.5 cm and 0.5 cm; argon plasma density ($n_0$) = $10^{11}$/cm$^3$; wave frequency ($\omega/2\pi$) = 2.45 GHz, axial magnetic field ($B_0$) in the range $0.85 \leq \omega_p/\omega \leq 1.35$. (The wave frequency has been chosen equal to the frequency of the microwave source in our heating experiments.)

Numerical computations show that the DR yields slow wave roots for: (1) $Y_1$ - real, $Y_2$ - imaginary (Type I root); (ii) $Y_1$ and $Y_2$ both imaginary (Type II root). Here $Y_1$, $Y_2$ are the two perpendicular wavenumbers inside the plasma region (see [1]).

Dispersion curves (plots of $V_{ph...}$ versus $\omega/\omega_p$) for the $m=0, \pm 1$ modes, for the parameters given above, are presented in Fig.2. All the waves are slow waves. For large diameter helices, the Type I and Type II roots have separate branches with the Type I roots going to resonance at $\omega=\omega_p$ and the Type II roots propagating up to low values of magnetic field. For smaller $r_h$ some of the Type I roots may transform into Type II roots for $\omega \leq \omega_p$, (as seen for the $m=0$ mode for $r_h=3.5$ cm). No dependence on the plasma radius is shown in these plots, since the dispersion curves are found to be highly insensitive to the plasma column radius.

The radial variations of the field amplitudes are shown in Figs. 3a-3c. The key feature revealed is that the Type I waves are all body waves. The Type II waves are surface waves at low $B_0(\omega/\omega_p \approx 1.17)$ and transform into body waves at
Fig 1. Plasma column of radius $r_p$ situated coaxially inside a sheath helix of radius $r_h$ in free space. $\psi (\cot^{-1} \frac{2\pi r_p}{p}, p$: helix pitch) is the pitch angle. The different regions are also specified.

**Fig. 2.** Dispersion curves ($V_{phase}$ versus $\omega/\omega_{ce}$) for the $m=0, \pm 1$ modes of a plasma column loaded inside a sheath helix. The plots are independent of $r_p$. The vertical line on the $m=0$ mode curve for $r_h=3.5$ cm shows the transition from Type I to Type II root.
Fig. 3: (a)–(c): Radial variation of the normalized O.M field components for the \( m=0, \pm 1 \) modes and Type I, II roots for different values of \( B_0 \).
(d): Radial variation of the index of polarization \( s \). All parameters are the same as given in Fig. 2.
higher values of $B_0$ ($w/w_0 \approx 0.88$). $E_z$ is the largest field component for all the modes; the other significant components are: For the Type I, $m = -1$ mode, $H_x$, $H_r$, and $E_z$ are seen to be the important components; $E_r$ and $H_r$ are seen to be significant for $m = 0$ while $E_r$ and $H_r$ are important for $m = +1$. For the Type II root at low magnetic fields ($B_0 \approx 750$G), $E_r$ and $H_r$ are the significant components for $m = \pm 1$ while $E_r$ and $H_r$ are the important components for $m = 0$. For the Type II root at high magnetic fields ($B_0 \approx 1000$G), $E_r$ and $H_r$ are the significant components for $m = \pm 1$ while $E_r$ and $H_r$ are important for $m = 0$.

The presence of surface waves at low magnetic fields (Type II root) may be exploited for plasma heating in a mirror machine, by coupling to these waves from an external antenna situated at the mirror mid-plane. As the waves propagate to higher B regions, they transform to body waves and the energy in the wave is transported into the plasma interior. At the resonance region ($w\approx w_0$), it is expected that mode conversion to a suitable mode will lead to significant plasma heating.

Fig. 3d shows the radial profiles of the index of polarization $s = |E_r|/|E_\phi|$: left (right) hand components) of the wave fields in the plasma, in the $x$-$y$ plane for the Type I, Type II roots at values of $B_0$ corresponding to those used for the field profiles in Figs. 3a-3c. It is seen that the $m = -1$ modes are RHP (or close to it), inside the plasma and are linearly polarized (LP) at the plasma edge and beyond it. The $m=0$ modes are approximately LP throughout, although the Type II root at $B = 1000$ G shows a tendency towards RHP. (This feature is enhanced for small diameter helices.) The $m=1$ mode is LHP on axis and changes to RHP either inside the plasma, or just outside it. These features of the waves will be of importance in coupling to them from outside the helix region along its axis.

References

ESTIMATION OF ELECTRON TEMPERATURE AND DENSITY
BY DECONVOLVING THE ABSORPTION PART OF
THE PLASMA DISPERSION FUNCTION

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INTRODUCTION

The analysis of the propagation of plasma waves could be
a useful way to estimate plasma parameters such as electron
temperature $T_e$ and density $n_e$. The plasma dispersion
function, $Z(q)$, which arises in the study of linearized
plasma wave propagation in Maxwellian, non-relativistic
plasmas with or without a magnetic field, contains inherently
two types of information which can be described as follows:
1) in the absence of non-linear phenomena, the frequency
spectrum of the wave contains a Lorentzian component which
provides information about the damping of the wave in the
plasma. This damping depends, in general, on $T_e$ and $n_e$;
2) the frequency spectrum of the wave also contains a
Gaussian component which gives information about the thermal
motion of the plasma electrons. Those two components are
convolved with each other. The information contained in the
function $Z(q)$, related to the electron temperature and
density of the plasma, can be extracted if one is able to
find the parameters that characterize this function. In this
work it is suggested that those parameters can be found by
deconvolving the absorption part of $Z(\gamma)$.

PRELIMINARY CONSIDERATIONS

In many spectroscopies, Lorentzian spectral lines determined by the lifetime of the quantum excited states are affected by external perturbations, such as instrumental instabilities and thermal motions. If those perturbations have a random origin, then the absorption line is the convolution of the Lorentzian profile with a Gaussian distribution that represents the perturbing effects. This convolution is proportional to the Voigt profile /1/:

$$H(a, \nu) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(\nu-y)^2 + a^2} dy,$$

where $\nu = [2(\omega - \omega_0)\sqrt{\ln 2}/\Gamma_G]$ is the dimensionless frequency variable, $\Gamma_G$ being the FWHM of the Gaussian distribution in frequency units, and $a = [\Gamma_L\sqrt{\ln 2}/\Gamma_G]$ is Posener's parameter /2/ that measures the relative contributions of the Gaussian distribution and the Lorentzian function to the Voigt profile; $\Gamma_L$ is the FWHM of the Lorentzian.

On the other hand, it has been shown /3/ that the Voigt profile (1) can be written in terms of the imaginary or absorption part of the plasma dispersion function:

$$H(\nu, a) = \frac{1}{\sqrt{\pi}} \text{Im} Z(\gamma),$$

where $\gamma = \nu + ia$. As seen from the definitions of the variable $\nu$ and the parameter $a$, the function $Z(\gamma)$ inherently contains both a Lorentzian contribution representing the line shape of a spectral line of central frequency $\omega_0$, and the perturbing Gaussian contribution that represents the thermal motions in the emitting sample and/or instrumental instabilities. Deconvolution of expression (2) yields the Lorentzian and Gaussian contributions through the values of $\Gamma_L$ and $\Gamma_G$ /3/. For those cases in which the Gaussian contribution is small i.e. $a\gg1$, deconvolution is performed
by using an approximation method /4/.

APPLICATION TO THE DETERMINATION OF
$T_e$ and $n_e$ IN A PLASMA

In this section, a way to determine $T_e$ and $n_e$ by
deconvolving the frequency spectrum of a wave of frequency $\omega_0$
that probes a thermal plasma, is suggested. The wave
amplitude should be small enough to avoid non-linear
phenomena. In this application the Lorentzian contribution to
(1) and (2) represents the line shape of a linearized wave of
central frequency $\omega_0$ that propagates in the plasma, and the
Gaussian contribution represents the Maxwellian velocity
distribution of the plasma electrons. Therefore the above
preliminary considerations, suggest a method to estimate
electron temperature $T_e$ and density $n_e$ of the plasma, based
on the deconvolution of expression (2) to find the values of
$\Gamma_L$ and $\Gamma_G$. As will be seen below, these parameters can be
expressed in terms of $T_e$ and $n_e$.

The complex variable $q$ in the argument of the plasma
dispersion function for wave propagation can be written as:

\[ q = \frac{(\omega - \omega_0) + i\alpha}{k v_{th}}, \]  

where $\omega_0$ is the frequency of the wave and $\omega$ is introduced to
allow for a frequency dispersion; $\alpha$ is the damping constant
of the wave in the medium, $v_{th} = (2k_B T_e/m_e)^{1/2}$ is the
electron thermal velocity and $k$ is the wave number. It is
easy to see that $v_{th}$ can be written as: $v_{th} = \Gamma_G'/2\sqrt{\ln 2}$
where $\Gamma_G'$ is the FWHM of the Gaussian in velocity units.
Therefore (3) is rewritten as:

\[ q = \frac{2(\omega - \omega_0)\sqrt{\ln 2}}{\Gamma_G} + i \frac{\Gamma_L\sqrt{\ln 2}}{\Gamma_G}, \]  

where $\Gamma_L = 2\alpha$, and $\Gamma_G = k \Gamma_G'$ now has frequency units.

According to the definitions of $\omega$ and $\alpha$, the variable (3)
can then be written as \( \zeta = \omega + ia \), i.e. the argument of \( Z(\zeta) \), has been explicitly written in terms of the Lorentzian and Gaussian contributions to the detected line profile of the probing wave.

Reported deconvolution methods /3,4/, can be applied to the transmitted frequency spectrum of the signal of central frequency \( \omega_c \), to obtain the values of \( \Gamma_L \) and \( \Gamma_G \). To complete the analysis and finally obtain the values of \( T_e \) and \( n_e \), what remains to be done is to provide expressions for \( \Gamma_L \) and \( \Gamma_G \) in terms of these parameters. An expression for \( \Gamma_G \) was given above in terms of the wave number \( k \) of the probing signal and of the electron temperature. To provide an expression for \( \Gamma_L \) in terms of \( T_e \) and \( n_e \), let it be assumed that the amplitude of the wave, decays in such a way that \( \Gamma_L \) is the Landau damping:

\[
\Gamma_L = 2\alpha = 2\omega_p \sqrt{\pi/8} \left( k_D/k \right)^3 \exp\left(-3/2 - k^2/2k^2\right), \quad k << k_D,
\]

(5)

where \( \omega_p \) and \( k_D \) are plasma frequency and Debye wave number respectively; these are given in terms of \( T_e \) and \( n_e \). As indicated in (5), this method is only valid if the wave number of the probing waves, to be used in the experiment, are in the interval \( k << k_D \). Experiments to test this proposal are planned to be performed at the Laboratorio de Física de Plasmas, ININ, México.

Kinetic Description of Modulated Electromagnetic Wave in Relativistic Plasma

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Abstract

Evolution of a modulated electromagnetic plane wave in a collisionless, neutral, unmagnetized plasma is investigated. Particles are assumed to move with relativistic speeds, corresponding to hot or fast moving plasmas, with macroscopic velocities slowly varying in space-time.

A kinetic description of the system on the slow scale is constructed from a Hamilton variational principle connected with the oscillation center transformation. Expansion in a small parameter corresponding to weak dispersion and weak nonlinearity is carried out. In the highest order a generalization of the nonlinear Schrödinger equation is obtained. Modulational instability due to particles resonating with the group velocity is analyzed.

The modulational instability results are compared with Kates and Kaup results [1] for the electromagnetic waves in a relativistic, multispecies fluid plasma.

Introduction

Recent interest in relativistic plasmas was our motivation to generalize Dewar's description of an electrostatic wave in nonrelativistic plasma [2] on a covariant analysis of a full electromagnetic wave and relativistic kinematics of plasma particles.

Canonical formulation of relativistic plasma theory [3] was a basis for construction of a systematic procedure eliminating fast oscillations.

The assumptions

A separation of space-time scales takes place. Four-vector of the electromagnetic field potential $A_0$ as well as particle space-time positions $x_0$ and four-momenta $\pi_0$ can be devided into slowly varying parts ($x$, $\pi$, $A$) and rapidly oscillating ones:

$$x_0 = x + \xi(x, \pi)$$
$$\pi_0 = \pi + D\xi$$
$$A_0 = A + A$$

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where \((x,\pi)\) are slowly varying phase space coordinates of a fictitious oscillation center (to be found), \(D\) is the derivative along the oscillation center trajectory.

The wave is nearly monochromatic, with weak dispersion and weak nonlinearity:

\[
A = \left[ cA_{11}(ex) + c^2A_{12}(ex) + c^3A_{13}(ex) \right] e^{i\theta} + c^2A_2(\varepsilon x)e^{2i\theta} + c.c.
\]

\[
\theta = k \cdot x,
\]

\(k = \text{const}(x)\) (4)

The self-consistent field \(A\) scales as: \(A = c^2A(\varepsilon x)\).

The goal is to determine evolution of the system on the slow scale, averaging out fast oscillations.

**Action principle for single particles and fields**

The Hamilton action principle is formulated in 8 dimensional phase space of canonical coordinates \((r,p)\) [3]:

\[
\delta S = 0, \quad S[\gamma] = \sum_{\text{particles}} \int[p \cdot dr - h(r,p)dr] + \int d^4x \ell^f
\]

where \(\tau\) is a parameter along a particle trajectory \(\gamma\) and \(\ell^f\) is the free field Lagrangian density.

The 8 dim Hamiltonian \(h\) has a form

\[
h(r,p) = \frac{1}{2m}[(p - \frac{e}{c}A_0)^2 + m^2c^2]
\]

The variational principle has to include constraints, namely, the confinement of a particle motion to the mass shell:

\[
h(r,p) = 0
\]

and the Lorentz gauge for the components of \(A_0\).

**Oscillation center transformation**

The oscillation center (OC) transformation \(T\) [4]:

\[
T : (x_0, \pi_0) \rightarrow (x, \pi)
\]

is a Lie transform, consisting of subsequent infinitesimal canonical transformations. They are determined by the requirement that the transformed OC Hamiltonian \(H\)

\[
T^{*-1} : \sum_{n=0}^{4} e^n h_n = h \rightarrow H = \sum_{n=0}^{4} e^n H_n
\]

is practically not oscillating (contribution from oscillations is of order \(\varepsilon^5\)). The form invariance of Hamilton equations under OC transformation gives the required result: transformed phase space coordinates of particles \((x,\pi)\) do not oscillate on the fast scale. These new coordinates
determine the 'averaged' particle positions in phase space, corresponding to positions of fictitious oscillation centers.

Electromagnetic fields are not transformed in this approach.

**Action principle for oscillation centers and fields**

The interaction part of the averaged action is obtained through the OC transformation and it preserves the same form as in Eq.(6), due to the properties of the canonical transformations. The field Lagrangian density $\mathcal{L}_f$ is picked up to be a space-time average, taken over the slow scale, of the well-known free field action $I_f$ from Eq.(6):

$$\delta S = 0, \quad S[\gamma] = \sum_{\text{OC}} \int [P \cdot dR - H(R, P) d\tau] + \int d^4x \mathcal{L}_f$$

where $\gamma$ is the OC trajectory. Variations are taken with respect to OC trajectories and field amplitudes $A_{1i}, A_{2i}, A_i, i=1,2,3$.

**Statistical approach**

The single oscillation centers approach is equivalent to the description in terms of one oscillation center Klimontovich distribution function $f(R, P)$.

The Klimontovich approach leads to the replacement

$$\sum_{\text{OC}} \int d\tau \xi(R, P) \rightarrow \sum_{\text{species}} \int d^4R \int d^4P f(R, P) \xi(R, P)$$

The Hamilton equations are replaced by the Ignatiev equation:

$$\{f, H\} = 0$$

with the 8 dimensional phase space Poisson brackets.

The next step is to generalize $f$ to be a smooth Vlasov OC distribution function. In this way (13) becomes the Vlasov equation.

Consistently, $f$ is expanded as $f = f_0 + \epsilon^2 \delta f$, where $f_0 = f_0(P)$ is the 'OC global equilibrium' distribution function.

This leads to the linearized Vlasov equation:

$$\frac{P}{m} \frac{\partial f_0}{\partial R} \frac{\partial \delta f}{\partial P} - \frac{\partial f_0}{\partial P} \frac{\partial H_2(R, P)}{\partial R} = 0$$

which can be formally solved for $\delta f$ in terms of the field amplitude $A_{1i}$.

**Evolution equations**

We finally get a system of equations for the quantities $A_{1a}(X_1, X_2), a = 1, 2, 3$.

The notation is:
\[ A_{11a} = e_a \cdot A_{11}, \quad e_a \cdot k = 0, \quad |e_a| = 1, \]

\[ X_1 = \varepsilon x, \quad X_2 = \varepsilon^2 x, \quad \varepsilon \partial_{\mu} A_{11a} = \frac{\partial}{\partial X_1} A_{11a}, \quad \varepsilon^2 \Delta_{\mu} A_{11a} = \frac{\partial}{\partial X_2} A_{11a} \]  

(15)

The equations are:

Dispersion relation: \[ D_{\varepsilon} A_{11a} = 0 \]

Propagation with a group velocity: \[ V_{\varepsilon}^2 \partial_{\mu} A_{11a} = 0 \]

The highest order equation:

\[ iV_{\varepsilon}^2 D_{\mu} A_{11a} + \sum_b H_{ab}^H \partial_{\mu} \delta A_{11b} + \delta D_{\varepsilon} \delta f] A_{11a} + \omega_a A_{11a} \sum_b |A_{11b}|^2 + \varepsilon A_{11a} \sum_b A_{11b}^a = 0 \]  

(16)

Explicit expressions for the coefficients are given in [5].

Modulational instability

The detailed analysis of Eq.(16) is given in [5]. It leads to the conclusion that particles resonating with the group velocity cause a generic modulational instability. This result is new in comparison with modulational stability obtained by Kates and Kaup [1] in their relativistic fluid plasma model.

The longitudinal and transverse waves are treated separately.

For the longitudinal polarization to get a modulational dispersion relation the standard Fourier analysis of \( A_{11} \) into coupled modes (the large carrier wave and two small sidebands) is carried out: \( A_{11a} = A_{11a}^0 + \alpha_a \exp(iK \cdot X_1) + A_{11a}^\ast \exp(-iK \cdot X_1) \)

For the transverse polarization the four-vector \( k \) is showed to be timelike. Thus \( D \) in Eq.(16) is real. Therefore the fields \( A_{11a} = |A_{11a}| \exp(i\theta_a) \) and the Eq.(16) can be simply linearized around a constant solution (of linear or circular polarization), and next Fourier transformed to give the modulational dispersion relation.

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References

Topic 11:

Partially Ionized Plasmas
EXPERIMENTAL EVIDENCE OF SELF-ORGANIZATION
AND CHAOS IN SOME PLASMA DEVICES

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The presence of well-localized globular luminous space charge structures (GLSs) and
their ability to act as energy sources for ordered and disordered phenomenon were already
discussed [1,2]. Here we comparatively present experimental results which were obtained on
three different plasma devices in which GLSs appear. The results suggest that the GLSs
.generated through a self-organization process) exist in all plasma devices where self-sustained
time dependent phenomena could be observed.

First, the old problem of the dc glow discharge property to generate periodical vari-
atations of the dc current [1] is resumed. We want to show, that in reality the solution of this prob-
lem is possible only involving a self-organization phenomenon while kinetic energy of the ac-
celerated electrons is transformed in energy stored

in the electric field localized in the SCS via quantum processes. Because the GLS becomes movable
and disrupt when the potential drop $V_{DL}$ across SSC riches the ionization potential $V_{i}$ of the gas, the
cumulated energy is released and could sustain time dependent phenomena in the electrical
circuit including the GLS. Thus the periodical modulation of the glow

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Static I-V characteristic including the negative branch (forming phase of the planar DL) and the suddenly increase accompanying the transition to an unstable globular DL.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Dynamical I-V characteristic which evidences the transition from a stable planar to an unstable globular DL.}
\end{figure}

discharge current known as anode instabilities observed under conditions when its positive parts are contracted in a GLS in reality is produced by oscillations taking place in a proper oscillatory circuit [1,2]
In which the GLS act as a negative resistance (NR). The forming phase of the GLS can not be evidenced in the I-V characteristic of a dc discharge produced in a glass tube because these simultaneously appears in its final (unstable) state, with the discharge ignition. However, using a metal tube whose potential is changed with respect to the anode one obtains the results shown in fig.1. The negative branch corresponds to the phase during which in front of the anode a planar one and then a stable GLS (Fig.2) is formed. The direct transition from a stable to an unstable GLS, which takes place when \( V_{DL} = V_i/1.3 \) is evidenced in the dynamic I-V characteristic presented in fig. 2. When \( V_{DL} = V_i \) the GLS disrupts after it has expanded and is periodically reformed /1,3,4/. Because during the unstable state of the spherical GLS free charges are produced, the dc suddenly increases (fig.1). The parameters of the observed oscillation are mainly determined by the reactive and ohmic circuit elements of the proper oscillating circuit /2/. In fig.3 we present the equivalent circuit which explains the oscillatory behaviour of a dc glow discharge.

Another self-organization phenomenon has been observed in a low pressure plasma column produced by a plasma source PS (radio-frequency glow discharge). The so obtained plasma diffuses through a grounded grid G into a glass tube GT, forming a plasma column (Fig.4). When a current collecting positive biased planar electrode A is placed at the end of the column, a quasi-stationary GLS appears at a certain distance from it. The space-time in-
vastigation of this structure, reveals a space-time ordered phenomenon for both the axial electric field and the light emitted from this structure which proves that the GLS behaves in a pulsating state (Fig.5).

This phenomenon was accompanied by oscillations of both the main current collected by the anode and the capacitive wall current. For certain voltages (60 V) and pressure (10^{-3} Torr), it was possible to obtain even two spherical structures. In this case a period doubling phenomenon could be observed in the current variations. The static current-voltage characteristics show three hysteresis regions (Fig.6), each of them referring to different states of the GLS.

The results prove that the GLS is surrounded by a double layer (DL) which is responsible for its space-time evolution. This evolution is essentially based on quantum processes such like excitation and ionization.

New experimental results about the connections between the chaotic behavior of a plasma device and the self-organization phenomena (DL structure) were obtained using the experimental device presented in fig.7. A single DL or a multiple one, in stable or unstable state, is formed at the contact region between the two negative glow plasmas, when
between them a dc voltage is imposed /3/.

In the I-V characteristic shown in Fig. 8 different behaviours of the device are pointed out (ST-stable state; T, 2T, CH single period, double period and chaotic oscillations respectively). The I(t) - I(t+\tau) phase plots for the ordered and disordered current variations together with the corresponding amplitude frequency characteristics are presented in Fig. 9.

The local temporal variations in the chaotic state of the device representing the potential of four capacitive probes placed in four different places in A_1A_2 region, are presented in (Fig. 10). The contact region between the two unstable DLs seems to be the localization of the "source" for such a chaotic behaviour. The period-doubling phenomenon is observed when the second DL appears after the first one and has a correlated dynamics with it /5/.

Fig. 10 Simultaneous time-dependences of the signals from four capacitive probes equidistantly placed in the A_1A_2 forming-DL region (I_{12}=3.5 mA, I_1-I_2=4 mA, P=0.07 Torr).

The chaotic behaviour appears when the formation and disruption processes of the two DLs are not correlated and are controlled by aleatory processes. The portions of negative resistance on the I-V characteristics prove this.

TOMOGRAPHIC TIME-RESOLVED RECONSTRUCTION OF THE THREE DIMENSIONAL ELECTRON VELOCITY DISTRIBUTION IN A PARTIALLY IONIZED RF PLASMA

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With a rotating plane probe immersed in a rf plasma, time variations of the electron velocity distribution in three-dimensional velocity space have been studied. The angular rotation of a plane probe at a spatial point in a plasma may provide sufficient information for numerically reconstructing the three-dimensional shapes of the functions by computerized tomography.

The knowledge of the velocity distribution functions of electron in gas discharges is important for fundamental understanding of plasma processes which are applied to make new materials and to fabricate microelectric devices in industry. Electrostatic probes (Langmuir probe) and electrostatic energy analyzers (Faraday cup) are widely used as valuable diagnostic tools for studying the properties of electrons and ions in plasma. Due to the lack of angular resolution, however, it is difficult to directly measure the velocity distributions of charged particles as functions of all three velocity-space coordinates. To overcome this difficulty, a technique of computerized tomography (CT) has been proposed by Lehner [1] and Iwama et al. [2]. The angular rotation of a usual type of analyzer at a spatial point in plasma may provide sufficient information for recovering the three-dimensional shapes of the functions with numerical processing.

The electron current of a plane probe with bias voltage \( V \) is expressed in terms of the projected (one dimensional) electron velocity distribution function \( f_{1D}(u; n) \) in a spatial direction of \( n \), as

\[
I_e(V) = Se \int \frac{u f_{1D}(u; n) du}{\sqrt{(2eV)/m_e}}.
\]

where \( u \) is the unit normal to the probe surface, \( S \) the probe area, \( e \) the electron charge and \( m_e \) the electron mass. Taking the first derivative with respect to \( V \), we have

\[
f_{1D}(u; n) = -\frac{m_e}{Se^2} \frac{d I_e(V)}{dV}.
\]

Here the function \( f_{1D}(u; n) \) is defined as the Radon transform of the three dimensional velocity distribution function \( f_{3D}(v) \):

\[\text{Fig. 1: Planar projection } f_{1D}(u; n) \text{; the integration of } f_{3D}(v) \text{ over a } n \text{-plane which is designated by } u = n \cdot v.\]
\[ f_{1D}(u; n) = \int f_{3D}(v) \delta(u - n \cdot v) \, dv \]  

with the Dirac delta function \( \delta() \) and schematically shown in Fig. 1. If \( f_{1D}(u; n) \) is obtained as a function of velocity \( u \) in each of the directions of normals \( n = n_j \) \((j=1,2,..., J)\), then the numerical recovering of \( f_{3D}(v) \) as a function \( v \) is analogously a problem of CT \( /3,4/ \) in order to reconstruct the three-dimensional object from its planar projections.

The experimental apparatus is shown schematically in Fig. 2. Two parallel plate electrodes (stainless steel, 12cm diameter, 4cm spacing) covered with Pyrex glass except for the discharge area are positioned in the center of the chamber (stainless steel, 16cm inner diameter, 50cm length). A plane single probe (molybdenum, 3mm diameter) which is rotational in the azimuthal direction is inserted at the mid center of discharge region, as shown in Fig. 3. The chamber is always pumped out by using a 200 liters/s diffusion pump. Argon gas is used whose flow rate is arranged so as to hold pressure at 0.4 Torr. Low frequency pulsed rf power whose frequency is 30kHz, repetition frequency 1kHz and duty ratio 1/1 is applied to the electrodes. An unbalance-balance transformer is used in order to reduce the potential difference between the probe and the plasma; plasma space potential is fluctuating with the doubled frequency and its magnitude is nearly equal to anode potential. Therefore, the rectified rf voltage applied to electrodes is superposed on the dc bias voltage of probe. To achieve a time-resolved probe diagnostics a sampling converter is employed; the phase to sample data is manually set, whose gate pulse is synchronous with a reference signal of the rf generator. Probe characteristics are digitized by using a 12 bits A/D converter and then recorded on a personal computer. The first derivative with respect to dc bias voltage is obtained numerically by utilizing smoothing technique of B-spline function.
Waveforms of the discharge potential and current of each rf electrode, and the probe current at different dc bias voltage are shown in Fig. 4(a), (b) and (c), respectively. The potential of each electrode is symmetrically alternating. The discharge voltage is equal to the potential difference between these electrodes. The phase difference between each discharge current and applied rf potential is almost zero. This indicates that the discharge characteristic is similar to the dc discharge one in which loading impedance is purely resistive. The probe current with dc bias -10V is corresponding to the ion saturation current and that with +10V corresponding to the electron retarding current. It is found that the superposed rf potential on probe dc bias forces the probe potential to follow well the time-varying local plasma space potential.

The experimental data analyzed below are obtained at the phase \( t=t_1, t_2 \) and \( t_3 \).

As illustrated in Fig. 3, the probe was rotated, over the range of \( 0 \leq \theta \leq 2\pi \) in the \( \phi=0 \) plane. For each of the 129-values of \( \theta_j = (j-1)2\pi/12 \) \( (j=1,2,...,12) \), probe characteristics were measured and numerically differentiated with respect to dc probe bias voltage. Since ion current was estimated to be very small and ion temperature to be very low, differentiated characteristics were approximately regarded as one-dimensional electron velocity distribution functions projected to \( \theta \) direction in the three dimensional velocity space. Combining a pair of the differentiated curves for \( \theta = \theta_j \) and \( \theta = \theta_{j+6} \) at the plasma potential, we get a \( f_{1D}(u; n_j) \) curve in one polar direction of \( \theta = \theta_j \) \( (j=1,...,6) \). Consequently, we obtained 6 series of projection data over the range of \( 0 \leq \theta \leq \pi \).

Reconstructed electron velocity distributions by the ART (Algebraic Reconstruction Technique) are shown in Fig. 5, and Fig. 6(a), (b) and (c). Contour plot at the phase \( t=t_1 \) during discharge current increases is drawn in Fig. 5, where outward islands are

**Fig. 4** Discharge characteristics and time varying probe current: waveforms of (a) potential and (b) current of each rf electrode, and (c) probe current at dc bias -10V and 10V, 0.4Torr(Ar), 30kHz.

**Fig. 5** Contour plot of three-dimensional reconstruction of the electron velocity distribution function by ART method. Sampling phase is \( t=t_1 \) when discharge current is growing with time, as shown in Fig. 4(b).
meaningless, reconstruction errors due to lack of projection data. It is found that the electron velocity distribution is not isotropic, extended toward the inverse direction of the rf electric field. This distortion appears clearly only at the phase during the rf discharge current increases.

The flow of electron beams move towards the electrode with positive potential is interpreted as that of primary electrons to cause ionization. Here, the dip on the top is considered to indicate that the plane probe can not collect the whole electron current near the plasma space potential.

Time-resolved three-dimensional drawings by wire frame are shown in Fig. 6(a), (b) and (c), where sampling phases are $t=t_1$ during discharge current increases, $t=t_2$ at the maximum current and $t=t_3$ during current decreases, respectively. It is found that the electron velocity distribution function changes at the twice frequency of the applied rf; in a half period, the distribution starts from a non-isotropic function and after reaching to an isotropic one relaxes into a low-temperature isotropic profile.

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Fig. 6 Reconstruction of three dimensional electron velocity distribution function, by CT on the basis of ART method. The profiles are plotted in the two-dimensional velocity space with coordinates $v_z = |v| \cos \theta$ and $v_r = |v| \sin \theta$. (a) $t=t_1$ during discharge current increases, (b) $t=t_2$ at the maximum and (c) $t=t_3$ during decreases.
LARGE AMPLITUDE STRIATIONS IN AN RF DISCHARGE

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1. Introduction

R. T. Schneider and P. H. Handel have observed pairs of luminous disks separated by a dark-gap and emission of fast neutrons in a low-power radio-frequency (RF) discharge in deuterium at a pressure range of 0.1 — 1 Torr /1/. The disks exist only in pairs. This structure has been interpreted as a plasma caviton, induced by a resonance between applied RF field and the ion oscillation /1, 2/. It is interesting to investigate such a plasma structure from the viewpoint of particle trapping and acceleration.

In this paper, we present experimental data on paired luminous rings (not disks) observed in an RF discharge.

2. Experimental Setup

Figure 1 shows the experimental setup. An RF voltage of \( V_{RF} = \pm 0.5 - \pm 1.7 \) kV is applied to electrodes (100 mm in width) surrounding a 100 mm diameter of pyrex glass tube containing hydrogen gas. The RF voltage is pulse-modulated with a variable pulse length \( \tau \) (typically \( \tau \approx 150 \) \( \mu \)s) and a repetition frequency. The RF frequency range is \( f = 0.8 - 5 \) MHz. In our experiments, the parameters are quite similar to Ref. 1 expect for the higher pressure (\( p = 1 - 50 \) Torr) and the smaller electrode separation (\( d = 20 - 100 \) mm).

3. Results and Discussions

When the pressure is increased, a pair of luminous rings (each of them is like Saturn’s ring) separated by a narrow dark-gap is produced at the middle of two electrodes (see Fig.1), at a critical value of pressure \( p_1 \). The dimensions of the rings observed by a scanning diode-array camera (SDAC) are as follows: the width of each ring is \( \sim 1 \) mm, the outer diameter is \( \sim 80 \) mm, the inner diameter is \( \sim 50 \) mm, and the dark-gap width of paired rings is \( \sim 2 \) mm. By increasing the pressure up to the next critical value \( p_2 \), two pairs of rings are abruptly produced. Photographs of 1, 2 and 3 paired rings at \( p = p_1, p_2 \) and \( p_3 \), respectively, are shown in Fig. 2. The rings are axially symmetric with respect to the midplane of two electrodes and produced only in pairs. Figure 3 shows the pressure dependence of the ring position which is observed by the SDAC. In the figure, the appearance of \( n \) pairs of rings at critical pressures \( p_n \) (\( n = 1, 2, ..., 9 \)) are shown.

Figure 4 shows a time evolution of the axial light intensity profiles when the pulse length of the RF voltage \( \tau \) is changed from 10 to 55 \( \mu \)s at \( p = p_9 \), the pressure at which 5 pairs of rings exist at \( \tau > 55 \) \( \mu \)s. No rings are observed at \( \tau < 40 \) \( \mu \)s [Fig. 4(a)]. By
increasing $\tau$, two paired rings appear near the both electrodes at $\tau = 40 \mu$s [Fig. 4(b)]. At $\tau = 45 \mu$s, another two paired rings appear inside the first two ring pairs [Fig. 4(b)]. Finally, the fifth pair appears at the midplane at $\tau = 55 \mu$s [Fig. 4(d)]. With further increase in $\tau$, the number of pairs remains at 5, i.e., the structure is in the stationary state.

It is found from Fig. 3 that the distance from the electrode-edge ($x = \pm 15$ mm) to the outmost ring, $L$, and the distance between neighboring pairs, $l$, are inversely proportional to the pressure $p$. The measured values of $L$ are very close to the calculated RF excursion length of electrons, $X_e = eE_{RF}/m_e\nu_e\omega \propto V_{RF}/p$, where $E_{RF}$ is the RF electric field strength and $V_{RF}$ is the RF voltage applied to the electrodes. The measured values of $l$ are at the same order as the ionization mean-free-path of electrons, $\lambda_{ionize} = 1/n_n\sigma_{ionize} \propto 1/p$, where $n_n$ is the neutral gas density and $\sigma_{ionize}$ is the ionization cross-section. These results show that the RF excursion length and the ionization mean-free-path might be important parameters in understanding the mechanism of the luminous ring formation.

When there are no rings, the probe characteristics give the plasma density $n \approx 10^9 - 10^{10}$ cm$^{-3}$, the plasma electron temperature $T_e \approx 6$ eV and the floating potential $V_f \approx -20$ V. Once the rings appear the floating potential becomes more negative. Axial profiles of $V_f$ with and without rings are shown in Fig. 5. The $V_f$ profile with a ring pair ($\tau = 500 \mu$s) shows a negative dip at the position of the dark-gap between paired rings. Assuming that the $V_f$ shown in Fig. 5 indicates the time-averaged plasma potential, the calculated space charge density $(n_e - n_i)$ at the dark-gap, where the potential profile has a negative dip, and at the rings, where the potential has a positive peak, are at the order of $-10^8$ cm$^{-3}$ (electron rich) and $10^8$ cm$^{-3}$ (ion rich), respectively.

In our experiment no neutrons were observed. This may be caused by the difference in the pressure regions and the electrode separation from Ref. 1.

References


Figure Captions

Fig. 1: The experimental setup.
Fig. 2: Photographs of 1, 2 and 3 pairs of rings.
Fig. 3: The pressure dependence of the axial ring position (electrode separation $d = 30$ mm, $f = 1.5$ MHz, $V_{RF} = \pm 1.0$ kV, $\tau = 150$ $\mu$s).
Fig. 4: The time evolution of the axial light intensity profiles ($d = 30$ mm, $f = 1.5$ MHz, $V_{RF} = \pm 1.0$ kV, $p = p_0 = 7.5$ Torr).
Fig. 5: Axial profiles of the floating potential with ($\tau = 500$ $\mu$s) and without ($\tau = 30$ $\mu$s) rings ($d = 30$ mm, $f = 1.5$ MHz, $V_{RF} = \pm 1.0$ kV, $p = p_1 = 6.15$ Torr).
IONIZATION FRONT STRUCTURE IN THE LATERAL SPREADING OF A D.C. DISCHARGE ALONG CYLINDRICAL COAXIAL ELECTRODES

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The lateral spreading of the breakdown ionization front (IF) in glow discharge, along cylindrical coaxial electrodes, was primarily studied by Steenbeck /1/ and later by Emeleus and von Engel /2,3/. An edged field model was taken into account to explain axial spread by axial drift of ions in the stray edge fields. Mather et al. /4/ extend such experiments to large currents and voltages, where the (IF) is comparable in axial extension with the main current sheet. Such structure might explain the rundown plasma stage by j x B force in so called snow-plough model. However, despite of large difference between the current intensities, in the above experiments performed in similar geometries of the electrodes and the same range of the gas pressure, the (IF) propagates along the axial structure with comparable velocity ($10^4$ m/s). That's why many reports pointed out that a better spatial and temporal resolution would be necessary to clarify the mechanism of lateral spreading.

Accordingly to this purpose Popa et al. /5/ started a set of experiments which have been lead to acquire an improvement on temporal and spatial resolution of the (IF) speed-up and on the accuracy of velocity measurements. The technique used to plot these results was the one proposed by Stirand /6/ for ionization waves studies.

Following the same line, in this paper, experimental results are presented, which have been obtained by electrical and optical means, in order to study the space-time structure of the (IF), to get detailed information about acceleration mechanism.

For this purpose, a segmented cathode was used (40 segments of 10 mm length each) and the axial distribution of
the discharge current with the above spatial resolution was registered by a box car technique, at different time intervals after the discharge ignition at one end of the structure.

Axial distribution of the light intensity was also measured by a similar technique, during axial spreading, for different gas pressures (0.1 - 2 mbar), discharge voltages (300 - 400 V) and external resistances (10² - 10⁴ Ω).

The experimental set-up is shown in fig.1: it consists of two cylindrical coaxial stainless steel electrodes (segmented cathode C - 10 mm inner diameter and anode A - 4 mm exterior diameter, both of 400 mm length) placed in a glass tube of 300 mm diameter.

A normal d.c. discharge is ignited by means of a trigger pulse (TP) at one end of the structure and spreads lateral along the two cylinders with a speed of about 10⁴ m/s. The repetition time of the discharge ignition was around 10 ms. Light emission from the discharge in visible range of spectrum had been observed through a narrow slit (1 mm broad) using a photomultiplier (FM) which had been moved along whole length of the cathode /5/. The current flowing through each one O-ring cathode had been measured independently and accordingly to the axial distance from the discharge ignition place, fig.1.

The measurements performed using a box car technique show a close connection between the spatial structure of the (IF), the spatial distribution of the total discharge current and the external working conditions. Two patterns of the (IF) evolution have been established, with the upper limitation of the total discharge current as a specification. A spatial structure of both, current flowing in the (IF) and light intensity have been observed.
Figure 2 shows a space-time diagram of the current, flowing through each D-ring cathode at different time, over whole discharge (40 cm run out). Total discharge current was limited by an external resistor (\( U = 380 \text{ V}, \ I \) less than 0.5 A). This current increases linearly with axial spreading of the discharge (see Fig. 3, curve I) a fact that shows that the current of each ring remains constant during lateral spreading of the discharge.

For this behaviour of the discharge the space-time dependence of the (IF) spreading is indicated in Fig. 4, curve I. The spread velocity is a monotonically decreasing function on the axial distance from the discharge ignition point. The (IF) exhibits a distinct spatial structure that remains compact during the axial spreading and could be a double layer. Figure 5 shows electrical (bottom trace) and optical (upper trace) signals, registered by the above mentioned technique at 15 cm from the discharge ignition point. Their shape shows an identical time evolution with a
peak in the (IF) region. This peak increases considerably when total discharge current increases, in comparison with radial current flowing and light emission behind the (IF). Whenever total current was limited both, by an external resistor and the power supply (see Fig. 3, curve II), spatial structure of the (IF) was considerably extended (see, Fig. 6). Time dependence of the (IF) spreading in this case is also changed (see Fig. 4, curve II). Both spatial structure of the (IF) and the range values where axial velocity of the (IF) lies ($10^4$ m/s) in lateral spreading of glow discharge with coaxial electrodes (discharge current less than 1 A), suggest that for such like experiments but at very large discharge currents (e.g. plasma focus, plasma guns) another mechanism than $j \times B$, in plasma spreading between coaxial electrodes, should be also involved.

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Plasma heating in low pressure RF discharges

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Introduction

In the present paper we consider the interaction of electrons with an oscillating sheath as a possible mechanism for plasma heating in low pressure radiofrequency discharges. The heating rate is found to be considerable and exceeds the rate of the energy loss due to electrons escaping across the sheath and recombining on the electrode surface.

Sheath model

We are interested in the range of discharge parameters where the field frequency \( \omega_{\text{RF}} \) is much greater than the ion plasma frequency \( \omega_p \) and much less than the electron plasma frequency \( \omega_{pe} \). The following assumptions are made about the particle behaviour in the one-dimensional RF sheath:

1. The electrons have a Maxwellian velocity distribution characterized by a temperature \( T_e \) and they follow the instantaneous electric field;
2. The ions respond only to the time-average electric field and their motion within the sheath is collisionless. It is assumed that as in the DC sheath model ions enter the sheath with the Bohm speed \( u_B = \sqrt{kT_e/M} \), where \( M \) is the ion mass.
3. We neglect ionization in the sheath, bulk recombination and secondary emission from the electrode.

In the calculation of the sheath potential we follow the work of Stekolnikov et al [1]. The potential waveform in the sheath is assumed to have the following form:

\[
\phi(x,t) = \phi(x) + \Phi(x) \sin \omega t
\]

The dimensionless variables \( \eta = -e\Phi / kT_e \) and \( \xi = x/\lambda_p \) are used for the potential and the distance. Taking into account the equations of the flux and energy conservation for ions, and also Poisson’s equation for the potential, we obtain (for details see reference [1]) the following system of coupled differential equations for the nondimensional time-average and oscillating parts of the sheath potential:

\[
\begin{align*}
\frac{d^2 \eta}{d\xi^2} &= (1 + 2\eta)^{-h} \exp(-\eta) I_h(-\eta) \\
\frac{d^2 \eta}{d\xi^2} &= -2\exp(-\eta) I_1(-\eta)
\end{align*}
\]
where \( I_0(x) \) and \( I_1(x) \) are the modified zero and first order Bessel functions, respectively.

The numerical solution of the system (2) is obtained starting from the analytical solution in the limit \( \eta \rightarrow 1 \). The boundary values at the electrode surface have been chosen to satisfy the condition of zero net current. The results for \( \eta = -50.0 \) are presented in Fig. 1.

**Electron motion in the sheath**

The electron motion in the time-dependent sheath potential is investigated by means of the numerical solution of the electron motion equation:

\[
m_e \frac{d^2 s}{dt^2} = -e \left. \frac{\partial \Phi(x,t)}{\partial x} \right|_{s,0}
\]

in the following potential:

\[
\Phi(x,t) = \Phi(x) + \Phi(x) \sin(\omega t + \theta)
\]

This equation has been solved for different values of the initial phase angle \( \theta \) and the initial electron energy \( \varepsilon = E/kT_e \) (it is used in the dimensionless form). \( E \) is the component of the electron kinetic energy which is associated with motion perpendicular to the sheath edge.

The computations show that the result of the electron interaction with the RF-sheath depends significantly upon the initial phase \( \theta \). Fig. 2 shows the trajectories of two electrons with the same initial energy \( \varepsilon = 1 \) which penetrate the sheath with different phases (\( \theta = 1.50 \pi \) and \( \theta = 0.45 \pi \) respectively). It can be easily seen that the time of the e-sheath interaction and the depth of penetration in the sheath are very different for these two electrons. It is also clear from the slopes of the trajectories that the first electron on leaving the sheath has almost the same energy as it had before the interaction, while the second one gains significant energy.

Fig. 3 shows the ratio of the electron energy on leaving the sheath to that on entering (curve 1) and the e-sheath interaction time (curve 2), as a function of the phase \( \theta \) for electrons with initial energy \( \varepsilon = 1.0 \). The energy of an electron can rise rapidly if it penetrates the RF-sheath at the moment when the retarding potential is very low. Such an electron spends a large time inside the sheath and penetrates deeply into it. (curve 2 in Fig. 2; peak on the curve 2 in Fig. 3). The energy of those electrons that face a large retarding potential on entry changes very little, and the time of the e-sheath interaction is short.

Finally, we have calculated the heating and cooling fluxes to estimate the energy balance in an RF-sheath. The expression for the heat flux can be written as follows:

\[
f_h = n_e \frac{(kT_e)^{3/2}}{(2\pi m_e)^{1/2}} \Delta \varepsilon \left( \frac{1 - N_{ph}(\varepsilon)}{N(\varepsilon)} \right) \exp(-\varepsilon) d\varepsilon
\]

where \( \Delta \varepsilon \) is the average over all \( \theta \) of the increase in the nondimensional energy of the
electrons on returning to the plasma; \( n_e \) is the electron concentration at the sheath-plasma boundary; \( N_w/N \) is an average over the phase angle of the ratio of the number of the electrons that recombine on the wall to the total number of the electrons with energy \( \varepsilon \) which enter the sheath.

Some electrons have enough energy to reach the electrode surface and recombine on it. The expression for the heat loss (cooling) flux due to these electrons has the following form:

\[
J_c = n_e \frac{(kT_e)^{3/2}}{(2\pi m_e)^{1/2}} \int_0^\infty \frac{N_\varepsilon(\varepsilon)}{N(\varepsilon)} (\varepsilon + 1) \exp(-\varepsilon) d\varepsilon
\]  

(6)

Fig. 4 represents the expressions under the integral signs for the heating (curve 1) and cooling (curve 2) fluxes. It can be seen that the total heating flux is significantly larger than the cooling flux in this parameter range. The input of energy to the ions is a different matter; it also occurs via the sheath [5].

Discussion

The results obtained differ from those in [2] where the energy gained in the e-sheath interaction is a small fraction of that lost by those electrons which escape across the sheath. The reason for this discrepancy is that the sheath in [2] is assumed to be weakly modulated like a sheath at a large grounded electrode, while the heating obtained here is associated with a strongly modulated sheath such as that at a driven electrode. The final conclusion that e-sheath interaction is an important electron heating mechanism agrees with the results obtained by other authors [3,4]. However, an estimate of the heating flux due to the secondary emission from the electrode surface shows that this mechanism can also be significant.

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References

Fig. 1. The sheath potential profile
1, $\eta(\omega t=3\pi/2)$; 2, $\bar{\eta}$;
3, $\eta(\omega t=\pi/2)$; 4, $\bar{\eta}$.

Fig. 2. The trajectories of two electrons with energy $\varepsilon=1$ entering the sheath with the phases
$\theta=1.50\pi$ and $\theta=0.45\pi$ respectively. $\omega_p/\omega_R=42$.

Fig. 3. The ratio of the electron energy on leaving the sheath to that on entering (curve 1) and the e-sheath interaction time (curve 2, in units of $\omega_{pe}$) as a function of the phase angle $\theta$. The initial electron energy $\varepsilon=1.0$.

Fig. 4. The nondimensional differential fluxes with respect to the nondimensional energy $\varepsilon$. Areas under the curves represent the total nondimensional heating and cooling fluxes respectively.
ION ENERGY DISTRIBUTION IN AN RF ARGON PLASMA

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1. Introduction

The energy of ions bombarding grounded surfaces in RF reactors is a critical parameter in many plasma processing applications. Previous experiments which have been reported in the literature on the structure of the ion energy distribution function (IEDF) at the surface of an electrode have dealt with a strongly RF modulated sheath with a large bias. In that case the large DC bias has allowed a relatively easy resolution of the multi-peak IEDF structure associated with the RF modulation of the sheath and the charge exchange collisions. The same effect (however, not so strong) is also present at the relatively large grounded electrode with a weak sheath, but is more difficult to resolve because of the smaller ion energies. In this paper we present measurements of the IEDF obtained with a retarding field ion energy analyzer mounted on the grounded electrode of an industrial etcher operating at 13.56 MHz frequency. The chamber has been filled with Argon at a pressure varying from 0.3 to 6 Pa. The experimental results are compared to the theoretical predictions based on the numerical solution of the Boltzmann Kinetic Equation for ions in the electric field of a time-varying sheath.

2. Experiment

The experiment has been carried out in an industrial etcher (Nordiko). The vacuum chamber is a cylindrical vessel made of a stainless steel (Fig. 1). The discharge is generated between two electrodes 19 cm. diameter spaced 7 cm apart. The powered electrode (lower) has a cup-like shape which is formed by inserting a copper cylinder of the same diameter as the electrodes and 5 cm height. The RF-voltage (13.56 MHz) is supplied to the electrode through a matching unit.

The grounded electrode (upper) is electrically connected to the chamber. The retarding field ion energy analyzer [5] is mounted in the surface of this electrode. A driven Langmuir probe [6] has access to the interelectrode gap through an orifice in the copper cylinder 3 cm above the powered electrode. It provides information about the plasma parameters (electron temperature, density and plasma potential).

3. Numerical model

To compare the experimental results with theoretical predictions we have calculated the ion transport through the RF-modulated sheath. The one-dimensional sheath model is similar to that developed in [1] and is subject to the following assumptions:
1. The field frequency is much larger than the ion plasma frequency and much less than the electron plasma frequency. So the ions respond only to the time-average electric field and their motion within the sheath is collisionless. Ions enter the sheath with the Bohm speed $u_b = \sqrt{kT_e/M}$.

2. The electrons have a Maxwellian velocity distribution and they follow the instantaneous electric field.

3. We neglect ionization in the sheath, bulk recombination and secondary emission from the electrode. These assumptions are applied only to the calculation of the sheath waveform, not to the consideration of the ion transport.

   The sheath potential profile is obtained by a numerical solution of the system of the conservation equations for ions and Poisson's equation (for details see [1]) and has the following form:

   $$\Phi(x,t) = \Phi_0(x) + \Phi(x) \cos \omega t,$$

   (1)

   where $\Phi_0(x)$ and $\Phi(x)$ profiles are presented in fig. 2 (the nondimensional units $\eta = -e\Phi/kT_e$ and $\xi = x/\lambda_D$ are employed).

   The ion transport through the RF-sheath with the potential waveform given by (1) has been calculated from a numerical solution of the Boltzmann Kinetic Equation:

   $$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{eE}{m_e} \frac{\partial f}{\partial v} = I_{col}(f),$$

   (2)

   where $f=f(x,v,t)$ is the ion distribution function in phase space. The collision integral $I_{col}(f)$ takes into account elastic and charge-transfer collisions; the cross-section data were taken from [2] and [3].

   In the case of the lower pressure ($P \leq 1\,\text{Pa}$) the scattering is weak, the ion mean free path is much larger than the sheath thickness, so it was assumed that ions undergo not more than one charge-transfer collision on their transit through the sheath. The solution of the weak scattering - transport equation is quite straightforward. When the pressure is higher and the ion mean free path is comparable to the sheath thickness a convective scheme [4] has been used to solve the Boltzmann Kinetic Equation. A major problem in the application of the convective scheme to the simulation of the low-pressure discharge is an artificial scattering. Not very important at the higher pressures, it can introduce a significant error when being used in the pressure range 2-10 Pa, so a special technique has been developed to suppress this effect and make it less than the elastic scattering. The calculations were performed on the 300 by 400 mesh with a nonuniform step along the v-axis. The ion velocity distribution on the sheath-plasma boundary is taken to be that calculated by Ingram and Braithwaite [5] (instead of the assumption that ions enter the sheath with the Bohm speed, which has been made for the sheath potential calculation, see above).

4. Discussion

   The ion energy distribution functions, measured and calculated for different pressures, are presented...
It can be seen that there is a reasonable agreement between the experimental and theoretical results, however some difference exists, especially at the higher pressures. The reason for this discrepancy is probably that the ion energy distribution function at the electrode surface is a sensitive function of the sheath potential profile, so a little variation in a sheath thickness (perhaps due to an error in the plasma density or the electron temperature measurements) can cause a significant shift of the peaks in the tail of the ion distribution function (Fig. 5). Further work is required to find the reason why the experimental results show that ions undergo more scattering than that predicted by the numerical model (Fig. 5).

Acknowledgment
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References
Fig. 2. A typical sheath potential waveform
1, $\tilde{\eta}$; 2, $\eta$.

Fig. 4. Ion energy distribution function (arb. units). Pressure = 1 Pa, electron density $n_e=4.1\cdot10^{15}m^{-3}$; $T_e=2.5$ eV. The solid line represents the experimental results; the dashed one - simulation. The curves shown represent $f(v)$ versus $E$.

Fig. 5. Ion energy distribution function (arb. units). Pressure = 6 Pa, electron density $n_e=1.8\cdot10^{15}m^{-3}$; $T_e=2.8$ eV. The solid line represents the experimental results; the dashed one - simulation. The curves shown represent $f(v)$ versus $E$. 
OBSERVATION OF COHERENT DRIFT STRUCTURES
IN THE TURBULENCE OF A DC DISCHARGE

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The experiment is performed in a magnetized neon hot-cathode dc discharge /1,2/. Small Langmuir probes give central values of \( k_B T_e = (4-7) \text{eV} \), \( n_e = (7-10) \times 10^{16} \text{m}^{-3} \). For the floating potential one has to take care of a frequency-dependent voltage division through cable capacities and sheath resistances /3/. The plasma potential is calculated from floating potential and temperature /4/. Radial electric fields are of the order of the ambipolar fields /1/. Digital signal analysis with paired probes is performed on a PC as described in literature /5,6/. Drift waves \((0 < k_r < k \) ) are expected at (1) and flute-type modes \((k_r = 0)\) at (2) and (3) /7/:

\[
\begin{align*}
\omega_1 &= -k_\varphi \frac{k_B T_e (1 - k^2 r_i^2)}{eB (1 + k^2 r_i^2)} - k_\varphi \frac{E_r}{eB} r_i^2 = \frac{k_B T_e}{e^2 B^2} \left( \frac{M_i}{M_i} \right)^2 r_i^2, \\
\omega_2 &= k_\varphi \frac{k_B T_e}{eB} - k_\varphi \frac{E_r}{eB}, \quad \kappa_\varphi = \frac{\varphi}{n_e} < 0, \quad \kappa_T = \frac{1}{n_e} \frac{\partial \varphi}{\partial r} = \frac{1}{T_e} \frac{\partial T_e}{\partial r}.
\end{align*}
\]

In the regime \( B < 40 \text{mT} \) oscillations appear \((5 \text{kHz} < f < 15 \text{kHz}, \text{short wavelengths} k_\varphi r_i = m r_i / r > 1)\) close to (1). Harmonics are of a non-resonant forced type /2/. The phase velocity is less than the electron diamagnetic speed - is equal for all harmonics /2,8/. With increasing \( B \) modes of short- and long-wavelengths establish a transition to turbulence /1,2/.

At \( B = 48 \text{mT} \) we observe waves propagating into the electron diamagnetic direction at frequencies of drift waves (1), potential (dashed lines) and density (solid lines) spectra being in agreement (Fig 1a). The \( m = 1 \) mode is found at \( k_\varphi r_i < 1 \), higher mode numbers appear at \( k_\varphi r_i > 1 \). We observe \( k_i = 6 \text{m}^{-1} \).

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Fig 1  Cross-power (a) \( B = 48 \text{mT} \), (b) \( B = 68 \text{mT} \) (density: solid lines, potential: dashed lines)
A dominant mode is observed at $u/v^* = 0.3$ with (1) predicting $u/v^* = 0.5$. The reduction of $u = \omega/k_p$ through nonlinearities was shown in former studies/2,8/. Fig 2a gives $k = k_p(\omega)$ for $B=48mT$. In contrast to (1) waves at $10kHz < f < 20kHz$ propagate with constant $u$, $u/v^* = 0.12$, indicating a phase-locked behaviour/9/.

With $B$ increased to $68mT$ (Fig 1b), the level of density fluctuation $\delta n/n$ decreases by ~50%, indicating a reduction of instability. Low frequency modes (<6kHz) are found propagating into the electron diamagnetic direction as given by (2), (3) with differences in density and potential expected /10/ and observed. Phase velocities $u$ are given for density and potential, (Fig 2b, coherency of density (+) and of potential (x)). Amplitudes in the regime of linear drift waves ($15kHz < f < 22kHz$) are reduced. The new features compared to previous results/2/ are coherent contributions with anomalous dispersive behaviour. A 9.6kHz-peak belongs to waves propagating into the ion diamagnetic direction with $3< |m| < 4$ and $u<0$ ($u/v^* = -0.1\pm 30\%)$, Fig 3b). Waves at 25kHz ($m=1$) propagate into the electron diamagnetic direction $u>v^*$, $u/v^* = 1.3$. Wave number spectra of density and potential do not agree (Fig 3a). With signals passed through a band pass filter (6kHz - 31kHz), the spectra agree perfectly (Fig 3b). Most of the disagreement in Fig 3a is produced by the low frequency components, which obviously are not drift-, but flute-type waves.

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**Fig 2** Dispersion characteristics and coherency c:
(a) $B=48mT, k = k_p(\omega)$; (b) $B=68mT$, $u = \omega/k_p$, $0<\omega<100\%$, I: flute modes, II: slow soliton-type, III: fast soliton-type, solid lines: $u$ from density, dashed lines: $u$ from potential.
Results of bicoherence analysis are different for both situations (Fig 4). For $B=(48-54)$ mT wave–wave–coupling processes occur between drift waves and between drift waves and flute-type modes, $b'=0.1-0.4$ (Fig 4a, b). At $B=68$ mT no significant coupling occurs (Fig 4c). The frequency doubling peak ($b'=0.15$) only accounts for a non-sinusoidal shape of the 9.6 kHz-wave.

Detailed analysis of solitons includes the rotation of the ion–fluid $v_0$ due to $T_i$ and $E_i/8,12/$. We consider hot Boltzmann-distributed electrons, ions in warm cross-field approximation, treat waves moving in slab geometry perpendicular to $\vec{B}$ and $(\kappa_\parallel, \kappa_\perp)\times /8,12/$. The limit of no periodicity gives the soliton solution ($\dot{\xi} = y - v_0^f$, $\tau = t$, $\Phi = e\phi/k_BT_\parallel = \delta n/n_0$):

$$\Phi = -\frac{3}{2} \Lambda \cdot sech^2 \left( \frac{1}{2} \sqrt{-\Lambda (\xi - u\tau)} \right), \Lambda = -\frac{1}{r_0^f} \left( 1 + \frac{v'}{u} \left( 1 + \frac{T_i}{T_e} \right) \right), S = e\frac{\omega_{ei}}{2k_BT_\parallel}\kappa_\parallel$$

Two types of solitons appear, fast ($u>v_0+v'$) and slow ($u<v_0$) ones. Taking $\vec{E}$ and $T_i$ into account we find the 9.6 kHz-peak corresponding to $u=490 m/s < v_0 = +140 m/s \pm 80\%$ and the 25 kHz-peak to $u>v_0+v'=4000 m/s$: We obtain coherent waves in the regimes ($u-v_0)/v'= -0.14 \pm 80\%$ and $(u-v_0)/v'=1.3$ (Fig 2b). In a gas of solitons it depends on the average energy density $<E_i> \sim |\kappa_\parallel |$ whether fast or slow solitons dominate. We have $<E_i> > 18$ and should expect ion diamagnetic, slow solitons most pronounced, in agreement with observations. Maximum activity is expected at $u/v'= -0.096$, which has to be compared with the observed
$u/v^* = -0.1 \pm 30\%$, or including $V_{te}$: $(u-v_0)/v^* = -0.14 \pm 80\%$ close to theory/11/, which, however, only holds for non-interacting solitons in the "KDV-limit" $(kpr)^2 < 1$. The 9.6kHz-waves are observed at $(kpr)^2 = 2-3$. The KDV-limit is fulfilled for fast contributions (25kHz), $(kpr)^2 = 0.13$.

Waves above $\omega_{ci}$ show a phase-locked behaviour as in Fig.2a and weak broadband coupling, too (Fig 5).

![Fig. 5](image-url)

In conclusion, we observe spectral contributions at velocities expected for solitons, complementary to periodic waves. The ion diamagnetic propagation cannot be explained by a reversed $E \times B$ -rotation of the column, since the $E \times B$ -rotation is positive and cannot explain shifts to negative velocities. Some azimuthal modes ($m=1-4$) remain at positive $u$. If the whole plasma rotates into the ion diamagnetic direction so that a $(3<m<4, f=9.6kHz)$-drift-mode occurs at negative $u$, the other $(m=3-4, f<9.6kHz)$-drift modes should be reversed, too. This is not observed. The very low frequency flute-type modes $f<6kHz$ occur at positive $u$, indicating positive $E \times B$ -rotation of the column, too. The low-frequency turbulence in a magnetized DC discharge consists of four type of waves: Broad-band drift-, flute-type and waves above $\omega_{ci}$ are found with finite coupling and coherent waves ($u$ close to soliton-type) without coupling. Flute-type modes produce a disagreement in spectra of density and potential.

EFFECTS OF MAGNETIC FIELD ON A MAGNETRON DISCHARGE

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Magneton discharge is one of the crossed-field discharges in which the efficient ionization occurs in relatively low pressure range and it is widely used as the sputtering devices for thin film formation. In the commercial devices, the magnetic field is applied by permanent magnets and then the field pattern is distorted strongly. This fact seems it difficult to clear the discharge mechanism in such devices. Description of the magnetron operation is still incomplete, although there are a number of studies [1,2].

In this paper, we describe the effect of magnetic field on the enhanced ionization in a planar magnetron device. We also discuss the mechanism for sustaining discharge and the field pattern required for the magnetron operation.

Experiments are carried out by using a planar magnetron device whose electrode separation and the magnetic field strength can be varied (see Fig. 1). The ring-shaped glowing plasma is produced in the region restricted by the magnetic field. The gases used are He and H₂ in the pressure range from 2 to 0.15 Torr. The variation of the discharge voltage with the magnetic field B is measured under the conditions of fixed discharge current Iₐ and various electrode separation dₕ.

The typical results are demonstrated in Fig. 2. As the magnetic field is increasing, the discharge voltage Vₐ rapidly falls at a critical value Bₐ of the field and the low impedance discharge mode (magnetron discharge mode) occurs at the magnetic field beyond Bₐ. This effect is distinguished in lower pressure range. The critical field Bₐ strongly depends on the electrode separation and is slightly affected by the.

Fig 1: Illustration of planar magnetron device.
discharge current, the species and the pressure of gases. The relationship between $B_c$ and $d_{AK}$ is shown in Fig. 3. Curves in Fig. 3 indicate that $B_c$ varies inversely with $d_{AK}$ and then the product $B_c d_{AK}$ is constant. In the large electrode separation ($d_{AK} > d^*$), however, $B_c$ is independent of $d_{AK}$ and $B_c$ takes a minimum value $B^*$. It seems to be caused by the nonuniformity of the magnetic field.

In general, the cathode fall $V_c$ for a fixed current is expressed as the following formula obtained from the plasma balance equations [3],

$$V_c = C(V_i/\gamma) F(n_{et})$$

where $C$ is the factor ($\geq 1$) associated with the plasma particle loss and energy loss, $V_i$ and $\gamma$ are the ionization potential and the coefficient of $\gamma$ process. $F(n_{et})$ means a factor ($\geq 1$) for the primary electrons trapped between electrodes (total primary electron flux)/(trapped primary electron flux) /%. If the potential drop across the plasma is small, $V_c$ is nearly equal to $V_d$.

In the low pressure diode discharge without the magnetic field, $F(n_{et})$ is much larger than unity. In the magnetron discharge the primary electrons are sufficiently trapped by the magnetic field and then $F(n_{et})=1$. The critical field $B_c$ is the lower limit of the field which satisfies the condition of sufficient trapping of primary electrons. If the magnetic field is uniform and the discharge is dense glow mode (cathode sheath thickness $d_c <<$ electron cyclotron radius $r_{ce}$), the condition is given by $r_{ce} = (mv/eB) \leq d_{AK}$. 

Fig 2: Variation of discharge voltage with magnetic field strength. Magnetron discharge occurs when $B > B_c$. $B$ is measured on the cathode at the center of magnet gap.

Fig 3: Relationship between the critical $B$-field and the electrode separation.
where \( v \) is electron velocity corresponding to \( V_c \). We easily obtain the relationship, \( B_c d_{AK} = \text{const.} \), from \( r_{ce} = d_{AK} \).

In this situation the primary electrons drift by reflection on the cathode sheath edge as shown in Fig.4. The drift motion is different from the ExB drift \( /4,5/ \), which occurs at lower density discharge (\( d_c > r_{ce} \)). The effect of the nonuniform field is discussed as follows.

For simplicity, we consider the magnetic field due to a dipole array of gap length 2a and the orbital motion of primary electron (accelerated up to \( eV_c \) and start from \( z=b \)) on the mid plane, where the field is given by \( B(z) = B_0/((1+z^2/a^2) \). From computer calculation we obtain the trajectory of electron motion. The distance, \( Z_T = (z_T - b)/r_{ce0} \), from the cathode sheath edge to the turning point changes with the effective magnetic field, \( \text{A} = a/r_{ce0} \) as shown in Fig.5, where \( r_{ce0} = \text{mv}/eB_0 \). When A is large (\( \text{A} > 1 \)), \( Z_T \) does not change with A very much and is found to be \( Z_T \approx 1 \). In the case of \( \text{A} \approx 1 \), however, the turning point moves far away even if A slightly decreases.

The electrons are trapped when \( Z_T \) is smaller than \( d_{AK}/r_{ce0} \). The magnetron operation will be obtained when the following conditions are satisfied \( /6/ \).

\[
Z_T \leq 1 \leq \frac{d_{AK}}{r_{ce0}} \quad \text{for} \quad d_{AK} < a, \quad (1)
\]

or

\[
A = a/r_{ce0} > 1 \quad \text{for} \quad d_{AK} > a. \quad (2)
\]

They are corresponding to \( B_c d_{AK} = \text{const.} \) for \( d_{AK} < d^* \) and \( B_c = B^* \) for \( d_{AK} > d^* \), where \( d^* \) is related to a.

The calculated values are in good agreement with the experimental results. In this model the electron drift is due to the reflection on the cathode sheath and/or grad-B. Further confinement of the primary electrons does not discuss but the electrons seem to be confined by the
magnetic mirror. The ring-shaped glowing plasma is produced by the bombardment of the trapped energetic electrons.

Equations (1) and (2) or Fig.5 are useful to design the field pattern and the electrode arrangement in magnetron devices. Equation (2) is same as the result obtained from a computer simulation by Goree et al. /7/.

Low impedance characteristics in the magnetron discharges is caused by efficient ionization due to the primary electrons which are accelerated by the cathode fall and trapped magnetically between the electrodes. The trapped electrons drift resulting in the reflection on the cathode sheath and/or grad-B and bounce between the mirror points.

/* In the simplest case, \( F(n_{et}) \) is given by the formula,
\[
F(n_{et}) = \left(1 - \exp\left[-\left(\frac{B_{c}\lambda_{0}}{B_{c}\lambda_{e}}\right)\sin^{-1}\left(\frac{B}{B_{c}}\right)\right]\right)^{-1}
\]
for \( B \leq B_{c} \),
where \( \lambda_{e} \) is m.f.p. of the primary electrons.
THE LANGMUIR PROBE STUDY OF THE ELECTRON ENERGY DISTRIBUTION FUNCTION IN THE FLOWING AFTERGLOW PLASMA

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The Flowing Afterglow Langmuir Probe (FALP, [1,2]) apparatus, with microwave discharge (2.5 GHz, 25 - 75 W) as a plasma source, was used to study evolution of the Electron Energy Distribution Function (EEDF) in the low pressure (50 to 85 Pa) argon flowing afterglow plasma. The polyatomic molecular reactants such as n-hexane and benzene were added to the argon carrier gas in order to study their influence on the EEDF and its relaxation. Some aspects of the determination of the plasma parameters from the Langmuir probe measurements are discussed.

Introduction

The flowing afterglow technique was developed by Ferguson and co-workers [3] and it has been extensively used to study ion-molecule reactions. The FALP version was used to study recombination, attachment and plasma relaxation processes [2,4,5,6]. In most of the measurements He was used as a carrier gas and as a common feature fast thermalization was observed. The situation is different when Ar is used as a carrier gas; Ar has higher atomic mass and the cross-section for the electron atom collisions has pronounced minimum just in the energy range 0.1 - 1 eV (Ramsauer effect)[7]. As a consequence an energy transfer between electrons and Ar atoms is two orders of magnitude less efficient as it is in the case of He atoms, so for the investigation of the processes influencing the EEDF Ar is more suitable as a carrier gas [8].

Experimental

The experimental data have been accumulated using the flowing afterglow apparatus of the FALP type (for details see [9]). The flow tube is 1 m long with inner diameter 0.06 m. The velocity of the argon carrier gas was 50 ms⁻¹, corresponding decay times (t) are up to 20 ms. Plasma parameters have been measured on the axis of the flow tube by means of an axially movable cylindrical Langmuir probe (platinum, length 4 mm, diameter 40 μm).

The probe current (Iₚ), as a function of probe potential (Vₚ), was measured. The EEDF (F(U)) was determined on the basis of its relation to the second derivative of the electron part (Iₑ) of the probe current [9]:

F(U) = Const × V₁/² × (d²Iₑ/dV²)

where Iₑ = Iₚ - Iᵢ, V = Vₚ - V_{PLASMA} and U is the electron energy in eV corresponding to V. The second derivative of Iₑ (I"ₑ) was calculated from the probe current and correction to the second derivative of the ion current was taken into account, for details see [10]. The plasma potential was taken as a probe potential at which the second derivative is changing its sign (inflection point; I"ₑ ≠ 0 and I"ₑ = 0).
Typical examples of the measured probe characteristics and the calculated second derivatives of the electron current for the different reactant gases are plotted in Fig.1a,b,c (absolute value $|I_p|$ and $|I_e|$ versus V in semi-logarithmic plot).

Near the plasma potential (V=0) the measured probe characteristic is influenced by processes on the surface of the probe and by fluctuations of the plasma potential; as a consequence the calculated second derivative is inaccurate for V up to $-2kT_e/e$ and an extrapolation has to be used. In the presence of molecular reactant gases the determination of the plasma potential can be influenced by the plasma enhanced deposition on the probe surface. To avoid this problem the high negative voltage has been applied to the probe in order to clean the probe surface by ion bombardment. The high voltage was applied after each change of the probe voltage prior to the measurements of the probe current.

If the EEDF is Maxwellian the semi-logarithmic plot of $d^2I_e/dV^2$ versus V is linear with a slope given by the electron temperature $T_e$ (see Fig.1a). When the "body" of the EEDF is near to Maxwellian there is a linear part of semi-logarithmic plot of $d^2I_e/dV^2$ versus V (see Fig.1b) and "effective" (approximative) electron temperature can be estimated. Semi-logarithmic plots of $I_e$ versus V in pure Ar do not have in general a linear part (see Fig.1c) and the effective temperature cannot be estimated. The direct calculation of a mean energy of electrons from measured EEDF is inaccurate due to the mentioned inaccuracy of the obtained $I_e$ in the low energy region.

The shape of the EEDF has to be considered when determining the plasma density. The probe theories usually suppose the Maxwellian EEDF, so they should be used with care. Presented electron concentrations are calculated, when possible, from the probe current at plasma potential and temperature or effective temperature is used.

Evolution of the EEDF in flowing afterglow

The evolution of the EEDF has been studied along the flowtube in pure Ar and in Ar with small admixture of polyatomic molecular reactant gas (benzene and n-hexane) introduced downstream of the microwave discharge. N-hexane and benzene have been used because of the effect of polymerization and effective
deposition in a plasma [12]. The preliminary analysis for benzene shows that the rate of the deposition, refractive index and electric conductivity of the deposited films depends on the position in the flow tube.

For the evaluation of deviation of the measured EEDF from the Maxwellian one \( F_m(U) \) corresponding to the same temperature it is useful to plot the ratio \( F(U)/F_m(U) \) as a function of \( U/kT_e \). In Fig.2a,b \( \Pi_l \) and the ratio \( F(U)/F_m(U) \) are plotted for different positions along the flow tube in pure Ar (distance from the reactant entry port is given in cm, 5 cm corresponds to 1 ms in time scale). It is evident that the EEDF is not Maxwellian and the deviation is increasing with increasing decay time; plasma is not thermalized even after 8 ms. The electron gas is heated mainly on the expenses of the energy of the metastable argon atoms and by the dissociative recombination of \( \text{Ar}_2^+ \) ions. The Druyvesteyn distribution is a good approximation for the measured EEDF in pure argon. This is typical of Ar flowing afterglow [6, 13]. For purposes of Fig.2b the effective temperature as a rough approximation has been estimated from the \( \Pi_l \) Vs V semi-log plot.

In Fig.2c,d the same dependences as in Fig.2a,b are plotted the only difference being the admixture: 0.35% of n-hexane in the flow tube. The corresponding evolutions of \( T_e, N_e \) and EEDF along the flow tube are plotted in Fig.3. As it can be seen, in presence of n-hexane, the plasma is thermalized rapidly and as a consequence the diffusion losses are smaller and the concentration changes slowly. Immediately after the entry port the \( \text{Ar}^+ \) ions are converted to molecular ions and then they are removed by dissociative recombination. The estimated recombination rate coefficient is \( 4.7 \times 10^{-10} \text{ cm}^3 \text{s}^{-1} \).

The same dependences for admixture of 0.33% of benzene in Ar are plotted in Fig.2e,f. The deviation from the Maxwellian distribution due to the production of energetic electrons in the Penning ionization of benzene in energy range \( 1.0 - 1.5 \text{ eV} \) is evident. It is also seen from Fig.2f that this group vanishes with increasing afterglow time.
Conclusion

The evolution of the EEDF in the flowing afterglow has been studied. It was possible to show how the EEDF, electron density and effective temperature in an Ar afterglow are influenced by the addition of benzene and n-hexane.

References

RF glow discharges and other processing plasmas are used extensively in the microelectronics industry. Self-consistent fluid equations have been used recently to study the structural features of RF and DC glows\textsuperscript{1,2}. However, since these discharges are inherently complex and the particle distributions are non-Maxwellian, there has been a considerable effort in developing self-consistent kinetic models without making any assumptions on the distribution functions\textsuperscript{3,4}. 

In order to use particle-in-cell simulation codes for modeling collisional plasmas and self-sustained discharges it is necessary to include interactions between charged and neutral particles. Monte Carlo methods have been used extensively in swarm simulations\textsuperscript{5,9}. In many Monte Carlo schemes, the time (or distance) between collisions for each particle is calculated from a random number. This allows for efficient algorithms, especially when the null collision method is used\textsuperscript{10}. This technique is however, not compatible with PIC simulations, since all particle trajectories are integrated simultaneously in time. Hence, the collision probability for the \textit{i}th charged particle is calculated from

\[ P_i = 1 - \exp(-v\Delta t) \]

where \( v = n \sigma_f(E_i) v_i \), \( \sigma_f \) is the total collision cross section, \( E_i \) is the kinetic energy of the particles and \( n \) is the neutral density. A collision takes place if a uniformly distributed random number on the interval [0, 1] is less than \( P_i \). The null collision method can be incorporated into the collision model by picking a constant collision frequency \( v' \) such that,

\[ v' \geq (n \sigma_f v)_{\text{max}} \]

which greatly reduces the computational cost, by not evaluating \( \sigma_f(E_i) \) for each particle. The fraction of the total number of particles (chosen at random) in the simulation that experience collisions is given by

\[ P = 1 - \exp(-v'\Delta t) \]

The collisions are assumed to take place at the current positions of the particles. It should be noted that the choice of \( \Delta t \) will affect the accuracy of the collision model. For instance, \( v, \Delta t \) should not be much larger than simulation length scales of interest (e.g. grid spacing, \( \lambda_m \)) and \( \Delta t, \sigma_f(E_i)n \) should be about 0.1 or less\textsuperscript{12}, which means that less than 10\% of the charged particles
collide in one time step. Once a collision occurs, the type of collision, the energy of the ejected electron (for an ionizing collision) and the direction(s) are determined with new random numbers. These quantities are related back to the system coordinate axes. The procedure for electron-neutral collisions is described in detail by Boeuf and Marode\textsuperscript{7}, and by Thompson \textit{et al.} (1988) for ion-neutral collisions. Expressions for differential cross sections that are analytically integrable are useful, as then the computational cost of determining scattering angles and energy redistribution in ionizing collisions is minimized\textsuperscript{11,12}.

A Monte Carlo collision handler as described above, including the null collision method, has been developed as an addition to the PIC scheme as shown in Fig. 1. The full three dimensional character of a collision is modeled with three velocity components. The neutrals are assumed to be uniformly distributed between the boundaries with a constant density and a Maxwellian profile. The model is still valid if the neutral density is a weak function of position and time (small variations across the mean free path and collision times). This scheme can also be extended to model Coulomb collisions between charged particles.

![Flow chart for an explicit bounded PIC scheme with the addition of the collision handler, called PIC-MCC\textsuperscript{9}.](image)

RF discharge modeling displays many physical time scales, e.g. $\omega_x < \omega_{RF} \ll \omega_{pe}$. With a PIC model including an electrostatic response, the highest frequency that must be resolved by the explicit numerical methods used to solve the particle and field equations is $\omega_{pe}$. If $\omega_{pe} \Delta t > 2$, numerical instabilities can occur for explicit methods\textsuperscript{13}. To observe most of the physics of interest, one needs to resolve the RF timescale only, and therefore much computing time is wasted resolving the electron plasma oscillation time scale.

Implicit particle simulation\textsuperscript{14} has been developed to relax the numerical stability constraint. Implicit particle movers advance the position of the $i$th super particle by the equation

$$x_i^{n+1} = \alpha E_{+}^{n+1}(x_i^{n+1}) + x_i^{n+1}$$
where $x$ is the portion of the position advanced, dependent on quantities known at present and previous time levels, $\alpha' = \beta \Delta t^2 q/m$, and $\beta$ depends on the particular implicit scheme. Linearizing the locations $x_i^{n+1}$ about the locations $x_i^n$ in the superparticle-to-grid weighting equations gives the numerically implicit Poisson equation

$$\partial_x [1 + \alpha \tilde{\rho}^{n+1} \partial_x \phi^{n+1}] = -\tilde{\rho}^{n+1} / \epsilon_0$$

where $\alpha = \alpha'/\epsilon_0$.

Simple boundary conditions for the RF discharge are a zero potential at the left wall and an RF source voltage at the right wall. The electric field, at the left wall is then derived by taking a Gaussian pill box about the wall,

$$[(1 - \alpha \tilde{\rho}) E]_{\text{left}} = \sigma \Delta x / \epsilon_0 + \tilde{\rho} \Delta x / 2 \epsilon_0$$

A similar procedure is used for the right wall. The equations may be generalized to include more complicated boundary conditions including external circuit elements.

Many accuracy constraints still remain. One important constraint is that the fastest particle species should resolve spatial gradients in the field, i.e., $v_{\text{max}} \Delta t / s < 1$ where $s$ is the gradient length. Another important constraint is that all particles should sample the field on the grid in a continuous manner over a single time step. This gives $v_{\text{max}} \Delta t / \Delta x < 1$. A problem with implicit methods is excessive numerical cooling which is due to poor sampling of fast particles in simulations with large time steps. Resolving fast particles is particularly important in RF discharges because it is the fast electrons which maintain the discharge through ionization collisions with the neutrals. A possible way out of this problem is to do multi-scale simulations. That is, the few fast electrons that maintain the discharge may be pushed with a small time step while the remaining slow particles, essentially residing in the bulk plasma, may be pushed with a large implicit time step. Based on these algorithms, which have been implemented in PDPl, we have been able to speed up our bounded PIC-MCC code by factors of 5-20 times, when simulating RF Discharges with laboratory parameters.

We will present comparisons of results obtained from simulation with those published by Godyak et al.16,17. Our simulation results show that the electrons have a two-temperature distribution in a 2 cm gap argon RF discharge as measured by Godyak et al.18. The Figure on the next page shows an instantaneous electron velocity distribution in the system as a function of position.

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ON MODELLING OF STEADY STATES IN TOROIDAL MACHINES FOR BASIC PLASMA PHYSICS
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Introduction. Toroidal machines like ACT-I (1), BLAAMANN (2) and TORELLO (3) have a strong toroidal magnetic field, and may under steady state operation produce a plasma continuously for instance between a hot filament and an anode placed diametrically opposite in the toroidal direction. Charged particles spiral in the toroidal direction, collide in between themselves and with the background gas and drift and diffuse normal to and along the magnetic field giving rise to particular density, temperature and electric potential profiles over the circular cross section. Details of measured global plasma behaviour (2), are of interest because they reflect fundamental transport mechanisms that are important in these machines. Some typical parameters of the plasma in BLAAMANN are: Major radius, R₀=60cm, minor radius a=15cm, toroidal magnetic field, Bₚ=0.1 Tesla, electron and ion densities, nₑ,i=10⁹-10¹⁰/cm³, neutral particle density, N₀=10¹²/cm³, electron temperature, Tₑ=1-10eV, ion temperature, Tᵢ, typically 10% of Tₑ. For an argon-plasma this gives the for the electron- and ion cyclotron frequencies \( \Omegaₑ=1.8 \times 10^{10}/s \) and \( \Omegaᵢ=2.4 \times 10^5/s \), and corresponding Larmor radii, \( rₑ=8 \times 10^3/cm \) and \( rᵢ=0.6 \) cm, for electron-ion collision frequency, \( vₑᵢ=1.4 \times 10^7/s \), for electron-atom collision frequency, \( vₑᵣ=2 \times 10^7/s \) and for ion-atom collision frequency, \( vᵢᵢ=8 \times 10^2/s \). The BLAAMANN machine has been equipped with additional coils for producing small components of magnetic field in the vertical and/or major radial directions (of order 1% of Bₚ). In particular some effects of small magnetic field components on the plasma behaviour are discussed in this paper. Classical transport theory is assumed valid, (4), (5).

Transverse versus longitudinal B particle transport. Transverse a strong magnetic field, ions diffuse freely faster than electrons, while along the field electrons diffuse freely.
faster than ions. This means that in ambipolar transport of the plasma the ions are hindered by
the electrons in transverse B movement, while along the field the electrons are hindered by the
ions. Adding small components of B in the z and/or the R-directions give rise to some
peculiarities: With gradients in the z and R directions, and with a small component of magnetic
field in the z direction, cf. fig.1, we have typically for the free electron diffusion along (l) the
field, $\Gamma_{\text{Free}}^{z \perp} = -D_\perp \frac{\partial n}{\partial t \perp} = -\frac{x_T \in B}{m_e v_{\text{rec}}} \frac{\partial n}{\partial z}$, where $\frac{\partial n}{\partial z} = \sin \epsilon \frac{\partial}{\partial z} = \frac{B_T}{B_e} \frac{\partial}{\partial z}$. This diffusion has a component
in the z-direction, $\Gamma_{\text{Free}}^{z} = \left(\frac{B_T}{B_e}\right)^2 \frac{x_T \in B}{m_e v_{\text{rec}}} \frac{\partial n}{\partial z}$. For the ion transverse (l) diffusion we have typically
$\Gamma_{\text{Free}}^{z \perp} = -D_\perp \frac{\partial n}{\partial z \perp} = -\frac{x_T \in B}{2m_i \Omega_i^2} \frac{\partial n}{\partial z \perp}$, and with $\frac{\partial}{\partial z} = \cos \epsilon \frac{\partial}{\partial z} = \frac{1}{2} \epsilon z \frac{\partial}{\partial z}$, the z component of this diffusion
becomes $\Gamma_{\text{Free}}^{z \perp} = \left(\frac{B_T}{B_e}\right)^2 \frac{x_T \in B}{2m_i \Omega_i^2} \frac{\partial n}{\partial z \perp}$. Between these free diffusion components in the z direction we have
in the actual parameter regime the relation $\frac{\Gamma_{\text{Free}}^{z \perp}}{\Gamma_{\text{Free}}^{z}} = \left(\frac{B_T}{B_e}\right)^2 \frac{x_T \in B}{2m_i \Omega_i^2} \frac{\partial n}{\partial z \perp} = \frac{1}{4} 10^{10} \left(\frac{B_T}{B_e}\right)^2$. This
shows that even for a very small component of magnetic field in the z-direction the electron free
diffusion component in the z direction, almost perpendicular to the magnetic field, dominates
over the ion free diffusion. In turn this will influence ambipolar transport.

Particle transport equations. Equations transverse and along the magnetic field are for ions
(single ionized) and electrons

$$
e E_\perp + \frac{e}{c} (u_i \times B) = -\frac{1}{n_i} \frac{\partial}{\partial \perp} n_i \kappa T_i - \mu_{ia} V_{ia} u_{i, \perp} + m_e v_e \frac{n_e}{n_i} (u_e \perp - u_{i, \perp}) = 0$$

$$-e E_\perp - \frac{e}{c} (u_e \times B) = -\frac{1}{n_e} \frac{\partial}{\partial \perp} n_e \kappa T_e - \mu_{ea} V_{ea} u_{e, \perp} - m_e v_e \frac{n_e}{n_i} (u_e \perp - u_{i, \perp}) = 0$$

$$e E_{i \perp} - \frac{1}{n_e} \frac{\partial}{\partial \perp} n_e \kappa T_e - \mu_{ea} V_{ea} u_{e, \perp} - m_e v_e \frac{n_e}{n_i} (u_e \perp - u_{i, \perp}) = 0$$

$$-e E_{i \perp} - \frac{1}{n_e} \frac{\partial}{\partial \perp} n_e \kappa T_e - \mu_{ea} V_{ea} u_{e, \perp} - m_e v_e \frac{n_e}{n_i} (u_e \perp - u_{i, \perp}) = 0$$

The $\mu$'s are reduced masses. Acceleration terms have been neglected. Gradients are over the
cross-section. Toroidal symmetry is assumed. B = $e_0 B \cos \cos \eta + e_0 B \sin \eta + e R \cos \sin \eta$
where $e$ and $\eta$ are small angles between B and its horizontal projection and between the
horizontal projection and the toroidal direction, respectively.

We neglect electron-ion collisions in the following. For particle fluxes $\Gamma^{(\alpha)} = n_\alpha u_\alpha$ in a cross
section plane, $\alpha=(e)$electrons, (i)ons, we obtain

$$\Gamma^{(\alpha)} = -\frac{v_{\alpha a}}{\Omega_\alpha^2} \left(\begin{array}{cc}
\cos \eta^2 & 0 \\
0 & (\cos \epsilon)^2
\end{array}\right) d^{(\alpha)} - \frac{1}{v_{\alpha a}} \left(\begin{array}{cc}
\cos \epsilon \sin \eta & \cos \epsilon \sin \sin \eta \\
\cos \epsilon \sin \sin \eta & (\sin \epsilon)^2
\end{array}\right) d^{(\alpha)}$$

$$\frac{1}{\Omega_\alpha} \left(\begin{array}{cc}
\cos \epsilon \cos \eta \sin \eta & 0 \\
0 & -\cos \epsilon \cos \eta \sin \eta \sin (q_\alpha)
\end{array}\right) d^{(\alpha)}$$
Here $d^{(m)} = \frac{1}{m} \frac{\partial}{\partial r} n_\alpha k T_\alpha + \frac{n_\alpha}{m_\alpha} \frac{\partial}{\partial r} \phi$ when $E = -\frac{1}{\lambda} \frac{\partial}{\partial r} \phi$. Generally, these fluxes are substituted into the mass- and energy conservation equations for electrons and ions, which again are coupled to Poisson's equation in an electrostatic approximation.

Instead we specialize and assume quasineutrality in the main plasma volume, $n_e = n_i = n$, and ambipolar particle transport, $\Gamma_{(0)} = \Gamma_{(1)}$. The electric field is then also ambipolar. Due to lack of space it is omitted in the following. We mention a few special cases:

Case 1: $\frac{\epsilon^2}{\Omega}$ and $\frac{\eta^2}{\Omega}$ both $<< \frac{\nu_m}{\Omega}$ $<< \frac{1}{\Omega}$. Recall that $\frac{\nu_m}{\Omega} = 10^{-6}$ in the actual parameter regime we operate. Then the effects of $\epsilon$ and $\eta$ are negligible, and the limits $\epsilon \to 0, \eta \to 0$, are appropriate. The expressions for particle flux are a special case of,

$\frac{\nu_n}{\Omega} = 3 \cdot 10^{-3}$. The particle transport then becomes

$$\Gamma_R = \frac{1}{m \nu_{in}} \frac{\partial}{\partial r} \left( n \kappa (T_e + T_i) \right) + \frac{1}{m \nu_{in}} \frac{\partial}{\partial z} \left( n \kappa (T_e + T_i) \right)$$

$$\Gamma_z = \frac{1}{m \nu_{in}} \frac{\partial}{\partial z} \left( n \kappa (T_e + T_i) \right) + \frac{1}{m \nu_{in}} \frac{\partial}{\partial R} \left( n \kappa (T_e + T_i) \right)$$

The equations for Case 1 are obtained by letting $\epsilon \to 0$.

Case 3: $\epsilon = 0$, while $\eta << \frac{\nu_m}{\Omega}$. Then

$$\Gamma_R = \frac{1}{m \nu_{in}} \frac{\partial}{\partial r} \left( n \kappa (T_e + T_i) \right) + \frac{1}{m \nu_{in}} \frac{\partial}{\partial z} \left( n \kappa (T_e + T_i) \right)$$

$$\Gamma_z = \frac{1}{m \nu_{in}} \frac{\partial}{\partial z} \left( n \kappa (T_e + T_i) \right) + \frac{1}{m \nu_{in}} \frac{\partial}{\partial R} \left( n \kappa (T_e + T_i) \right)$$

Case 4: $\frac{\nu_m}{\Omega} << (\text{both } \epsilon \text{ and } \eta) << \frac{\nu_n}{\Omega}$. Then

$$\Gamma_R = \frac{1}{D} \left( a_{11} + \eta^2 a_{12} \right) \frac{\partial}{\partial r} \left( n \kappa (T_e + T_i) \right) + \frac{1}{D} \left( a_{21} + \epsilon \eta a_{22} \right) \frac{\partial}{\partial z} \left( n \kappa (T_e + T_i) \right)$$

$$\Gamma_z = \frac{1}{D} \left( a_{11} + \epsilon^2 a_{12} \right) \frac{\partial}{\partial z} \left( n \kappa (T_e + T_i) \right) + \frac{1}{D} \left( a_{21} + \epsilon \eta a_{22} \right) \frac{\partial}{\partial R} \left( n \kappa (T_e + T_i) \right)$$

where $D = \frac{1}{\nu_{in} \Omega} + \frac{m}{m_n \nu_{in} (\epsilon^2 + \eta^2)}$, $a_{11} = \frac{\nu_n}{m \nu_{in}}$, $a_{12} = \frac{1}{m_n \nu_{in} \Omega_i}$, $a_{21} = \frac{\nu_n}{m \nu_{in} \Omega_i}$, $a_{22} = \frac{1}{m_n \nu_{in} \Omega_i}$.

In steady state the mass conservation equation becomes $\nabla \cdot \Gamma = 0$. When in particular electron and ion temperatures may be set constant we have typically in cases (1)–(4) for particle density the equation

$$A \frac{\partial^2 n}{\partial x^2} + 2B \frac{\partial^2 n}{\partial x \partial z} + C \frac{\partial^2 n}{\partial z^2} = -f(x, z)$$

We have simplified the geometry and neglected some effects using cartesian coordinates.

$f(x, z) = \frac{1}{xt_e + T_i} \left( \frac{\partial n}{\partial t} \right)_c$. In case (1) $B = 0$, $A = C = \frac{1}{m \nu_{in}}$, in case (2) $B = 0$, $A = \frac{m \nu_{in}}{m_n \nu_{in} \Omega_i}$, $C = \frac{1}{m \nu_{in}}$, in
case (3) $B=0$, $A=\frac{1}{m_{1}v_{1x}}$, $C=\frac{1}{m_{2}v_{2x}}$ in case (4) $A=\frac{1}{D}(a_{11}+\eta^{2}a_{12})$, $B=\frac{\eta^{2}a_{12}}{D}$, $C=\frac{1}{D}(a_{11}+\epsilon^{2}a_{12})$

where $D$, given above, also depends on $\epsilon$ and $\eta$.

We demonstrate in Figs. (2)-(4) some effects of $\epsilon$ and $\eta$ on solutions of $n$ over a quadratic cross section, $-a \leq x \leq a$, $0 \leq z \leq 2a$ ($a=15$), in the cases (1)-(3). Shown are level curves of $n-n_0$, where $n_0$ is the same constant, unspecified density on the boundary ('outside' a boundary sheath). The same plasma production term $\left(\frac{\partial n}{\partial t}\right)_{e}$, proportional to $\delta(x+\frac{z}{2})\left(\sin\frac{2\pi}{a} - \frac{1}{3}\sin\frac{8\pi}{a}\right)$, has been used,

and the same numbering indicates the relative density distributions.

A ONE-DIMENSIONAL Q-MACHINE MODEL TAKING INTO ACCOUNT CHARGE-EXCHANGE COLLISIONS

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1. Introduction and model. The Q-machine\(^1\) is a nontrivial bounded plasma system which is excellently suited not only for fundamental plasma physics investigations but also for the development and testing of new theoretical methods for modeling such systems. However, although Q-machines have now been around for over thirty years, it appears that there exist no comprehensive theoretical models taking into account their considerable geometrical and physical complexity with a reasonable degree of self-consistency. In the present context we are concerned with the \textit{low-density, single-emitter} Q-machine, for which the most widely used model is probably the (one-dimensional) "collisionless plane-diode model",\(^3\)\(^4\) which has originally been developed for thermionic diodes.\(^5\)\(^-\)\(^7\) Although the validity of this model is restricted to certain "axial" phenomena, we consider it a suitable starting point for extensions of various kinds. While a generalization to two-dimensional geometry (with still collisionless plasma) is is being reported elsewhere,\(^8\) the present work represents a first extension to collisional plasma (with still one-dimensional geometry).

The model geometry is shown in Fig. 1. Both ions and electrons leave the emitter ("hot plate", located at \(x = 0\) and assigned potential \(V = 0\)) with half Maxwellian distribution functions corresponding to the emitter temperature \(T\). All particles reaching either the emitter or the collector ("cold plate", located at \(x = L\) and biased against the hot plate with potential \(V_c\)) are absorbed. Since this work represents a first step toward the self-consistent kinetic modeling of collisional bounded plasma systems, the ions are assumed to undergo symmetric charge-exchange collisions with the neutral background gas,\(^9\)\(^10\) whereas the electrons are still treated as collisionless. Moreover, we restrict ourselves to dc states with monotonically decreasing potential distributions, so as to avoid the occurrence of trapped particles.

In Sec. 3, our iterative analytic-numerical method for calculating self-consistent dc states of the above configuration is briefly outlined, and some typical numerical results on the effect of charge-exchange collisions are presented in Sec. 3. Full details are given in Ref. 11.

2. Outline of the method. Starting from a suitable first guess \(V_0(x)\) for the potential distribution \(V(x)\) (with \(V_0\) best taken from one-dimensional theory\(^4\)), the velocity
distribution functions must be found by solving the Vlasov equation for the electrons,
\[
\left[ v \cdot \frac{\partial}{\partial x} + \frac{q_e}{m_e} \left( E(x) + v \cdot B \right) \right] f_e(x,v) \equiv \frac{d}{dt} f_e(x,v) = 0
\]
and an appropriate linearized Boltzmann equation for the ions,\(^{10}\),
\[
\left[ v \cdot \frac{\partial}{\partial x} + \frac{q_i}{m_i} \left( E(x) + v \cdot B \right) \right] f_i(x,v) \equiv \frac{d}{dt} f_i(x,v) = \mathcal{G}_i(x,v)
\]
as described below. For the ion–neutral collision term \( \mathcal{G}_i \) we use the linearized form
\[
\mathcal{G}_i(x,v,f_i) = \mathcal{P}(x,v,f_i) - \nu(x,v) f_i(x,v),
\]
where
\[
\mathcal{P}(x,v,f_i) = \int d^3 V d^2 \Omega f_i(x,v') F(x,V') v_{rel} \sigma_d
\]
is the gain term and \( -\nu \mathcal{P}_i \) is the loss term, with
\[
\nu(x,v) = \int d^3 V F(x,V) v_{rel} \sigma_d
\]
the ion collision frequency. For the differential and total cross–cross sections we choose the well–known analytic approximations\(^9,10\)
\[
\sigma_d(v_{rel}, \theta) = \frac{\sigma_{0\theta}}{2\pi} \delta(1+\cos \theta), \quad \sigma_{tot}(v_{rel}) = \left[ a - b \log(v_{rel}) \right]^2.
\]
The distribution functions \( f_e(x,v) \) and \( f_i(x,v) \) are determined via integration of Eqs. (1) and (2), respectively, with \( \frac{d}{dt} \) denoting the Lagrangian time derivative along the species–a trajectory \((a = e, i)\) in the given dc potential. For the one–dimensional situation considered here, the trajectory integral of Eq. (1) is simply
\[
f_e(x,v) = f_e(0,v_0) = A_e \exp \left[ -m_e v_0^2 / 2kT \right] U(v)
\]
with \( m_e v_0^2 / 2 = m_e (v_x^2 + v_n^2) / 2 - eV(x) \) the electron’s energy at the emitter and \( U(v) \) the unit step function. Analogous formal integration of Eq. (2) would lead to an integral equation for \( f_i \), but in practice we solve that equation by direct numerical integration on a discrete spatial grid:
\[
f_{i,k+1} = f_{i,k} \left[ 1 - \Delta t_k (v_k + v_{k+1}) \right] + \Delta t_k \left( P_k + P_{k+1} \right),
\]
where the subscript \( k \) indicates quantities at the \( k \)th gridpoint and \( \Delta t_k \) is the ion’s time of flight between gridpoints \( k \) and \( k+1 \). In so doing, two types of ion trajectories must be distinguished, namely (i) those originating at the emitter with positive \( v_x \)–values (for which the initial condition is the half Maxwellian emission distribution), and (ii) those originating at the collector at negative \( v_x \)–values (for which the initial \( f_i \) equals zero).

With the velocity distributions at a given position \( x \) thus determined, we calculate
the electron and ion number densities and, from them, the space-charge density according to

\[ \rho(x) = e\left[n_e(x) - n_i(x)\right] = e\left[\int d^3v f_e(x,v) - \int d^3v f_i(x,v)\right] \]  

(9)

where the velocity-space integration is performed analytically for the electrons and numerically for the ions. With this charge-density distribution, Poisson's equation,

\[ \nabla^2 V(x) = -\rho(x)/\varepsilon_0 \]  

(10)

is solved approximately by means of a well-known relaxation scheme, the result being an improved potential distribution \( V_1(x) \).

This improved potential distribution is now used for calculating new distribution functions, new particle densities, a new space-charge distribution and, from it, a new potential distribution \( V_2(x) \), et., until a sufficiently accurate and self-consistent "final" potential distribution \( V(x) \) is obtained.

3. Results and discussion. In Figs. 2(a–d) we show some representative macroscopic quantities for a Q-machine (or thermionic diode) with length \( L = 5 \text{ mm} \) at two different background (K) densities \( n_b \) ("low": \( 10^{16}\text{ m}^{-3} \), marked by "1", and "intermediate": \( 10^{17}\text{ m}^{-3} \), marked by "2"). The other parameters characterizing the system are: \( V_c = -2\text{V}, T = 2100\text{K}, T_b = 500\text{K}, \) electron emission density \( n_{e0} = 1.5\times10^{15}\text{m}^{-3} \), and neutralization parameter \( \alpha \equiv n_{i0}/n_{e0} = 0.01. \) This system is "short" in the sense that there is no significant quasineutral plasma region between the emitter and collector sheaths. (It has been found that "long" systems are prone to numerical instability, and proper treatment of these will require further development of the method presented here.)

While Figs. 2(a) and 2(c) show the potential and space-charge distributions, Figs. 2(b) and 2(d) show their differences against the collisionless case. According to these results, charge-exchange collisions have the following basic effects on the system: They tend to lower the ion average velocity and, hence, to raise the ion density. Hence, the region of positive space charge is widened, and the "plasma point" (defined by exact space-charge neutrality, marked by a small open circle) is shifted to the left. These tendencies become stronger with increasing background density.

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Fig. 1

Fig. 2
DETERMINATION OF COMPOSITION AND PHYSICAL PROPERTIES OF PARTIALLY IONIZED PLASMAS IN THE FUNCTION OF TEMPERATURE

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1. INTRODUCTION
The investigations of various kinds of partially ionized plasma were conducted for the pressure of 0.1 MPa and in the range of temperature of 298.15 K to 24000 K. The physical properties of various kinds of partially ionized plasma depend mainly on their composition and temperature. The composition of particular kinds of partially ionized plasmas varies also in the function of temperature. Simultaneous going on of physical and chemical processes in plasma is the reason of difficulties in the calculations of plasma’s physical properties. The use of the laws of macroscopic thermodynamics for to calculations of physical properties of partially ionized plasma is impossible. There are enough exact methods for measuring of physical properties of partially ionized plasma. For these reasons the theoretical method using the base of statistic physics was used to calculate the composition and physical properties of various kinds of partially ionized plasma.

2. PRINCIPLE OF THE METHOD
The composition of partially ionized plasma, in the state of local thermodynamic equilibrium (LTE), in the elaborated method, is determined by solving a corresponding system of nonlinear algebraic equations. The number of the equations in this system is equal to the number of assumed components of plasma. For the assumed n components of plasma, that is made up of k elements, the system of equations is composed of n equations:

- n-(k+1) equations of equilibrium constants of dissociation, ionization and formation of negative ions reactions in plasma,
- k-1 equations of the balance of elements,
- 1 equation of the balance of partial pressures of particular components of plasma,
- 1 equation of the balance of electric charges.

The equilibrium constants of chemical reactions in plasma were defined by the relation

\[ K_p = \left( \prod_{i=1}^{1} \left( \frac{Q_{B_i}}{N} \right)^{b_i} \right) \left( \prod_{i=1}^{j} \left( \frac{Q_{A_i}}{N} \right)^{a_i} \right) \exp \left( - \frac{\Delta e}{kT} \right) \]  

(1)

where
- \( Q_{A_i}, Q_{B_i} \) - temperature functions of the sums of states of the plasma components \( A_i, B_i \),
- \( a_i, b_i \) - number of molecules, atoms, ions or electrons taking part in the given chemical reaction,
- N - Avogadro’s constant,
- k - Boltzmann’s constant.
- \( \Delta e \) - energy of activation of the chemical reaction at the temperature 0 K,
- T - temperature of the plasma.

The relations, describing the temperature functions of the sums of
states of particular components of the plasma, are given below:

- Atoms, one-atom ions and electrons

\[ Q_1(T) = N \left( \frac{2\pi M_1}{h^2 N} \right) \frac{T}{k} \sum_{j=0}^{\infty} q_{ij} \exp \left( - \frac{E_{ij}}{kT} \right) \]  

- Linear polyatomic molecules and ions

\[ Q_1(T) = N \left( \frac{2\pi M_1}{h^2 N} \right) \frac{T}{k} \sum_{j=0}^{\infty} q_{ij} \exp \left( - \frac{\nu_{ij}}{kT} \right) \]  

- Nonlinear polyatomic molecules and ions

\[ Q_1(T) = N \left( \frac{2\pi M_1}{h^2 N} \right) \frac{T}{k} \sum_{j=0}^{\infty} q_{ij} \exp \left( - \frac{\nu_{ij}}{kT} \right) \]  

where

- \( M_1 \) - mass of the "i" component of gas,
- \( h \) - Planck's constant,
- \( E_{ij} \) - "j" electronic energy level of the "i" gas component,
- \( q_{ij} \) - statistic weight of the "j" electronic energy level,
- \( \nu_{ij} \) - frequency (energy) of fundamental vibration of i-th molecule or ion,
- \( A_1, B_1, C_1 \) - constants connected with the moments of molecule or ion inertia,
- \( \sigma_i \) - coefficient of molecule or ion symmetry,
- \( m \) - number of atoms in molecule or ion.

<table>
<thead>
<tr>
<th>Molecules or ions</th>
<th>Linear</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( j(m) )</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

On the basis of determined composition of plasma physical properties of plasma were defined. The relations, describing the physical properties of plasma components, in the function of temperature, were determined by their sums of states.

The total enthalpy of plasma is the sum of its physical enthalpy and chemical energy and was calculated according to the relation
\[
\begin{align*}
\bar{h} &= \frac{1}{\sum_{i=1}^{n} M_i P_i} \sum_{i=1}^{n} P_i \left[ \Delta H_i(T_o, T_p) + \Delta H_i(T_o) \right] \\
\end{align*}
\]

where

- \( M_i \) — mass of \( i \) kmol of "i" plasma component,
- \( P_i \) — partial pressure of "i" plasma component,
- \( n \) — number of plasma component,
- \( \Delta H_i(T_o, T_p) \) — physical molar enthalpy increment of "i" plasma component between the reference temperature \( (T_o) \) and the plasma temperature \( (T_p) \) determined in Section 3,
- \( \Delta H_i(T_o) \) — chemical energy of "i" plasma component in the reference temperature \( (T_o=298.15 K) \).

The increment of the physical enthalpy of the particular components of plasma \( (\Delta H_i(T_o, T_p)) \) have been determined from relation

\[
\Delta H_i(T_o, T_p) = R \left[ T^2 \left( \frac{\text{d}Q_i}{\text{d}T} \right) - T_0^2 \left( \frac{\text{d}Q_i}{\text{d}T} \right) \right]_{T=T_0}
\]

where

- \( Q_i \) — sum of states of the "i" plasma component (relations 2, 3 and 4).

The mean specific heat of plasma in the function of temperature, can not be determined as an averaged function of the temperature functions of specific heats of the particular components of plasma. It is due to simultaneous proceeding of physical and chemical processes in plasma. Elaborated method takes into account variation of composition and chemical enthalpy of plasma in the function of temperature. Taking all this into account mean specific heat should be determined with the help of following formula

\[
c_{pm} = \frac{1}{\sum_{i=1}^{n} M_i P_i} \sum_{i=1}^{n} P_i c_{pi} + \frac{1}{2\Delta T} \left[ \frac{1}{\sum_{i=1}^{n} M_i P_i} \sum_{i=1}^{n} (P_{i1} - P_{i2}) \Delta H_i(T_o) \right]
\]

where

- \( M_i \) — mass of 1 mole of the "i" component of plasma,
- \( n \) — number of plasma components,
- \( P_{i1} \) — partial pressure of "i" component of plasma in temperature \( T \),
- \( P_{i2} \) — partial pressure of "i" component of plasma in temperature \( T+\Delta T \), in the calculations assumed \( \Delta T = 3 K \),
- \( c_{pi} \) — molar specific heat of "i" component of plasma in temperature \( T \) as a function of sum of states,
- \( \Delta H_i(T_o) \) — molar chemical enthalpy of "i" component of plasma in the reference temperature \( T_o \).

The mean molar mass, isentropic exponent, density and gas constant for various kinds of partially ionized plasmas were calculated with the help of relations valid for gaseous mixture.

Concentrations of electrons in plasma was calculated with the help of equation
where $p_e$ - partial pressure of electrons in plasma (calculated).

The formula for electrical conductivity of partially ionized plasma was presented in [1]. The other electrical properties of partially ionized plasma were calculated with the help of electron concentration in plasma ($n_e$) and collision cross-sections for electrons.

3. CALCULATIONS

Calculations were carried out with the help of computer programme elaborated on the basis of the mathematical model of physical and chemical proceeding in plasma. Some results of calculations are shown in the function of temperature in figure 1 and figure 2.

4. REFERENCES


Figure 1. Dependence of specific heat of argon plasma, helium plasma hydrogen plasma and nitrogen plasma on temperature

Figure 2. Dependence of degree of ionization of argon plasma, helium plasma, hydrogen plasma and nitrogen plasma on temperature
THERMODYNAMIC FUNCTIONS FOR PARTIALLY IONIZED PLASMAS
IN THE MEAN SPHERICAL APPROXIMATION

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Abstract

We present a free energy model for partially ionized plasmas of heavier elements, which is essentially based on the mean spherical approximation (MSA) and the reduced-volume hypothesis, and discuss its limiting behavior and numerical examples for xenon.

Free energy model

Our total Helmholtz free energy density of the plasma,

\[ \frac{F}{V} = f_N + f_D^X + f_{\text{MSA}} + f_{\text{xc}} + f_{\text{ie}} \]  (1)

consists of the usual Boltzmann part for atoms and ions, of the Fermi-Dirac part for free electrons in the reduced volume /1/, and of several interaction contributions which are explained in the following. MSA labels the excess contribution of the interaction among the heavy particles /2/, relative to the contribution of uncharged hard spheres /3/. By using the averaged value \( \langle F(z) \rangle = \frac{\sum_{n=0}^{\infty} n \cdot F(z) }{\sum_{n=0}^{\infty} n} \), we may write in compact form

\[ \beta F_{\text{MSA}} = \frac{K_i^2}{3\pi} - 1 \left( K_i n \langle \frac{z^2}{1 + K_i d_z} \rangle + \frac{R^2}{W} \right), \quad l = \frac{\beta e^2}{4\pi\varepsilon_0}, \\
R = n \langle \frac{zd_z}{1 + K_i d_z} \rangle, \quad W = \frac{2}{\pi} + \frac{1}{3} n \langle \frac{2 - K_i d_z}{1 + K_i d_z} \rangle. \]  (2)

The inverse screening length \( K_i \) of the heavy particles has to be obtained as the iterative solution of the nonlinear equation

\[ K_i^2 = n \langle \left( \frac{z - \frac{R}{W} d_z}{1 + K_i d_z} \right)^2 \rangle. \]  (3)

The exchange-correlation contribution of free electrons in the reduced volume is an analytic formula, derived from a Padé-type fit to numerical results for the internal energy in the Singwi-Tosi-Land-Sjölander scheme /4/ and then scaled according to

\[ f_{\text{xc}}^X(n_e, \eta) = f_{\text{xc}}(n_e^\star), \quad n_e^\star = \frac{n_e}{1-\eta}, \quad \eta = \frac{\pi}{6} n \langle d_z^3 \rangle. \]  (4)
Finally we constructed the excess contribution of the interaction between heavy particles and electrons in the same manner as it was done for point-like charges in the Debye limit /5/,

$$\beta f_{ie} = \frac{2}{3\pi} \left( K_i^3 + K_e^3 - K_e^3 \right).$$  \hspace{1cm} (5)

The inverse Thomas-Fermi screening length $K_e$ /6/ of free electrons is determined by the Fermi-Dirac part of their chemical potential, where $\beta_F$ is the Fermi integral and $\Lambda = \hbar (2\pi m_e / \beta)^{1/2}$,

$$K_e^2 = \pi \left( \frac{2}{\Lambda^3} \right) I_{-1/2} \left( \beta_F \right) + I_{1/2} \left( \beta_F \right) = \frac{1}{\gamma} n_e^2 \Lambda^3,$$ \hspace{1cm} (6)

and the sum of both screening contributions, Eqs. (3) and (6), defines the inverse total screening length $K$ according to

$$K^2 = K_i^2 + K_e^2.$$ \hspace{1cm} (7)

**Limiting behavior of the Coulomb contributions**

In the case of a dilute plasma, $\eta \to 0$, with classical electrons, $K_i$, $K_e$, and $K$ approach one half of the corresponding inverse Debye screening lengths, and for the Coulomb contributions to the free energy in Eq. (11) we recover the Debye values so that

$$\beta \left( f_{\text{MSA}} + f_{\text{MC}} + f_{ie} \right) \rightarrow -\frac{\gamma^2}{12\pi}, \quad \gamma^2 = 4\pi \left( n<z^2> + n_e \right).$$ \hspace{1cm} (8)

Deviations from this behavior at higher densities are caused by the finite size of the heavy particles and by degeneration and coupling effects of the free electrons. As an example, we give the MSA excess contribution up to the lowest-order correction,

$$K_i \rightarrow -\frac{1}{2} \chi_i - \pi \left( n<z^2_d> \right) = \chi_i^2 = 4\pi \left( n<z^2> \right),$$ \hspace{1cm} (9)

$$\beta f_{\text{MSA}} \rightarrow -\frac{\chi_i^3}{12\pi} + \frac{\chi_i^2}{4} \left( n<z^2_d> \right).$$

In the opposite case of a dense plasma, $\eta \to 1$, with degenerate electrons, the atoms and ions cannot contribute to the screening any longer, $K_i \rightarrow 0$, and hence $K \rightarrow K_e$ and $f_{ie} \rightarrow 0$. Further,

$$K_e \rightarrow \frac{1}{a \left( \alpha^2 \right)_{1/2}}, \quad \alpha = \left( \frac{3}{4\pi n a z^3} \right)^{1/3},$$ \hspace{1cm} (10)

$$\beta f_{\text{MSA}} \rightarrow -\frac{\chi_i^2}{4\pi d} \quad \text{for} \quad d \approx d \quad \text{and} \quad <z^2> \approx <z^2>.$$  

Note that the neutral atoms, though generally present in the MSA, do not appear in neither of the limits, Eqs. (9) and (10).
Application to partially ionized xenon plasma

Motivated by the permanent interest in the equilibrium properties of heavy noble gas plasmas [7], we applied the free energy model as outlined in Eqs. (1)-(7) to fluid xenon up to 50,000 K over a broad density range in the region of partial ionization. The hard-sphere diameters of the heavy particles were estimated by the simple scaling relation $d = 2a_0/\sqrt{T}\text{eV}/8$, where $T$ is the unperturbed ionization energy, for lack of more reliable values especially for the higher charged ions.

The limiting behavior of the inverse screening lengths (Fig. 1) and the relevance of the various contributions to the chemical potentials (Fig. 2) of the atoms and ions, $\mu_i = \partial f/\partial n_i$, and of the free electrons, $\mu_e = \partial f/\partial n_e$, are conveniently studied by assuming an arbitrarily fixed plasma composition, for which we chose two-fold charged ions with the density $n = n_2 = \frac{1}{2}n$. The spatial extension of the ions leads to a substantial decrease of their inverse screening length compared to the Debye value, cf. Eq. (9). In the dense fluid range $K$ exhibits a maximum and the inverse screening length of the free electrons reaches the Thomas-Fermi value, Eq. (10). The behavior at still higher densities cannot be adequately described due to the limitation of the MSA to the fluid hard-sphere system. The vanishing of $K$ and, consequently, of the ion-electron interaction contributions to the free energy, $f_i$, and to the chemical potentials, $\partial f/\partial n_i$ and $\partial f/\partial n_e$, only occurs very close to the unphysical limit $\eta^2 = 1$ and, therefore, has no significance. Moreover, the simple construction of Eq. (5) actually gives the Debye value, Eq. (8), but does not provide an appropriate treatment of the ion-electron interaction up to $\eta \approx 0.5$. Above a certain density it starts to yield qualitatively wrong results what is indicated by the change of sign in $\partial f_{\text{ie}}/\partial n_i$; see curve ie in Fig. 2.

![Fig. 1 (left): The inverse screening lengths and some of their limits. i - ionic contribution from Eq. (3); e - free-electron contribution, Eq. (6), and its degenerate limit, see Eq. (10); inverse total screening length, Eq. (7), and its Debye limit.](image1)

![Fig. 2 (right): The chemical potentials of the ions (i) and of the free electrons (e) and the ion contributions $\partial f_{\text{M}}/\partial n_i$, $\partial f_{\text{M}}/\partial n_i$, $\partial f_{\text{M}}/\partial n_i$, $\partial f_{\text{M}}/\partial n_i$, and $\partial f_{\text{M}}/\partial n_i$.](image2)
Fig. 3 (left): The detailed plasma composition at one temperature in terms of the relative abundances $\alpha = n_i/n$ of atoms and ions, $i = 0, 1, 2, 3, \ldots$, versus total heavy-particle density $n$. 

Fig. 4 (right): Thermal equation of state: Pressure isotherms versus total heavy-particle density for three temperatures.

Then we determined the ionization equilibrium (Fig. 3) and the thermal equation of state (Fig. 4) /7/. As in /9/, the pressure isotherms do not show unstable branches above $\approx 8,800$ K so that recent estimates /10/ of the critical temperature of the plasma phase transition /11/ seem to be far too high. However, this finding is strongly influenced by the ion-electron interaction, for a former choice of $f_e$ /5/ shifts our $T_c$ towards 14,200 K. This illustrates the need for further improvements of the model before a new phase diagram for fluid xenon can be calculated.

Acknowledgements. We are very grateful to Professor W. Ebeling and Dr. A. Förster (both Humboldt University, Berlin) for suggesting the application of the MSA to non-ideal plasmas and for several useful discussions during the preparation of this paper.

References

DIELECTRIC CONSTANT OF WEAKLY-IONIZED MAGNETIZED PLASMAS

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Independently of the very large activity in the field of the low-frequency instabilities and turbulent state of magnetized plasmas /1,2/ and the current direction to investigating nonlinear phenomena in weakly-ionized plasmas /3,4/, the complicate situation in the experiments in the gas-discharge plasmas submits many questions concerning the identification of the observed instabilities. One of the reasons for these complications is the anomalous behaviour of the ambipolar electric field in the range Ω_i ≈ ν_i, ρ_i ≈ R where the so-called low-frequency drift-wave instability associated with the finiteness of the ion gyroradius ρ_i /5-7/ exists. (Here Ω_i is the ion gyrofrequency, ν_i is the ion-neutral collision frequency and R is the plasma column radius.) The impossibility for explaining our recent experimental results /8/ in terms of the dispersion relation of the low-frequency drift wave instability /6/ has stimulated a discussion on the dielectric function of weakly-ionized inhomogeneous strongly-collisional plasma in crossed magnetic and electric fields. The result we present here for the plasma dielectric function rectifies that given in /6/ and appears as a next stage which generalizes the previous derivations of the permittivity tensor presented in /9/.

In the equilibrium state we have in mind a d.c. gas discharge plasma imposed in a steady-state magnetic field \( \mathbf{B} = (0, 0, H) \) with density inhomogeneity in transverse direction \( n(x) \) and steady-state electric fields \( \mathbf{E}_o = (E_{ox}, 0, E_{oz}) \), i.e. an ambipolar field and a field sustaining the discharge. For describing the plasma behaviour we start from the Boltzmann equation for the particle distribution function \( f_\alpha (\alpha = \text{e}, \text{i}) \) with a collision integral
which accounts for the collisions of the charged particles with the neutrals in BGK-approximation /9/:

\[ S_t (\phi) = -\nu_\alpha (f - n \phi_\alpha) \]  

where \( \phi_\alpha = (m_\alpha / 2\pi T_\alpha)^{3/2} \exp(-m_\alpha v^2 / 2T_\alpha) \).

The effect of the collisions (\( \nu_\alpha \)) is taken into account not only in the equilibrium state (as it is done in /9/) but in the equilibrium state, as well. The ambipolar field \( E_{ox} \) leads to a \( E \times B \)-drift \( u = -cE_{ox}/H_0 \) and the influence of the \( E_{oz} \)-field can be taken into account in the final result for the electron contribution through a Doppler shift \( kU_\phi \) of the frequency \( (U_\phi = -eE_{ox}/m_\alpha e) \). The equation describing the distribution function \( f_\alpha \) in the equilibrium state is:

\[ \frac{\partial f_\alpha}{\partial t} + \Omega_\alpha (v_y - u) \frac{\partial f_\alpha}{\partial v_x} - \Omega_\alpha v_x \frac{\partial f_\alpha}{\partial v_y} = -\nu_\alpha f_\alpha (v) + \nu_\alpha n \phi_\alpha (v) \]  

and it is solved after consecutive transformations to frames: \( v_x = v_{xc} = v \cos \varphi, v_y - u = v_{yc} = v \sin \varphi, v_z = v_z \). The set of the characteristic equations of eq. (2) give the integral:

\[ \Omega_\alpha x + v_{yc} = \text{const.} \]  

and the following result for \( f_\alpha \):

\[ f_\alpha = n_\alpha (x)f_\alpha (1 + av - bv) \]  

where \( f_\alpha = (m_\alpha / 2\pi T_\alpha)^{3/2} \exp(-m_\alpha (v_x^2 + v_z^2) / 2T_\alpha) \),

\[ a = \frac{\Omega_\alpha}{\Omega_\alpha^2 + \nu_\alpha^2} \left[ 1 - \frac{m_\alpha \nu_\alpha^2}{\Omega_\alpha} \right], \quad b = \frac{\nu_\alpha}{\Omega_\alpha^2 + \nu_\alpha^2} \left[ 1 + \frac{m_\alpha \Omega_\alpha}{\nu_\alpha} \right], \]

\( \Omega_\alpha = eH / mc \) and \( \nu = (1/n) \left( \frac{dn}{dx} \right) \). The last two terms in expression (4) describe the contributions of the plasma inhomogeneity, of the collisions and of the particle drift in the ambipolar electric field. The characteristic equations of the first order approximation of the Boltzman equation:

\[ \frac{\partial f}{\partial t} + (\hat{v} \cdot \hat{v})f_1 + e\hat{E} \frac{\partial f}{\partial \hat{v}} + \frac{e\hat{E}}{m_\alpha} \frac{\partial f}{\partial \hat{v}} + \frac{e\hat{E}}{m_\alpha} \frac{\partial f}{\partial \hat{v}} \]
used for describing quasi-static perturbations \( E_i = \nabla \phi_i \) of the
type \( f_i \alpha f_i(x) \exp(-i\omega t + ik_y y + ik_z z) \) lead to the integral
(3) and to:
\[
\frac{d}{d\rho} + i \frac{1}{\Omega_\alpha} \left[ \omega + iv_i \alpha - k_y y_c - k_z z_c \right] f_i \alpha
\]
\[
= -i \frac{m_\alpha}{\Omega_\alpha} \int_{-\infty}^{\infty} \right\{ \gamma \n_i \exp(- \frac{m_\alpha}{2T_\alpha} (u^2 + 2uv_y)) \}
\]
\[
-i \frac{e_\alpha}{m_\alpha} \int \left[ k_{\alpha} \gamma \exp(1 + av_y - bv_x) - k_{\alpha} \right]
\]
\[
+k_{\alpha} \frac{m_\alpha}{T_\alpha} \gamma \left( 1 + av_y - bv_x \right) \}.
\]

Levich calculations performed according to the method of integration over \( \phi / g \) gives a solution of eq. (6) of the form:
\[
\frac{n_{z\alpha}}{n_0} \beta = -\frac{e_\alpha}{T_\alpha} (A_\alpha + C_\alpha)
\]

which results into the following expression for the
longitudinal plasma permittivity:
\[
\varepsilon = 1 + \sum \frac{\omega_p^2 \alpha}{k^2 v_\alpha^2} \frac{A_\alpha + C_\alpha}{B_\alpha}
\]

where \( v_\alpha = (T_\alpha / m_\alpha)^{1/2} \), \( \omega_p = (4\pi e^2 n_0 / m_\alpha)^{1/2} \) and
\[
A_\alpha = 1 - \omega' - k_{\alpha} \gamma \frac{A_{\alpha}}{n_{\alpha}} \left( \frac{A_{\alpha}}{v_\alpha^2} \right)
\]
\[
B_\alpha = 1 - i\nu_{\alpha} \sum_{n=-\infty}^{\infty} \frac{A_{\alpha} (1-\beta_{\alpha}^2)^{1/2}}{\omega' - n\Omega_{\alpha}} J_0 (\beta_{\alpha}) \left( \frac{1 - \beta_{\alpha}}{1 + \beta_{\alpha}} \right)^{n/2}
\]
\[
C_\alpha = -i \frac{b\omega_{\alpha}}{k_y} \left\{ (\omega' - i\nu_{\alpha}) \sum_{n=-\infty}^{\infty} \frac{A_{\alpha} (1-\beta_{\alpha}^2)^{1/2}}{\omega' - n\Omega_{\alpha}} J_0 (\beta_{\alpha}) \right\}
\]
\[
+i \frac{\nu \omega'}{\nu^2} (1 - \nu) - \nu \Omega_n \sum_{n=\infty} - \frac{\nu_n(z_\alpha)}{\nu^2_n(z_\alpha)} \beta_n \lambda_n(z_\alpha) \frac{\nu_n(z_\alpha)}{\nu^2_n(z_\alpha)} J_n(\beta_n \lambda_n) \}
\]

with \( \omega' = \omega + i \nu - k u, \) \( \beta_i = u \Omega / k v^2, \) \( v = k^2 \rho^2, \)
\( \beta_n = (\omega - n \Omega) / |k|v, A_n(z_\alpha) = \exp(-z_\alpha) I_n(z_\alpha), \)
\( J_n(x) = x \exp(-x^2/2) \int \exp(\tau^2/2) d\tau. \)

In the results (7) and (8), \( B_\alpha \) accounts for the effect of the collisions when the collision integral is included only in the equation for \( f_{1\alpha} \) whereas \( A_\alpha \) with the first term in a (expression (5)) accounts for the net effect of the inhomogeneity. The contribution to \( A_\alpha \) given by the second term in a together with \( C_\alpha \) are the new terms (in comparison with /9/) which are due to the combined effect of the collisions, of the ambipolar field and the inhomogeneity on the distribution function in the stationary state. These terms differ from those obtained in /6/ where a relation which should correlate to (7) is obtained by using the method of integration over the trajectories. The calculations we have made by using the same method confirm the our result (7).

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/7/ M.I. Belavin, A.V. Timofeev, B.N. Shvilkin, Fizika Plasmy 6, 705 (1980)
ION BERNSTEIN WAVES IN A TOROIDAL
STEADY-STATE DEVICE

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INTRODUCTION

In recent years the Ion Bernstein Waves Heating (IBWH) has been subject of great interest, as it allows a direct heating of the bulk ions of tokamak plasmas. IBWs can be indirectly excited via Electron Plasma Waves (EPW) mode transformation or directly launched into the plasma. In this paper we present some experimental results on IBW propagation in a steady-state magnetized plasma. The aim of the experiment is the study of the basic phenomena involved in the IBW propagation characteristics. Moreover the dispersion relation of IBWs is strongly dependent on the ion temperature $T_i$ and hence it offers the possibility of an indirect diagnostic of this plasma parameter. This method was already used in a previous work /1/ where the dispersion relation of Neutralized Ion Bernstein Waves (NIBWs) was experimentally reconstructed. The use of IBWs should confirm the validity of this diagnostic method.

EQUIPMENT SET-UP

The experiment was performed on the Thorello device, which has the following main characteristics: major radius $R = 40$ cm, minor radius $r = 9$ cm, on-axis magnetic toroidal field $B = 2.3$ kG /2/. The steady-state magnetized plasma was produced in hydrogen neutral gas by means of hot-filament electron emission and voltage-induced acceleration. The electron density and temperature were measured with electrostatic Langmuir probes.


Typical parameters of the plasma are as follows: density $n_e = (10^8 - 10^{10})$ cm$^3$, electron temperature $T_e = 1$ eV, ion temperature $T_i = 0.3$ eV, neutral gas pressure $P = (10^{-5} - 10^{-4})$ mbar. The capability of operating in different discharge regimes makes the weakly ionized plasma of Thorrello particularly suitable to study plasma-waves interaction phenomena. In particular Ion Bernstein Waves are analyzed at the second harmonic of the ion cyclotron frequency, typically $f = (3-6)$ MHz.

The waves are excited by means of a launching system located at the plasma edge, in the low field side. The antenna consists of four thin metallic blades lying in a vertical plane, along the direction of B and fed with a relative phase of 90°. In this configuration the antenna system excites waves with $k_\parallel \leq 0.1$ cm$^{-1}$; the wavevector component perpendicular to B, $k_\perp$, was measured by means of an interferometric system.

The signal propagating in the plasma is detected by means of a double r.f. probe moving radially in the plasma; it is compared to the input signal with a mixer. The interferometric trace, proportional to the wave electric field, obtained by filtering the mixer output, in order to suppress time-dependent terms, is sent to the Y-channel of a digital oscilloscope.

**EXPERIMENTAL RESULTS**

The experimental results were compared with the numerically solved full electrostatic dispersion relation in the range of the ion cyclotron frequency /3/.

The IBW are indirectly excited via EPW mode transformation in the range of the second ion cyclotron harmonic frequencies $f = 2f_i = 6$ MHz. The EPW is coupled to the plasma edge by means of a slow-wave antenna, with a good efficiency $\leq 50\%$ at low power level ($\leq 1$ W). The EPW to IBW transformation process is located in the resonant region where the injection frequency is equal to the Lower Hybrid frequency ($\omega = \omega_{hi} = \omega_{pi}$).

The experimental interferogram shown in Fig. 1, gives the dependence of the wave electric field versus the probe distance from /3/ Petrillo V., Maroli C., Fasoli A., Galassi A. and Longari C., Il Nuovo Cimento 11D, 1337 (1988).
the antenna system, obtained in Hydrogen plasma at the frequency $f = 1.85 f_i$ ($f_i$ ion cyclotron frequency). The wave signal has been detected with an r.f. probe located at a toroidal angle of 100° with respect to the antenna system.

The lower hybrid resonance region is very close to the antenna surface and the conversion process is located at few millimeters from the antenna; therefore the only IBW appears in the interferometric trace. By varying the transmitted frequency and measuring at a fixed radial position the wavelength, the dispersion relation can then be experimentally reconstructed.

In Fig. 2 the experimental values of the $k_{\perp}$ real component, obtained in the range of the frequencies between 1.8 $f_i$ and 2.0 $f_i$, at 3 cm from the antenna, have been compared with the theoretical dispersion relation. The 10% experimental systematic error at lower and higher frequencies is due to the plasma disomogeneities along a wavelength distance.

The strong dependence of the backward branch wavelength with respect to the ion temperature makes possible to obtain an estimate of the local ion temperature. In the case of figure 2, the estimated ion temperature is 0.3 eV.

CONCLUSIONS

The propagation of IBW has been experimentally investigated in hydrogen plasmas produced in a steady-state toroidal device Thorello. It results that a slow-wave antenna efficiently launches the EPW, which propagates along the magnetic field and then is converted in IBW. The experimental reconstruction of the dispersion relation has been obtained by means of an interferometric system. Comparing the experimental relation dispersion with the theoretical one, it has been possible to perform an indirect diagnostic of the ion temperature.

ACKNOWLEDGMENTS

We would like to thank Prof. G. Lampis, dr. A. Cardinali and dr. S. Alba, for the very helpful discussions and Mr. G. Braga for the technical assistance.
Fig. 1. - Plot of experimental interferogram in a. u., versus the plasma radius, obtained in H2 plasma for $n_e = 10^{10}$ cm$^{-3}$, $T_e = 1$ eV, $T_i = 0.3$ eV, neutral gas pressure $P = 6 \times 10^{-5}$ mbar, $f = 5.5$ MHz; the limiter is located at the origin $x = 0$.

Fig. 2. - Fitting of experimental data by theoretical dispersion relation in H plasma with $n_e = 10^{10}$ cm$^{-3}$, $T_e = 1$ eV, $T_i = 0.3$ eV, neutral gas pressure $P = 6 \times 10^{-5}$ mbar, $f = 5.5$ MHz.
Study of Low Frequency Flute Type Oscillations and their nonlinear Interaction in Toroidal Plasma

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Magnetically confined low-\beta plasma is subjected to basically two types of low-frequency (\(\omega \ll \Omega_i\), where \(\omega\) is the observed frequency of the wave and \(\Omega_i\) is the ion gyrofrequency) flute (k-B = 0) instabilities viz., Kelvin – Helmholtz instability driven by velocity shear and curvature induced Rayleigh – Taylor instability (R–T). A number of experimental investigations pertaining to such instabilities have been carried out in different magnetic field geometries (Sugai et al., 1977; Komori et al., 1978; Bora 1989 and references therein).

In all the above experiments the study of instability is confined to identification of the self excited modes and comparison with theoretical dispersion relations. In this paper we report a detailed study of the curvature induced R–T instability in a plasma confined by a pure toroidal magnetic field with its evolution from a coherent state to a turbulent state. The experiments have been carried out in toroidal device BETA and diagnostics and data acquisition system is described in detail by Prasad et al. (1992). The data is analysed using standard FFT technique as outlined by Smith et al., (1974) in order to obtain information of the cross power, mode number and coherence spectrum of the fluctuations.

The plasma density and floating potential profiles at 200 gauss of toroidal magnetic field is shown in figure 1. The density and potential profiles exhibit sharp gradients between radial locations from 0 to ±3 cm and the rest of the regions have weak gradients. The density gradient scale length (\(L_n = \left[1/|\partial n/\partial r|\right]^{-1}\)) as calculated from figure 1 between radial locations 0 to ±3 cm is \(\approx 6\) cm and between radial locations ±3 to ±9 \(\approx 10\) cm. The electric field between radial locations 0 to ±3 cm is \(\approx 6.3\) V/cm and between radial locations ±3 to ±9 cm is \(\approx 0.6\) V/cm. The electron temperature varies from 13 to 18 eV between radial locations 0 to ±3 cm and has a value between 5 to 9 eV at other radial locations. Plasma has magnetised electrons (\(\nu_{en}/\Omega_e \ll 1\)) and ions (\(\nu_{in}/\Omega_i \ll 1\)).

The instability results presented here are obtained from measurements made in the regions away from the sharp density and electric field gradient locations. The measured value of \(\tilde{n}_{en}/n_e\) at 200 gauss of toroidal magnetic field varies from 10 to 50\%. The fluctuations have \(\tilde{n}/n_e \approx c\phi/T_e\) at all radial locations. The curvature of the applied
magnetic field represents by effective gravity \( g = C_s^2/R \) where \( C_s \) is the ion-sound speed and \( R \) is the major radius. The plasma in bad curvature region is subjected to R–T instability. In our experiment electric field varies nonuniformly with the radial locations \( (E(r) \) as shown in figure 1). This gives rise to a velocity shear \( \perpendicular \) to \( B \), creating a possibility of exciting velocity shear instability. Due to curvature effects in curved magnetic fields, velocity shear instability tends to localize in the bad curvature region (Sugai et al., 1977). However, Guzdar et al.(1984) investigated the influence of velocity shear on R–T instability and arrived at a conclusion that the presence of velocity shear increases the growth rate of long wavelength portion of the R–T instability and reduces the growth rate of short wavelength portion of the R–T instability.

At low toroidal magnetic fields (\( \leq 200 \) gauss) the fluctuations observed in floating potential and density are coherent. Cross power spectra of the oscillations picked up in bad curvature region of the plasma for various magnetic fields are shown in figure 2. The spectrum at 200 Gauss exhibits oscillations at frequency \( 2.8 \pm 0.3 \) kHz together with peaks at integer multiples of this frequency with mode number \( m = 1, 2, 3, 4 \). The frequencies of these modes are less than \( \Omega \). The phase spectrum of these fluctuations indicates that the waves travel in azimuthal direction with a phase velocity \( \simeq 2.4 \times 10^5 \) cm/sec. The coherence spectrum \( (\gamma_{12}(f)) \) of fluctuations picked by two probes one of which was located in the good curvature region and other in the bad curvature region at a radial location 8 cm on the equatorial plane exhibits \( \gamma_{12} \approx 0.9 \) for the frequencies where the cross power peaks. This indicates that the same fluctuations are observed in both the regions. The phase difference between the density and floating potential fluctuations as measured by two probes separated by 5 mm exhibits a phase difference of \( 165^\circ \) up to frequency \( 15 \) kHz which suggest that the observed modes are of flute type. The

Figure 2: Cross Power spectra for density fluctuations at different magnetic fields. The vertical scale for each trace is shifted for clarity.
measurement of radial and parallel wave numbers indicate that $\lambda_\parallel \gg \lambda_r$. The localisation condition $k_r L_n \approx 1$ is marginally satisfied.

In this geometry we may have basically two types of flute instabilities viz., velocity shear instability, due to $V_{\theta}(r) = E(r)/B$ where $r$ is minor radius, and curvature induced instabilities. In a curved magnetic field the instabilities excited due to velocity shear and due to curvature of the magnetic field are coupled and modify one another. The velocity shear instability (Suagai et al., 1977) due to the curvature effects is localised in bad curvature region, which is also the favourable region for excitation of curvature induced Rayleigh–Taylor instability. The velocity shear scale length in our experiment is smaller than the azimuthal wave length, the radius of plasma and is nearly equal to the density scale length. The empirical value of the observed frequency for velocity shear instability for mode number $m$, as given by Perkins et al., (1971), is $\approx 0.2$ to $0.5$ times the maximum value of $\omega_E$. Here $f_E = mE/(2\pi Br)$. In our experiment for $m = 1$, $r = 8$ cm, and $E/B \approx 3 \times 10^6$ cm/sec, $f_E \approx 13$ kHz to $32$ kHz which is very large compared with the observed frequency. The real part of the frequency calculated using the dispersion relation given by Sugai et al. (1977) for velocity shear instability gives $f\approx 50$ kHz for $m=1$ mode, which is again very large compared with the observed frequency. The real part of the frequency for R–T instability (Komari et al. 1979) has dominant contribution from $k_{\phi} E/B$. In our experiment it is $\approx 4.7$ kHz. Guzdar et al. (1984) have discussed in detail the effect of velocity shear on the R–T instability. They suggest that pure R–T instability favours the excitation of $k_{\perp} L_n > 1$ modes while pure Kelvin–Helmholtz instability driven by velocity shear favours the excitation of $k_{\perp} L_n < 1$ modes. For generalized Rayleigh–Taylor instability, in which both these instabilities coexist, the velocity shear preferentially excites the long wave length modes of R–T instability. All these arguments indicate that the observed peak at $3$ kHz with $m = 1$, may be due to the curvature induced R–T instability favoured by the presence of velocity shear. The observation of similar fluctuations in the good curvature region in our experiment cannot be explained by linear theories. The reason may be that the large $E \times B$ motion of the plasma might carry these fluctuations, from the bad curvature region where they are generated, to the good curvature region. It is seen from figure 2 that the coherent fluctuations gradually disappear as the magnetic field is increased and the spectrum becomes turbulent beyond 600 Gauss.

The coherent modes observed at low magnetic field of 200 Gauss, besides being harmonics of the fundamental frequency at $3$ kHz, also satisfy the resonance conditions on $\omega$ and $k$ viz. $\omega_0 = \omega_2 + \omega_1$ and $k_0 = k_2 + k_1$. Bispectral analysis (Kim and Powers, 1979) is used to identify the spontaneously excited modes and nonlinearly coupled modes in self excited oscillations. Bicoherence is a measure of degree of phase coherence between the waves which is defined as follows:

$$b^2(\omega_1, \omega_2) = \frac{|B(\omega_1, \omega_2)|^2}{P(\omega_1)P(\omega_2)P(\omega_1 + \omega_2)}$$  \hspace{1cm} (1)$$

where $P(\omega)$ is autopower spectrum. If $b^2 = 0$ the three spectral components are completely independent and randomly phase mixed. The power at $\omega_0$ then has source other than the coupling between $\omega_1$ and $\omega_2$. $b^2 = 1$ will indicate a high degree of phase coherence, in which case the wave at $\omega_0$ will be produced by efficient coupling between the waves at $\omega_1$ and $\omega_2$. The variance on bicoherence is estimated by

$$\text{var}(b) = \frac{(1 - b^2(\omega, \omega))}{M}$$  \hspace{1cm} (2)$$
The strength of the coupling for the interaction is estimated by coupling coefficient which is defined as follows

\[ A_{1,2} = \frac{\mathcal{B}(\omega_1, \omega_2)}{\mathcal{P}(\omega_1)\mathcal{P}(\omega_2)} \]  

The bicoherence spectrum for the case of fluctuations at 200 Gauss is shown in figure 3a and the contour plot of the same is shown in figure 3b. The bispectrum has one prominent peak at \((f_a, f_a)\) with bicoherence of ~0.8 indicating that \(f_a\) is first harmonic of \(f_0\). The bicoherence spectrum has a small peak with bicoherence of 0.45 indicating that half the power is contributed by the nonlinear interaction of waves at \((f_a, 2f_a)\). There are strong peaks at interaction frequencies \((f_a, 3f_a)\) and \((f_a, 4f_a)\) with a bicoherence of 0.7 which cannot be analysed by using bispectral analysis.

To summarise, an experimental investigation of flute type electrostatic fluctuations in toroidal low-\(\beta\) plasma is reported. The transition from coherent state to turbulent state as a function of toroidal magnetic field strength is described. The fluctuations are coherent at low toroidal magnetic field values \((\leq 200 \text{ Gauss})\). The coherent mode with mode number \(m = 1\) and \(f \approx 3 \text{ kHz}\) is identified as due to curvature induced R-T instability favoured by velocity shear. When the toroidal field is increased the fluctuations become turbulent.

References

We report here rather astonishing observations in the motion of charged particles (electrons) in an (in general) inhomogeneous magnetic field in the presence of a retarding electrostatic potential. The motivation for this study is a theoretical prediction made for this system, based on a formalism developed by Varma. The comparison of the experimental results with theory is presented in the later part of the paper.

To study the above motion electrons from an electron gun are injected from one end of an SS vacuum chamber (3m long and 27cm dia) permeated by an axial magnetic field, with a small pitch angle (≤5°) to it. At the other end of the chamber is a Faraday cup detector which can be moved along the axis of the chamber so as to vary its distance from the gun. The Faraday cup itself consists of a grounded collector plate with a grid, at 10mm in front of it which can be raised to any desired potential. The electron gun, capable of producing a beam in the energy range zero and 3KeV, gives an electron beam which gets focussed to a fine spot (= 2mm dia) in the presence of a magnetic field, at points along the magnetic field at distances, \( L = \frac{2\pi n v_\parallel}{\Omega} \), away from the gun (\( v_\parallel \) = parallel velocity component and \( \Omega = eB/mc \), the gyrofrequency). The magnetic field in the chamber is produced by a set of 35 current carrying coils equally spaced at a distance of 8.5cm. The system is shown as a scale drawing in Fig 1.
The experiment is carried out by first fixing a certain distance \( L = 30 \text{cm} \), between the gun and the detector and choosing an electron energy \( E = 600 \text{eV} \), and a spatially averaged magnetic field \( B = 170 \text{gauss} \). The detector grid is then raised to a negative potential \( \Phi_m < \left( \frac{e}{c} \right) \). All the beam electrons are thus stopped by the retarding potential and the plate current is then recorded as the grid voltage is allowed to drop from \(-\Phi_m\) to zero. The record of the plate current as a function of the applied negative grid potential actually obtained in the experiment is shown in Fig 2a. The plot is clearly far from monotonic and indeed exhibits a number of maxima and minima whose separation decreases monotonically with the retarding potential \( \Phi \). Moreover, the envelope of the oscillations also exhibits a modulation with the potential.

However, before we discuss the various conventional possibilities that may conceivably lead to these effects we shall first see whether the allowed, observed energy peaks in Fig 2a can be fitted into the relation (1) given below which is supposed to describe these peaks according to the theory:

\[
E_j = \frac{1}{2} m \left( \frac{3 \Omega L}{2 \pi} \right)^2 \left( j + \frac{1}{4} - \frac{\phi}{2 \pi} \right)^2; \tag{1}
\]

where \( E_j \) is the energy corresponding to an integer \( j \) labelling the peak, \( j \) is an analogue of a quantum number, \( L \) is the distance between the electron gun and the detector, and \( \phi \) is an undetermined phase.

We choose for simplicity every third peak starting from a peak labelled as 'a' in the figure. The energies \( E_j \) corresponding to these peaks and the values of \( (j + 1/4 - \phi/2\pi) \) as calculated from (1) and the \( j \) (integer) values so identified are given in the following Table.
It is found that within the accuracy of the determination of the average magnetic field, the integers \( j \) do differ from each other by three in surprising conformity with the choices made. Lest it should be regarded as accidental, we have found such integer identifications for energy peaks with other values of the magnetic field and length \( L \). Of course, the \( j \) values for a particular neighbourhood of an energy value would be smaller or larger depending on the smaller or the larger value of \( \Omega L \). The important point, however, is that they do differ by unity for consecutive peaks; this is particularly significant when large \( j \) values (~35-50) are identified. It is, of course clear that the lower values of \( j \) would correspond to higher energies, for a given value of \( \Omega \) and \( L \), so that a higher energy beam is required to explore lower 'quantum numbers'. This has also been verified to be the case.

Since the observed behaviour is so unexpected à la the standard paradigm, it is desirable to try to rule out various conventional possibilities as its explanation. The first of these is the question whether the periodic structure of the grid may be playing a role. This was ruled out, both by measuring the grid current (besides the plate current) as well as by removing the grid and applying the bias to the plate itself as we measure the plate current. The observed effect persists in the latter case, while the grid current also (Fig 2b) shows a behaviour in phase with the plate current. Thus grid could not be playing any role in causing this effect. The grid current profile obtained for a different value of magnetic field \( B \) (= 210 gauss) is shown in Fig 2c.

Another possibility is to argue that the dips in the transmitted current may be caused by the scattering of the beam electrons by electrostatic fluctuations generated by some beam instability. However with the beam current as low as ~ 0.1 \( \mu \)A, and a vacuum of ~ 5.10^7 torr, the beam number density \( n \sim 10^4-10^5 \) cm\(^{-3}\), falls far too short for the beam instability condition to be satisfied.
It should finally be mentioned that perhaps, the most striking feature of the observed result, as indeed of the relation (1), is the dependence of the allowed energy peaks on the length $L$ of the box - the distance between the gun and the detector. Such a dependence signifies a wave-like nonlocality which is contained in the formalism of Ref 1, and hence the relation (1), but which is difficult to understand in terms of the standard initial value paradigm of classical mechanics.

In summary it is important to highlight again the following facts that we have demonstrated:

(i) That the discrete allowed states of motion do exist in the domain of parameters where one would use classical mechanical equation of motion to determine the motion.

(ii) That the energies of these allowed states are well represented by the relation (1) which in fact, formed the basis of the prediction. This relation is obviously non quantal, as there is no Planck quantum $\hbar$ in it.

(iii) That the allowed states $E_j$, form a hydrogen-like sequence for which 'quantum numbers' $j$ and the phases $\phi$ can be identified as shown in the Table.

(iv) That the allowed energy values $E_j$ and the associated quantum numbers $j$ depend in a continuous manner on the length $L$ of the 'box'. This is a manifestation of wave-like nonlocality which is not known to be a characteristic of the standard initial value paradigm of classical mechanics.

Finally, astonishing as it may sound, it is clear that because of the above mentioned characteristics of the observed effect, nonlocality in particular, the solution of the classical mechanical Lorentz equation of motion with appropriate initial conditions would not be able to reproduce the observed behaviour.

References:


It was shown that using a small and narrow transverse magnetic field applied near the hollow cathode of a d.c. glow discharge in He, different states of nearly globular luminous space charge structures (GLSs) formed in front of the anode can be obtained /1/. Depending on the intensity ($B_1$) and axial position ($d_{m1}$) of this magnetic field from the cathode (see also fig.1), the spatial dimensions and the transition from a regular to the chaotic behaviour of the GLSs can be determined.

The experimental device shown in fig.1 was used with regard to the influence of secondary electron emission at the discharge tube wall on the GLS stability /2/. Therefore, a small hollow anode $A_1$ was connected through the variable resistor $R_v$ with a stainless steel mesh grid $A_2$, which covers the inner surface of the glass tube wall and acts as a second anode /3/.

Thus, without changing the discharge current $I$, the current-voltage $I_1-U_{12}$ characteristic of the circuit between the small anode and the large one, can be plotted as in fig.1.

It is seen that, almost independent on the discharge current $I$, the $I_1-U_{12}$ characteristics are showing two different branches with negative slopes (indicated by the
dotted segments). The first one is started at very small currents $I_1$ through the small anode and potential differences to the large one of about $U_{12} = 30$ V, when a small but stable GLS is also pelling off from the small anode cavity. For an increase of the variable resistance with approx. 50% from its initial value of $R_o = 2$ kΩ, this first negative slope disappears at currents of $I_1 = 2$ mA and potential differences of at least $U_{12} \geq 24$ V. By the further increase of the resistance $R_o$ to about $\sim 10$ kΩ, the spatial dimensions of a general unstable GLS are gradually increased up to the conditions when the second negative slope on the same $I_1$-$U_{12}$ characteristic is set in. These slopes are obtained for currents $I_1$ which are approx. 75% of the discharge current $I$ and potential differences which are not exceeding $U_{12} \leq 40$ V. During the run through this second negative slope, the spatial dimensions and structure of the GLSs are suddenly changed together with their temporal behaviour. Finally, the maximum $I_1$-$U_{12}$ values are obtained for $R_o \geq 100$ kΩ when the large anode returns to its usual floating potential. Obviously, the value of the negative resistances, which correspond to these two negative slopes, are also differing by an order of magnitude.

In the same figure the values of the fundamental frequency for the current and potential fluctuations in the anode circuit are indicated by dots. Those amplitudes can attain up to 20% of their d.c. values, but the fundamental frequencies remain nearly constant over the $I_1$-$U_{12}$ domain situated above the second negative slope.

Therefore, to obtain a better insight into the correlation between the features of the $I_1$-$U_{12}$ characteristics and the unstable behaviour of the GLSs, we have analyzed the frequency spectra of these current and potential fluctuations for different He pressures and transverse magnetic fields. Thus, for an increasing He pressure (fig.2) and magnetic field intensity near the cathode (fig.3) or the GLS (fig.4), the chaotic behaviour of the unstable GLSs and also the second negative slope on the $I_1$-$U_{12}$ characteristics become more evidently. This is true under the conditions corresponding to
the $I_1-U_{12}$ values situated above the second negative slope and also to the floating anode $A_2$. The unstable GLSs with a rather regular behaviour are obtained for lower gas pressures and magnetic field intensities, usually for $I_1-U_{12}$ values which are situated below the second negative slope.

As is also seen in the figs. 2, 3 and 4, for these last conditions the initial negative slope is gradually changing into a positive one in spite of the regular instability of the GLSs. This can be due to the decrease of the negative resistance value below those of the $R_v$ ones with decreasing He pressures or/and magnetic field intensities. It must be noted that under our experimental conditions the regular behaviour of the unstable GLSs is rather an exception. Only by using the proposed anode circuit we can enlarge the He pressures and magnetic field domain where this regular behaviour of GLSs can be obtained. As a limit situation, at very small currents $I_1 \leq 1 \text{ mA}$ typical relaxation oscillations with a very low frequency ($\sim 1 \text{ kHz}$) are found for higher magnetic fields nearly independently of the He pressure.
In conclusion, the presented data prove that:

1) The deviation of the fast electrons originating from the cathode by the help of a transverse magnetic field allows us to control the GLSs appearance and

2) The currents collected by an usual anode and a very large one placed at the discharge tube wall are able to control the reciprocal dynamics of the double layers (DL) which are involved in the GLSs instability in order to be correlated.

In this manner it was possible to determine the transition from an ordered to a disordered (chaotic) behaviour of this d.c. glow discharge at higher pressures and to emphasize that the same processes which are implied in the generation and disruption of the GLSs stand also at the origin of the spatio-temporal structures (oscillations, noise and chaos) which are observed as a rule in several plasma devices.

ON THE ELECTRODYNAMIC EXPLANATION OF THE RETROGRADE MOTION OF THE ELECTRIC ARC
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1 Introduction

The retrograde motion of the cathode spot in a transverse magnetic field is one of the more intriguing phenomena of the electric arc. Although the phenomenon has been known for nearly ninety years since its discovery by Stark [1] and has stimulated numerous investigations which result in many models giving explanation from different points of view, there is still no theory that can account both qualitatively and quantitatively for all the observations.

Most of the explanations of the retrograde motion involve the study of cathode processes to give the preferential formation of new cathode spots along the retrograde direction [2]. One line of explanation, which is rather different from the others, is based on electrodynamics [3,4]. In this approach the retrograde motion is treated as an electrodynamic event.

The present paper develops the theory suggested by Robson and von Engel [3,4]. A more complete model is proposed and studied in detail by means of electromagnetic field theory. The results obtained not only show that the retrograde motion can be explained by the electrodynamics, but also confirm that the average current density on the cathode spot must be around the order of \(10^{12} \text{A/m}^2\). Recent studies of spot current density have shown values of this order [6].

2 The Model

The physical model to be investigated is shown in Fig.1, which is based on the following facts (i) that the radius of the cathode spot, which is no greater than \(10^{-3}\text{cm}\) [6,7,8], is much smaller than the dimensions of the cathode; (ii) that the distance from the anode to the cathode is much larger than the radius of the cathode spot and (iii) that the arc column is deflected in the Lorentzian direction [3,9,11,20].

According to Robson and von Engel [3,4] that the magnetic field acting upon the cathode spot is the sum of the applied field \(B_a\) and a self-field \(-\mu_0 I/R\), where \(I\) is the arc current and \(R\) an effective radius of curvature of the arc column where it joins the spot (see Fig.1). The direction of motion is then determined by the direction of the field \(B\), given by

\[
B = B_a - \mu_0 I/R
\]

(1)

Retrograde motion occurs if the self-field exceeds the applied field.

Smith [11] has criticised this model on the grounds that the self-field is unable to exceed the applied field according to his calculations. However, Gille and Seeker [14] pointed out that the magnetic field values deduced by Smith are open to doubt inasmuch as the current density in a discharge varies considerably over the cross section so that the calculation of the forces is somewhat complicated and the fields cannot be determined in the simple way employed by Smith.

We believe as Robson and von Engel, that the retrograde motion can be understood in terms of electrodynamics, but we do not agree with their criterion for occurrence of the retrograde motion because the cathode spot in reality is not just a point and over it the fields change from point to point, so do the forces. Therefore the retrograde motion cannot be judged by the field given by Eqn. (1).

Following the above arguments, we suggest that the direction of motion is determined by

\[
F_R + F_L = \int \int_{\text{spot}} f(\mathbf{r}) \cdot \mathbf{a}_L \, dS
\]

(2)

where \(f(\mathbf{r})\) is the electrodynamic force density; \(\mathbf{a}_L\) is the unit vector parallel to the spot surface but perpendicular to the applied magnetic field; \(F_R\) and \(F_L\) represent the resultant forces per unit length exerted on the spot in the retrograde and the Lorentzian directions respectively. If \(|F_R| > |F_L|\), retrograde motion occurs.
3 Theory

The model used here is a steady-state one employing Ohm's law, $J(r) = \sigma E(r)$, so that

$$\nabla^2 V(r) = 0 \quad \text{where} \quad E(r) = -\nabla V(r)$$

(3)

The magnetic induction is given by

$$B(r) = \nabla \times A(r) \quad \text{where} \quad A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r - r'|} \, dv$$

(4)

where $r'$ is the source position vector. Hence the magnetic force per unit volume can be calculated

$$f(r) = J(r) \times (B(r) + B_a)$$

(5)

where $B_a$ is the known constant magnetic induction applied. Eqn. (3) is the familiar Laplace's equation. It is obvious that if the scalar potential is known the other field quantities of interest can be determined.

The solution of Laplace's equation can be found in terms of the integration equation

$$V(r) = \frac{1}{c} \int_{\Gamma} \left( G(x,x') \frac{\partial V}{\partial n} - V \frac{\partial G(x,x')}{\partial n} \right) \, d\Gamma$$

(6)

where $c = 1$ if $r$ is inside the homogenous region $\Omega$ enclosed by the boundary $\Gamma$ and $G(x,x')$ is the Green function. The boundary element method (BEM) is employed to solve Eqn. (6) for various shapes of the arc column. The details of the solution will be the subject of another paper and will not be discussed further here.

4 Results and Discussion

For our investigation, the plasma column is bent step by step as Fig.2 shown. The field components in which we are interested are computed and plotted in Fig.3, where the normalized current density is given by $j_a = J_a / A$, the average current density of the cathode spot; the normalized magnetic induction is defined by $B_a / \mu_0 A_0$, and the normalized force density results from the multiplication of the normalized current density and the normalized magnetic induction. It should be noted that the applied magnetic field is not involved in Fig.3.

One can see from Fig.3 that the numerical results for arc 1 (refer to Fig.2) are in good agreement with the analytical one denoted by the dotted lines. As the plasma column becomes more bent, the numerical results for the bent arc change smoothly in the direction indicated by the arrows. It turns out that as the plasma column is bent in the Lorentzian direction due to the applied magnetic field, the current density over the cathode spot is no longer uniform and becomes stronger on the Lorentzian side while weaker on the retrograde side. The similar tendency for changing the distributions of the self-magnetic induction and the force density due to the self-field is obtained. These distributions for a typical bent arc can be clearly seen in Fig.4.

It is obvious that in the absence of the applied magnetic field, the cathode spot of the bent arc would always move in the retrograde direction. However, it should be kept in mind that the bending of the plasma column is attributed to the applied field which is in favour of the motion in the Lorentzian direction. According to our model, therefore, there exist the two opposite motion tendencies within the cathode spot region and the spot will move as a whole in the direction in which the force is dominant.

To demonstrate our model, we chose a set of experimental data from different sources and inserted them into our theory. Because of the limitation on space only one set of results is tabulated in Table 1. One can first see that the retrograde motion can indeed be predicted by our model if the average current density $j_a$ is of the order of $10^3$ m$^2$, which is in agreement with the values suggested by recent studies of spot current density [6,8,15]. A number of the reported features of the retrograde motion can be explained by our model as below.

- The retrograde motion is temperature dependent. With increasing cathode temperature, the retrograde velocity decreases [18]; and the retrograde motion can be reversed if the temperature is sufficiently high [13]. As the cathode temperature is increased, the spot current density is observed to decrease [18]. In our model the ratio of $|F_R/F_L|$ decreases with decreasing spot current
Table 1. $F_s = 20 A$. $B_0 = 0.3$ Tesla, copper cathode.
Motion observed: retrograde, ref. [6]

<table>
<thead>
<tr>
<th>$I_m$ ($A/cm^2$)</th>
<th>$T_{R}^{2}$ (nC)</th>
<th>$T_{R}^{2}I_{m}^{2}$ (nC^2)</th>
<th>$T_{R}^{2}I_{m}^{2}$ (nC^2)</th>
<th>$T_{R}^{2}I_{m}^{2}$ (nC^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.73 x 10^5</td>
<td>7.00 x 10^5</td>
<td>4.09 x 10^-5</td>
<td>retrograde</td>
</tr>
<tr>
<td>5</td>
<td>3.66 x 10^6</td>
<td>7.00 x 10^5</td>
<td>3.03 x 10^-5</td>
<td>retrograde</td>
</tr>
<tr>
<td>4</td>
<td>3.97 x 10^6</td>
<td>7.00 x 10^5</td>
<td>2.02 x 10^-5</td>
<td>retrograde</td>
</tr>
<tr>
<td>2</td>
<td>6.56 x 10^6</td>
<td>7.00 x 10^5</td>
<td>1.01 x 10^-5</td>
<td>retrograde</td>
</tr>
</tbody>
</table>

Fig. 1. Physical model of an arc

Fig. 2. Deformation of arcs

Fig. 3. Fields of arcs

Fig. 4. Field distributions
density; thus the retrograde velocity decreases with increasing cathode temperature. If the cathode is heated to a sufficiently high temperature, which can vary with the cathode material, the spot current density can be so low that $|F_R/F_L| < 1$ and the motion is reversed.

- Lower background gas pressures favour the retrograde motion and the motion can be reversed if the gas pressure rises above a critical value. The bending of the arc also depends on the background gas pressure. The lower the pressure, the more bent is the arc column. As a result, the ratio of $|F_R/F_L|$ increases and so does the retrograde velocity. On the contrary, if the pressure is increased, the arc column becomes straight and the ratio of $|F_R/F_L|$ decreases and can be smaller than one above a certain pressure so that the direction of motion changes to the Lorentzian direction.

- The retrograde velocity is generally found to increase with an increase in the magnitude of the applied magnetic field. However, reversal of the motion occurs with a very strong field. The model proposed clearly shows that the two simultaneous effects of the applied magnetic field, namely (a) bending the arc column in the Lorentzian direction; (b) enhancing the force component $F_L$. Because the first effect is in favour of the retrograde motion and is opposite to the second one, the resultant effect of increasing the applied magnetic field will depend on the dominant one of the two effects. As the applied magnetic field is increased from a lower level, the arc column is expected to be bent at first rapidly and then less so due to a limit of the bending. Hence, the first effect is dominant and the retrograde velocity increases before this limit is reached. Then, the second effect becomes dominant and the motion will be reversed when $|F_L| > |F_R|$.

- The critical pressure increases with an increase of the applied magnetic field. As previously stated an increase in the applied magnetic field will cause the arc column to bend before reaching a limit. On the other hand, an increase in the background gas pressure makes the arc column straighter. At a reversal position of the arc column, the effects of the increase in the two quantities can cancel each other out which directly leads to the observed result.

- In vacuum or low pressure the retrograde velocity was found to increase with an increase in arc current [16,17,19]. According to the proposed model it can be shown that the ratio of $|F_R/F_L|$ increases with an increase in arc current, so does the retrograde velocity.

- The retrograde motion depends on the material and surface condition of the cathode [14,22]. The connection of the present model to this feature is twofold. First, the cathode current density is found to be material-sensitive, so is the force predicted by the model. Secondly, the spot is a plasma source of dense metal vapour evaporated from the cathode material so that the plasma mass density of the spot will also affect the spot motion.

References

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It is well known that the charged particles of a weakly ionized plasma in their random thermal motion from one region of the gas to another tend to transport with them the properties of the region from they come. In particular if these macroscopic properties are nonuniform, the particles become themselves out of equilibrium with properties where they arrive. In this context we analyze, in cylindrical coordinates, these transport phenomena in the presence of nonuniform static electric field generated between a cylindrical conductor, considered as point form, and a planar conducting wall (this field is known as corona or also marconian dipole field). A strong steep temperature gradient is considered present between the hot charged conductor and the cold wall.

The analysis method applied here is based on the integral form of the Boltzmann equation of the test particles, especially electrons, linearized and with the effects of the boundaries taken into full account. This formulation may be considered valid for an arbitrary ratio of the scale length to the mean free path $l$. It is known that this integral Boltzmann equation may be reduced to an inhomogeneous system of differential equations by choosing a proper degenerate (elementary) scattering kernel. The associate homogeneous system may give a dispersion determinantal equation for diffusion waves along with the stability and resonance conditions for the wave propagation.

In particular as a first approach, a BGK model is presented, analyzed and harmonic eigenmodes calculated. An experimental test based on the measurements of the electron density and current distributions in a low density and temperature plasma is here proposed.

We consider the linearized integral Boltzmann equation with a scattering kernel of elementary type as follows

$$K(y' \to y) = \sum_{i=1}^{N} \phi_i(y') \Psi_i(y)$$

where $\phi_i(y')$ and $\Psi_i(y)$ are two proper sets of functions convergent in
v-space and with the moment of the distribution function

\[ \xi_1(x,t) = \langle \Phi_1 | f \rangle = \int_{R^3} \Phi_1(v') f(v') \, dv'. \]

Then we may write the Boltzmann equation for test particles of mass \( m \) in the following integral form

\[ f = h + K_1 \sum_{t=1}^{N} \psi \, \xi_1 - K_2 \sum_{t=1}^{N} \left( \frac{F/m}{t} \frac{\partial \xi_1}{\partial t} \right), \]

where

\[ h = f(x(L),v(L),t-L) \exp \left[ -\int_{0}^{L} v \, dt \right] + \int_{0}^{L} Q(x(t),v(t),t-t) \exp \left[ -\int_{0}^{t} v \, dt' \right] \, dt', \]

\[ Q \] and \( F \) are the external source term and force respectively,

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial x}, \]

\( v \) is the collision frequency generally considered velocity dependent, \( L \) is the boundary phase of the temporal solution to the characteristic equation

\[ \frac{1}{2} \frac{F}{m_e} t^2 - vt + x = 0 \]

where \((x,y)\) is a point of the phase space,

\[ K_1 = \int_{0}^{L} \exp \left[ -\int_{0}^{t} v \, dt' \right] \, dt, \quad K_2 = \int_{0}^{L} \exp \left[ -\int_{0}^{t} v \, dt' \right] \, dt, \]

are two collisional moments dependent on the phase \((x,y,L)\). By operating a scalar product in v-space, eq. (1) may be reduced to a linear system of differential equations, as follows

\[ \sum_{j=1}^{N} (\Phi_j, K_2 \frac{\partial}{\partial y} \psi) \xi_j + \sum_{j=1}^{N} \frac{F}{m_t} (\Phi_j, K_2 \frac{\partial}{\partial y} \psi_1) \xi_j - \sum_{j=1}^{N} (\Phi_j, K_1 \psi) \xi_j + \xi_j = (\Phi_j, h), \]

the BGK solution for electron dynamics. In this case, we assume the collision frequency \( \nu = \text{const} \), \( N = 1 \) in (3), \( \psi_1(v) = \nu(t) \exp(-\Gamma^2 v^2) \]

where \( \Gamma = m_0/(2T) \) and \( \Phi_1 = 1 \). The external force in cylindrical coordinates has the following polar components

\[ F_r(r,\theta) = \frac{q^2}{r^3} \cos \theta, \quad F_\theta(r,\theta) = \frac{q^2}{r^3} \sin \theta \]

of an electric dipole where, as shown in Fig.1A, \( q = -Ze \) is the electric charge located in \((1/2,0)\) of the cylindrical conductor (point form) generating the electric field with respect to the line \( G(O,\pi/2) \) which
is supposed to zero potential. The point charge \( q \) and the line \( G \) represent, in the \((r,\theta)\) plane, respectively the section of a line and a plane both perpendicular to the \((r,\theta)\) meridian plane and parallel to the z-axis (Fig. 1B).

For a steady state equilibrium \((\partial/\partial t = 0)\) the linear system (3) is reduced to the following linear differential equation,

\[
\frac{\partial^2 \xi}{\partial r^2} + \left( \frac{1}{r} \frac{\partial}{\partial r} \right) \frac{\partial \xi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \xi}{\partial \theta^2} - \left( 1, K_1 \Psi \right) \xi + \frac{F_r}{m} \left( 1, K_2 \frac{\partial \Psi}{\partial r} \right) + \frac{F_\theta}{m} \left( 1, K_2 \frac{\partial \Psi}{\partial \theta} \right) + \xi = 0
\]

(4)

\[
\xi = (1, h),
\]

that may be given in the synthetic form

\[
\Lambda_1 \frac{\partial^2 \xi}{\partial r^2} + \Lambda_2 \frac{\partial \xi}{\partial r} + \Lambda_3 \xi = C
\]

By taking \( \xi(r, \theta) = \xi_1(r) \xi_2(\theta) \), the solutions to this equation are the following,

\[
\xi_1 = \exp \left[ -\int_0^r \frac{P_1^2}{2} \right] \left\{ \exp \left[ -\int_{\theta_1}^\theta \frac{P_2^2}{2} \right] Q \right\} dr' + C_1
\]

\[
\xi_2 = \xi_2(0) \exp \left[ -\int_0^\theta \frac{P_3^2}{2} \right]
\]

where \( m^2 \) is the separation constant, dependent from boundaries, \( P(r, \theta) = (A_2 + m^2)/A_1 \), \( Q(r, \theta) = (C_2^2)/A_1 \). The solution may be easily expressed in terms of series of hyperbolic Bessel functions by taking

\[
\exp \left[ -\int_0^r \frac{P_1^2}{2} \right] = \sum \sum (\pm)^n I_n (p_2 r^{-2}) J_n (p_3 r^{-2}) \exp \left[ \mp 1 \right] (m+n) \theta
\]

where

\[
P_1 = \frac{(1, K_1 \Psi)^{-1}}{(1, K_2 \Psi)^{-1}}, \quad P_2 = \frac{F_r(1, K_2 \Psi / \partial r)}{2m (1, K_2 \Psi / \partial \theta)}, \quad P_3 = \frac{F_\theta(1, K_2 \Psi / \partial \theta)}{2m (1, K_2 \Psi / \partial r)}.
\]

The source term \( h(x, \nu) \) is composed by two sources of electrons:

1) the charged conductor of density \( n_1 \) and at temperature \( T_1 \),

2) the surface \( G \) at temperature \( T_2 \) and density \( n_2 \). It is written as,

\[
h(x, \nu) = n_1 \left[ \frac{1}{\pi} \right] g_1 \exp \left[ -\nu \Psi(x) \right] - \nu \left[ \frac{1}{\pi} \right] g_2 \left[ \frac{2}{\pi} \right] \exp \left[ -\nu \Psi(x) \right] - \nu \left[ \frac{2}{\pi} \right]
\]

The weights \( g_1(\tau_1, \tau_2, L) \) and \( g_2(\tau_1, \tau_2, L) \) are calculated considering the
boundary conditions on the temporal solutions of the characteristic equation (2) where $L$ that may be $\tau_1$ or $\tau_2$, is the phase $t$ of the point $(x,y,t)$ respectively from the boundary sources 1) or 2) and $L=t$ from a point inside the medium $(x,y,t)/1/$. The BGK eigenmodes are determined by solving the determinantal equation of the associate homogeneous equation to (4), considered as time-dependent equation with a solution

$$\xi = \xi_0 + \xi_1 \exp[i(k \cdot x - \omega t)], \quad \xi_1 \ll \xi_0,$$

where $k \cdot x = k r \cos \theta$. In particular the condition for the Cherenkov resonance is considered. The determinantal equation to solve is the following

$$i \left[ (1, K_1, \psi_1, k \cos \theta - (1, K_2, \psi_1, k \sin \theta - \omega (1, K_1 \psi_1) \right] + (1, K_1 \psi_1 -$$

$$- \frac{F}{m_e} (1, K_2 \psi_1 / \nu \phi) - \frac{F}{m_e} (1, K_2 \psi_1 / \nu \phi) = 1$$

which, in this approximation, is linear and of the type

$$\omega = F(\theta) k + \lambda_0,$$

in general complex, which, in principle, implies also backward modes.

In order to verify the approximation of this simple BGK model we consider an experiment in which the diffusion plasma is generated by a DC discharge between an hot filament (cathode) and a planar conductor (anode) where it is possible to measure the electron density and the electron current density distribution and, as well as, the electron temperature. For this test write the linearized BGK Boltzmann equation (1) in the suitable form

$$f = h + \left[ K_1 n - K_2 v \cos \theta \left( \frac{nF}{T_e} - |Vn| \right) \right] v (T_e/\pi)^{3/2} \exp(-T_e v^2).$$

In particular, in the case of plasma at low temperature and density with a low degree of ionization by detecting, experimentally via Langmuir probe, the first and second derivatives of the Langmuir I/V characteristic it is possible to derive the experimental electron distribution function.

References.

EXPERIMENTAL STUDY OF THE AXIAL STRUCTURE OF AN ARGON PLASMA COLUMN SUSTAINED BY AN ELECTROMAGNETIC WAVE IN THE PRESENCE OF AN EXTERNAL MAGNETIC FIELD

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A typical feature of the plasma columns sustained by travelling azimuthally-symmetric electromagnetic waves is their axial inhomogeneity. An imposed magnetic field decreases the axial gradient /1,2/. The goal of this study is to measure the axial density profiles at different discharge conditions and compare them with the theory.

The experimental equipment used is shown in Fig. 1. It consists of a

![Diagram](image)

Fig. 1. Scheme of the experimental setup
glass tube of radius R, thickness d and permittivity ε surrounded by a metal enclosure, a HF generator of frequency ω/2π = 510 MHz, a HF power meter, a wave launcher of type "surfatron", eight Helmholtz coils creating the external axially homogeneous magnetic field, a coupler giving the reference signal for the vector millivoltmeter and a movable along the z-axis antenna.

The relevant experimental procedure is as follows: The vector millivoltmeter measures the phase difference φ between the reference signal and the signal from the movable antenna as a function of the axial coordinate z. Next, the wave number k(z) is obtained as k = dφ/dz. Then the phase diagram gives the plasma density n as a function of k. It is convenient for easier comparing with the theory to introduce the following dimensionless quantities: axial coordinate ζ = vz/ωR (v being the collision frequency for momentum transfer), wave number x = kR and electron number density N = n/n₀, where n₀ is the cut-off plasma density, n₀ = mω²/4πe². Since the derivative dφ/dz has been used, small fluctuations in the raw experimental data give great deviations of N. That is why the φ(z) curves must be fitted with smooth ones.

The experiment has been carried out in argon in two tubes with radii of 0.82 cm and 2.05 cm (identified by the numerical parameter σ = ωR/c = 0.088 and 0.219 respectively), at pressures of 20, 50, and 100 mTorr and magnetic field inductions of 146 G and 218 G corresponding to ω/c = 0.8 (weak magnetic field) and ω/c = 1.2 (strong magnetic field) /2/. The wave power ranges from 10 to 70 W. In order to be able to compare the experimental data with the theory we have to set the power level in such a way that the end of the plasma column does not reach the end of the tube; Otherwise the propagation of reflected waves is quite possible. It is worth noting also that in contrast to the case of an isotropic ionized medium (when the column ends relatively sharply), the end of a magnetized wave produced plasma column tapers off, resembling the tip of a well sharpened pencil. With further increasing of the magnetic field the plasma column shrinks and reduces to a thin plasma wire along the tube axis and the theoretical models /1,2/ are not applicable. Also the lower the pressure, the smaller the contraction magnetic field.

By using this method the plasma density can be measured at arbitrary number of points. In our opinion 15 to 20 points are enough. Figures 2 to 5 show the experimental data compared with the corresponding theoretical
profiles. These profiles are derived from an improved numerical code of [2] taking into account the presence of the dielectric tube and the metal enclosure.

Fig. 2. Theoretical axial plasma density profile and experimental points for $\Omega=0.8$, $\sigma=0.088$ and pressure $p=50$ mTorr

Fig. 3. Theoretical axial plasma density profile and experimental points for $\Omega=1.2$, $\sigma=0.088$ and pressure $p=100$ mTorr

The experimental error can reach 30% mainly because for large $N$ the phase diagrams are very steep and a small deviation of the measured wave-number cause large deviation of the obtained plasma density. We think that the agreement between theory and experiment is acceptable.
Fig. 4. Theoretical axial plasma density profile and experimental points for $\Omega=0.8$, $\sigma=0.219$ and pressure $p=20$ mTorr

Fig. 5. Theoretical axial plasma density profile and experimental points for $\Omega=1.2$, $\sigma=0.219$ and pressure $p=100$ mTorr

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References

Electromagnetic Surface Waves for Large-Area RF Plasma Productions between Large-Area Planar Electrodes

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Recently, large-area plasma production has been tested by means of a 13.56 MHz radio-frequency (RF) discharge between a pair of large-area planar electrodes, approximately 0.5 m x 1.4 m, as one of the semiconductor technologies for fabrication of large-area amorphous silicon solar cells in the "Sunshine Project" of the Agency of Industrial Science and Technology in Japan/1/. We also confirmed long plasma production between a pair of long electrodes/2/.

In this paper, normal electromagnetic (EM) waves propagating in a region between a planar waveguide with one plasma and two dielectric layers are analyzed in order to study the feasibility of large-area plasma productions by EM wave-discharges between a pair of large-area RF electrodes larger than the half-wavelength of RF wave. In conclusion, plasmas higher than an electron plasma frequency will be produced by an odd TMoo surface mode.

§ 1. Normal Mode Analysis

In this analysis, as shown by an one-dimensional waveguide model in cartesian coordinates (x, y, z) in Fig. 3 (b), the plasma layer (-a ≤ x ≤ a) is separated from the metal electrodes by dielectric layers with thickness d (= b-a) and dielectric constant Kg so as to support surface waves. The EM waves are assumed to propagate in the z-direction in the form of exp(j(ωt-βz)), and the wave fields are uniform in the y-direction (d/dy = 0), where ω and β are wave frequency and wave number, respectively. The metals (b ≤ |x|) and the plasma (-a ≤ x ≤ a) are assumed cold, lossless and uniform as expressed by equivalent dielectric constants Km and Kp, respectively: Km = 1 - (ωm/ω)^2 and Kp = 1 - (ωp/ω)^2, where ωm = √(Nme^2/ε_m) and ωp = √(Npe^2/ε_p), Nm and Np are electron density numbers in metal and in plasma, respectively, and the other notations are standard.

We employed standard procedures to solve Maxwell's equations. The results are summarized as follows; (1) the EM wave usually separates into two independent TM and TE waves/3/. (2) Each of the TE and TM waves splits into
two modes, even and odd modes. (3) Each even or odd mode splits further into three modes: (I) surface wave mode, (II) EM mode in dielectrics and (III) EM mode in plasma. All the fields are represented by either a hyperbolic or sinusoidal function in each layer, as shown in Fig. 1 for an odd TM surface mode.

Boundary conditions for wave fields at two surfaces $x = \pm a$ and $\pm b$ gives dispersion equations, for examples, for odd TM$n$ modes as follows:

$$Dm \cdot Dp = ((Kp/Up) - (Kg/Ug)tanh(Upa))((Km/Um) - (Kg/Ug))exp(-2Ugd), \quad (1)$$

where $Dm = (Km/Um) + (Kg/Ug)$, $Dp = (Kp/Up) + (Kg/Ug)tanh(Upa)$,

$$Um^2 = \beta^2 - \beta m^2 \equiv \beta^2 - ko^2Km,$$  
$$Up^2 = \beta^2 - \beta p^2 \equiv \beta^2 - ko^2Kp,$$  
$$Ug^2 = \beta^2 - \beta g^2 \equiv \beta^2 - ko^2Kg,$$  
$$ko^2 = \omega^2 \varepsilon_0 \mu_0,$$

$U_m, p, g$ are wave numbers in the $x$-direction, in the metal, in the plasma and in the dielectric, respectively. Subscripts $m$ and $n$ denote mode numbers in plasma and in dielectric regions, respectively.

Thus, we have ten normal modes in total because the TE wave has no surface wave solutions. However, the most efficient mode for plasma production is an odd TM$00$ surface mode/4/, so that we omit here all the other normal modes.

§ 2. Dispersion Characteristics

Equation (1) contains two dispersion relations for surface waves propagating along the plasma and metal surfaces $x = \pm a$ and $\pm b$. Thus, eq. (1) expresses the coupled dispersion equation of the two odd surface waves. When the plasma thickness $2a$ becomes zero, dispersion relation for an odd metal surface wave (MS$0$) along $x = \pm b$ is derived from eq. (1) as follows:

$$\left(\frac{Km}{Um}\right) + \left(\frac{Kg}{Ug}\right)\coth(Ugb) = 0 \quad \text{(for odd MS$0$)}, \quad (2)$$

Whereas, when the metals are perfect conductors, another dispersion relation for an odd plasma surface mode (PS$0$) along $x = \pm a$ is derived from eq. (1) as:

$$\left(\frac{Kp}{Up}\right)\coth(Upa) + \left(\frac{Kg}{Ug}\right)\coth(Uga) = 0 \quad \text{(for odd PS$0$)}, \quad (3)$$
Dispersion characteristics for MSo and PSo modes are obtained qualitatively from graphical inspections of eqs. (2) and (3), as shown in Figs. 2 and 3, respectively, where \( \lambda \) and \( \lambda g \) are wavelengths corresponding to \( \beta \) and \( \beta g \). The results are summarized as follows: (I) both surface waves have slow phase velocities \((\lambda g/\lambda)^2 \leq 1\), and (II) the density \( Nm \) and \( Np \) go to infinity when \((\lambda g/\lambda)^2\) approaches 1, and (III) go to \(1 + Kg\) when \((\lambda g/\lambda)^2\) approaches infinity.

\[
\left(\frac{\omega_p}{\omega}\right)^2 = \left(\frac{\omega_p}{\omega}\right)^2 (1 + \eta^2)
\]

Next, we take into account lossy plasmas in dielectric constant \( Kp: Kp = 1 - (\omega_p/\omega)^2/(1 + \eta^2) - j\eta(\omega_p/\omega)^2/(1 + \eta^2) \), where \( \eta = \nu c/\omega \) and \( \nu c \) is an electron-gas collision frequency for momentum transfer. The dispersion curves in Fig. 3 are modified to some extent, depending on the value \( \eta \), to shift toward the upper side of \((\omega_p/\omega)^2\) and/or to shift toward the larger side of \((\lambda g/\lambda)^2\), as is known from eq. (3). Furthermore, when the PSo mode is coupled with the MSo mode as in eq. (1), the dispersion curve in Fig. 3 will also go down somewhat to the bottom side, but the value \((\omega_p/\omega)^2\) is more than 1.

§ 3. Theory for Plasma Productions and Discussions

For this purpose, we begin with the case when a ready-made DC plasma is set up for the plasma layer. RF voltage \( V_{\text{exp}}(j\omega t) \) is then applied to the metal electrodes at an arbitrary point defined as \( z = 0 \), using a coaxial cable as is the standard procedure.

First, in order to simplify the theory, eq. (1) is rewritten, using a
quasistatic approximation ($\beta \rightarrow \text{large}$), as follows:

$$(K_p + K_{\text{tanh}}(\beta a))(K_m + K_g) = (K_p - K_{\text{tanh}}(\beta a))(K_m - K_g) \exp(-2\beta d), \quad (4)$$

whereas, the profile of the wave potential is the same as $E_z$ in Fig. 1. Thus, one can define an RF wave voltage $V$ between metals; that is, $V = 0$ for even TMoo mode, and $V \neq 0$ for odd TMoo mode. Therefore, this odd mode is directly coupled with an applied RF voltage $V$ at $z = 0$. Then, this odd TMoo mode can propagate inducing the RF voltage in the form of $V = V \exp(j\omega t - \beta z)$. These results also shows that the odd MSo mode will play a role of modifying the odd FSo mode and exciting it at the position far away from the $z = 0$. When we take into account finite dimension of electrodes, standing wave resonances will occur within the electrode area.

Next, we change the focus of our interest to RF plasma production. Even in such circumstances, all the concepts and results are basically the same as in the above case of DC, as long as we use coaxial cables. Thus, the most efficient mode for plasma production is identified an odd TMoo surface mode.

The present theory suggests the feasibility of large-area or long plasma production that may be required for mass production of large-area silicon solar cells at the ground level $/1/ or in a space factory in the near future. The remaining problems in terms of these purposes are now being prepared for publications.

§ 4. Conclusions

When we use coaxial cables for the generation of large-area plasma between a pair of large-area planar electrodes larger than the half-wavelength of the TEM wave $\lambda g/2$, (I) the RF plasmas are produced by an odd metal and plasma surface mode TMoo with the wavelength range $\lambda < \lambda g$, and (II) then the produced plasma density is higher than the critical value of an electron plasma oscillation ($\omega_p^2 \geq \omega^2$).

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CO_2 -TE LASER PRODUCED CADMIUM PLASMA IN VACUUM
EXPERIMENT AND KINETICS

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The rapidly transient character of the plasmas generated under the action of pulsed, high-intensity laser radiation may lead especially through relaxation processes to important deviations from the thermodynamic equilibrium distributions and sometimes to population inversions and lasing action.

Related to the investigation of lasing action in the visible region, our results on laser emission at \( \lambda = 5378 \) A recombination line in CO_2 -TE laser produced cadmium plasma in vacuum, were previously reported /1/.

A detailed further study of the visible emission spectrum and the related parameters of the laser produced cadmium plasma are now presented. For the plasma production we have used a CO_2 -TE laser with a typical pulse shape (a peak of 200 nsec. and a tail of 2 \( \mu \)sec.).

The analysis of the emission spectrum has put into evidence lines belonging to Cd I and Cd II species, but the presence of the recombination lines of Cd II ions indicates also the existence of Cd III ions in plasma, in its early evolution stages at least. These lines were not directly detected because Cd III ions have not emission lines in the visible region.

The plasma electron temperature was obtained using the ratio of the relative line intensities of subsequent ionisation stages, /2/ taking into account that for times between 0.2 and 1 \( \mu \)sec. in its evolution, plasma can be considered in local thermodynamic equilibrium.

The electron density was not directly measured. This is the reason why the electron temperature calculations were considered for two electron densities: \( 10^{17} \text{cm}^{-3} \) and \( 10^{18} \text{cm}^{-3} \).
For a moment situated at 0.2 μsec. after the plasma ignition, the corresponding temperature values are 1.4 eV and 1.7 eV; after 0.7 μsec. the temperature decreases slowly to 1.3 eV and respectively 1.5 eV. The determined temperatures are averaged over the plasma length at the considered times.

The theoretical values for temperature, were calculated using a hydrodynamic model /3/. The parameter α from this treatment takes into account the two limiting cases for the ion temperature: α = 1 for an ionic temperature equated to zero and α = (z +1)/z for an equal temperature of the ions and electrons (z is the plasma charge). In the case of α = 1, the results give a value of 37.72 eV for the maximum electrons temperature and a corresponding electron density value of $6.37 \times 10^{18}$ cm$^{-3}$. The calculated values for temperature as a function of the distance from the target surface, which correspond to different moments in plasma evolution, are greater than the spectral estimated temperatures for a region situated near the target, becoming lower for greater distances. The corresponding values for temperature and density at 5 mm in front of the target are 1.30 eV and respectively $4 \times 10^{16}$ cm$^{-3}$.

Taking into account that the interaction time of the laser pulse is about 2 μsec., we have considered useful to do some considerations about the absorption coefficient (K) in plasma /4/. With the same hydrodynamic model we obtained a value of $K = 17 \text{ cm}^{-1}$ in the period of plasma formation, after which the value decreases rapidly at $K = 0.064 \text{ cm}^{-1}$ at 5 mm from the target surface, indicating that the tail of the laser pulse is weakly absorbed in the expanded plasma.

The number of heavy particles in plasma produced by laser ablation, considering that the process is most powerful under the action of the laser peak, is $4 \times 10^{15}$ particles for a value of $2 \times 10^8 \text{ W cm}^{-2}$ for the laser intensity. This indicates an ionisation coefficient for the maximum calculated electron density of 28 $\%$, considering that the number of ions is equal to the number of electrons. In the real case (a medium charge
in the plasma of about $z = 2$), the number of ions represent approximately 14%.

In order to analyse kinetics of the laser-produced plasma, the collisional excitation and - using the detailed equilibrium principle - de-excitation rate coefficients were calculated at different temperatures, taking into account two lines: $\lambda = 5378$ Å laser recombination line and another line, which has not a recombination character, $\lambda = 4415$ Å line. The calculated values of the rate coefficients put into evidence that they are much greater for $\lambda = 5378$ Å line, than for $\lambda = 4415$ Å line.

The results were obtained using Bates-Damgaard approximation /5/ (for the first line) and Van Regemorter semiempirical formula /5/ (for the second line), the temperatures involved in the calculations being of interest for the "hot" period of the plasma evolution. The values are presented in table I.

### Table I

<table>
<thead>
<tr>
<th>$\lambda$ (Å)</th>
<th>$T = 4$ eV</th>
<th>$T = 8$ eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>4415.63</td>
<td>$K_{ex}$ (cm$^3$s$^{-1}$)</td>
<td>$K_{ex}$ (cm$^3$s$^{-1}$)</td>
</tr>
<tr>
<td>2.99(-8)</td>
<td>9.88(-9)</td>
<td>1.10(-8)</td>
</tr>
<tr>
<td>5378.13</td>
<td>$K_{dex}$ (cm$^3$s$^{-1}$)</td>
<td>$K_{dex}$ (cm$^3$s$^{-1}$)</td>
</tr>
<tr>
<td>2.64(-5)</td>
<td>2.38(-5)</td>
<td></td>
</tr>
<tr>
<td>5.58(-5)</td>
<td>4.90(-5)</td>
<td></td>
</tr>
</tbody>
</table>

We mention that the transitions analysis was made in the case of a single electron involved in the transition.
References:


Topic 12:
Elementary Processes in Plasmas
MEASURED AND CALCULATED CHARGE DENSITY IN Ce and Cu VAPOR PRODUCED BY AN ELECTRON GUN

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Abstract
A measure of electron density by Langmuir probe is reported and compared with a calculation taking into account electron-atom collisions.

1. Introduction
The charge rates of cerium and copper vapor, created by an electron gun, were measured. A Langmuir probe measured the charge density $n_e$ and a quartz crystal measured the flux of atoms. The atomic density $n_{at}$ is deduced, by the knowledge of the atomic velocity along the vertical axis /1/.

To interpret this experimental charge rate $\tau_i = n_e / n_{at}$, we solve the rate equations for each class of electrons present in the vapor, taking into account electron impact ionisation, excitation, deexcitation processes and radiative decay.

2. Experimental Set-Up
Evaporation was produced using a commercial electron gun of 11 kV and power $P$ varying between 500 W and 5 kW (see fig(1)). The beam, deflected by 270°, heated a metal contained in a water cooled copper crucible, which can move along the vertical axis. The vapor, generated at the source (S), is collimated in A. The Langmuir probe is located 50mm above (S) for Ce and 80mm for Cu, out of the electron beam. The flux of atoms is measured with a quartz crystal above A.

3. Electronic population
A proportion of the incident electrons is backscattered (see fig. 2). Their energy is slightly smaller than the beam energy. The backscattering coefficient is 0.28 for Cu and 0.40 for Ce /2/. Energetic electrons create "vapor electrons" in the vapor, along the beam direction. According to /3/ and /4/, we estimate that the energy distribution of the "vapor electrons" is initially
maxwellian, with a temperature $T_1$ equal to 22eV and 42eV for Ce and Cu.

In addition, secondary processes between beam and metal produce secondary electrons with a mean energy of a few ev. Due to the high temperature $T_S$ of the source ($\approx 2500K$), thermoionic electrons can be present at the point of beam impact.

4. Model

We describe the states of copper by 7 levels /5/, and the states of cerium by 9 levels (2 states and 7 pseudo-states).

We solve the stationary rate equation for ion density versus the altitude $h$ above the source:

$$v \cdot \frac{dn_i}{dh} = \text{Loss} + \text{Gain} \quad (a)$$

where $v$ is the mean velocity of the atoms $n_i$ is the population of the state $i$ and Loss and Gain terms are the contributions of radiative decay, excitation, deexcitation and ionisation by electron impact.

For the ionization cross section, we use the Drawin semi-empirical formula /6/, which reproduce well the cross section data of /7/ and /8/, if we adjust the parameter $D$ (see Fig 3).

We first solve equation (a) for energetic electrons. The reduction of their energy ($\approx 11$keV) by collision with the vapor atoms is negligible. These electrons create 60% and 38% of multicharged $A^{n+}$ ions for Ce and Cu respectively.

Then, to calculate the contribution of "vapor electrons", we solve equations (a) and (b):

$$\frac{d(n_e \cdot 5/2T_e)}{dh} = L + G \quad (\text{see /9/}) \quad (b)$$

where $L$ and $G$ energy loss and gain terms due to the same sort of collisional processes than in equation (a).

The $h$ variation temperature $T_e$ shows a decrease from $T_1$ to less than 1eV in few micrometers.

Secondary electrons contribution is negligible, since they are very cold. The thermoionic charge density
calculated by the Saha equation is too high (0.08%) to yield a good agreement with the Ce experimental rate. We don't take it into account.

5. Results and conclusion

In order to perform a more realistic calculation of charge rate versus power, it would be necessary to know the evolution of the surface S (impact of the beam on the source with power). We estimate the radius $R_S$ of S at approximately 0.3cm.

We checked two simple situations, following the model described in part 4: one where $P/S$ is constant, and one where S is constant.

Fig. 4 shows the experimental (see part 2) and calculated charge rate. In the case of Ce, the agreement between experiment and calculation is good enough if $S=Cte$, except for lowest beam power. For Cu, the calculation reproduce experiment, but the fluctuations of the rates are too small to see a difference between the two calculations.

With this very simple model, we reproduce the order of magnitude of charge density in terms of gun power for different elements, wherefore we conclude that the main processes are taken into account with right order of magnitude of rate coefficients.

![Fig 1: Experimental Set Up](image1)

![Fig 2: Electronic Populations](image2)
Fig 3: Ionization Cross Section of Cerium

Fig 4: Charge Rates

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Stokes Parameters for Compton Scattering in a Strong Magnetic Field

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1. Introduction

The role played by superstrong magnetic fields in astrophysical plasmas has attracted considerable attention. It is generally believed that enormously strong magnetic fields seem to be characteristic of collapsed stars. Exterior fields of $10^{12} G$ or even greater are required by models which try to relate pulsar observations to rotating neutron stars /1/. On the other hand, the observation of polarized light and X-rays from white dwarfs suggests that such stars also possess huge surface magnetic fields in excess of $10^8 G$ /2/. Moreover, the discovery of X-ray cyclotron emission from Her X-1 by Trumper et al. /3/ provides further direct evidence for strong magnetic fields of $10^{12} G$. More recently, the detection of effects due to electron cyclotron resonance, in the spectrum of mass-accreting X-ray pulsars from Ginga satellite provides further direct and accurate measurements for the surface magnetic fields of strength $(1 \sim 4) \times 10^{12} G$ /4/. Such huge magnetic fields can greatly affect the motion of electrons in the outer parts of the star by tending to confine their motion perpendicular to the magnetic field. The interaction of photons with electrons will be greatly altered whenever the magnetic field is sufficiently strong to restrict the electron motion to orbits parallel to the field. Thus, the magnetic field may possibly greatly reduce the opacity in such stars so that they may cool much more rapidly than is indicated by those calculations which ignore it.

In recent years, several theoretical investigations have been focused to study the dynamics of resonant and nonresonant scattering of photons by electrons in a plasma permeated by a superstrong magnetic field /5,6,7/. The most distinguishing features is the fact that the superstrong magnetic field greatly affects the Compton scattering process, resulting in resonances in the scattering cross-section /8,9,10/. The important consequences of these cyclotron resonances are the increase in the photon mean free path in the scattering regions, and greatly increased electron energy-loss rates /11/, strongly affecting the angular distribution, and polarization properties of the scattered photons /12/. More recently, analytical expression for the Compton scattering cross-section in an intense magnetic field has been presented by several authors /9, 13/. In particular, Lieu derived simpler and more usable expressions for the total cross-section which reduce to the magnetic Thomson cross-section in the low frequency and weak field limit (i.e., $\hbar \omega \ll mc^2, B \ll B_\theta$, where $m$ is the mass of the electron and...
$B_0 \equiv m^2 c^2 / \hbar k = 4.414 \times 10^{13} G$. This limiting cross-section obtained by Liu is essentially the same as the one previously given by Canuto et al. who derived it in the low density and weak field limit ($\omega \gg \omega_p, \omega \gg \omega_c$ where $\omega_p$ and $\omega_c$ denote respectively the plasma frequency and cyclotron frequency of the electrons). The indices of refraction for photons propagating in such huge magnetic fields was determined by Adler /14/ who showed that except for propagation along the external field direction, the vacuum polarized by an external constant magnetic field is birefringent. In 1979, Iacopini and Zavattini /15/ proposed to measure this birefringence in the laboratory by a precision determination of the induced ellipticity on a laser beam down to $10^{-11}$. More recently, Ni et al. /16/ even proposed schemes to detect the change of index of refraction in ultra-high sensitive interferometers.

Despite the possible significant laboratory and astrophysical applications of the scattering of photons by electrons in a strongly magnetized electron gas in connection with precision measurement in the laboratory, X-ray or $\gamma$-ray source and neutron star cooling, it appears that magnetic Compton scattering has not been treated thoroughly. In this paper we present a detailed analysis of the Stokes’s parameters for Compton scattering in a superstrong magnetic field by the method of relativistic quantum field theory. The Stokes’s parameters for the scattered photons are computed explicitly in terms of the state of polarization of the incident wave, the electron-cyclotron frequency, the angle of incidence and the angle of scattering. Various appropriate limits will be considered. Specific application to high energy astrophysics such as neutron star cooling and models for compact $\gamma$-ray source will be briefly discussed.

2. Results and Discussion

We consider the scattering of a photon by an electron located at the origin of a Cartesian coordinate system, in a uniform magnetic field $\vec{B} = B \hat{z}$ where $\hat{z}$ denotes the unit vector and $B$ represents the constant field strength. The propagation vector $\vec{k}$ of the incident photon makes an angle $\theta$ with the static magnetic field. We shall only be concerned with the zero-order approximation, which consists of the two graphs (Figure 1). We note that the effects of the magnetic field on the photon via vacuum polarization is at least of first order and can be ignored. For applications to compact stellar objects (such as in the vicinity of neutron stars) the radiation energy density is much less than the magnetic energy density for field strength of $10^{12}$ Gauss /17/. Using the standard prescription of relativistic quantum field theory, the formal expression for the S-matrix for magnetic Compton scattering may be written in natural units ($\hbar = c = 1$) as

$$S_{fi} = \frac{1}{2} e^2 \int \psi_f(x') \mathcal{A}(k', x') i S_{fp}(x' - x) \mathcal{A}(k, x) \psi_i(x) \bar{d}^4 x \bar{d}^4 x' \left( \text{cross term} \right)$$

where $\bar{d^4 x} = d^4 x/(2\pi)$, $f$ denotes final, $i$ initial, and $\vec{x} = (x, y, z, t)$ and $\vec{x}' = (x', y', z', t')$ are 4-vectors. $\mathcal{A} = \gamma^\mu A_\mu$ where $\gamma$ are Dirac matrices and $A_\mu(\vec{k}, \vec{x}) =$...
Using the standard method of quantum electrodynamics and the density matrix $\rho_{ij} \equiv e_i \cdot e_j^* = \rho_{ji}^*$, the Stokes's polarization parameters may be defined as $S_n \equiv (S_0, S_1, S_2, S_3)$ for the incident photon where $S_0 = \rho_{11} + \rho_{22}, S_1 = \rho_{12} + \rho_{21}, S_2 = \rho_{11} - \rho_{22}, S_3 = i(\rho_{12} - \rho_{21})$. Expressions for the scattered photons may be similarly defined as $S'_{n'} \equiv (S'_0, S'_1, S'_2, S'_3)$ etc. The first Stokes's parameter $S'_0$ represents the scattered radiation intensity and is proportional to the differential scattering cross section as follows $\frac{d\sigma}{d\Omega} = \frac{S'_0}{S_0} R^2$ where $R$ is the distance between the scattering region and the point of observation and $S_0$ refers to the incident photon. The other Stokes's parameters $S'_1, S'_2$ and $S'_3$ describe the state of polarization for the scattered photon. Without giving the details of the straightforward but rather tedious calculation we now write the Stokes's parameters $S'_{n'}$ after scattering in terms of the state of polarization of the incident photon $\omega$, the electron-cyclotron frequency $\omega_c$, the angle of incidence $(\theta, \phi)$ and the angle of scattering $(\theta', \phi')$.

$$S'_{n'} = \frac{e^4 S_0}{R^2} \frac{2m + \omega - \omega'}{2m} \frac{\omega'}{m + \omega(1 - \cos \theta \cos \theta') - \omega' \sin^2 \theta'} M_{n'}.$$  

The matrix $M_{n'}(n = 0, 1, 2, 3)$ is a very complicated expression and will not be given here /18/.

In the nonrelativistic limit $(\omega \ll m, \omega_c \ll m)$, our results for the total cross section reduce to the well known form for magnetic Thomson scattering; namely

$$\sigma = \sigma_T \left[ \sin^2 \theta + \frac{1 + \cos^2 \theta}{2} \left( \frac{\omega}{\omega + \omega_c} \right)^2 + \left( \frac{\omega}{\omega - \omega_c} \right)^2 \right]$$

where $\sigma_T$ is the canonical Thomson cross section /7,9/. The total cross section derived from $S'_0$ in this case may be cast in the form $\sigma(B) = \left( \frac{\omega_c}{\omega} \right)^2 \sigma(0)$ where $\sigma(B)$ and $\sigma(0)$ respectively denote the Thomson cross section with and without the presence of the magnetic field, $\omega_c$ represents the classical gyrofrequency for the electrons and $\omega$ is the frequency of the incident wave. It is clear that the criterion for the magnetic field to substantially affect the Stokes's parameters is that the photon frequency be less than the electron-cyclotron frequency. For instance, the Thomson cross section is significantly reduced and the photon mean free path is greatly enhanced for photons with $\omega \ll \omega_c$. On the other hand, in the absence of the external magnetic field, the differential cross section (derived from $S'_0$) becomes the famous Klein-Nishina formula. To conclude our discussion we note that although the nonrelativistic cross section shows only one resonance at $\omega = \omega_c$, the relativistic cross section exhibits a series of resonances as shown in Figure 2. Specific applications to high energy astrophysics in connection with neutron star cooling and models for $\gamma$-ray source are currently in progress and will be published elsewhere.
References

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CTMC STUDY OF COLLISIONS OF CARBON IONS WITH H AND HE, RELEVANT TO MAGNETICALLY CONFINED FUSION PLASMAS

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At the beginning of the last decennium it was well understood that the balance of the impurity content of a magnetically confined plasma is dominated by interaction between plasma particles and the material walls /1/. Accordingly, a systematic effort was oriented towards the study of the plasma-surface interaction and of the relevant atomic processes of ion-atom collisions. Atomic processes involving hydrogen, helium, carbon, oxygen and metallic impurities were of first priority in this domain. Systematic evaluation and compilation of cross sections relevant to fusion was already initiated by C.F. Barnett, founder of the ORNL Controlled Fusion Atomic Data Centre /2/. The status of the atomic data base for reactions occurring mostly in the plasma core or concerning mainly the plasma heating by energetic ion beams has been reviewed elsewhere /3/. Studies of H and He collisions involving metallic impurities /4,5/ or oxygen /6/ have been recently made available. The status of such data was recently reviewed in /7/ and /8/. Here we concentrate to our calculations in progress of collision cross sections for the carbon ions colliding with hydrogen and helium.

Figure 1 Our CTMC results. Plain curves: Experimental data from /10,1-

The CTMC method was used for calculating total ionisation and one electron capture cross sections for H - Cq+ collisions in the energy region of 10 to 5000 and of 1 to 1000 keV/amu correspondingly.

Three different potentials for the ions have been lately used with this method: An empirical
potential which has been used previously for studying hydrogen collisions with medium- and high-Z plasma impurity ions as in /4/, a modified version of it incorporating a detailed Thomas-Fermi radius and a Coulomb potential. As expected, in comparison with existing experimental and theoretical data, the results show that among them, only the Coulomb potential is giving acceptable cross section values; it is to be noted that this is the case for all low-Z ions colliding with hydrogen. This, together with previous high-Z results, allows for a better understanding of the model potential influence on the CTMC calculations for all the species of relevance to fusion research. For light atom-ion collisions, specific model-potentials taking into account their atomic structure were also developed /9/; use of these potentials is in principle giving the possibility to improve the CTMC results, although the computation time increases drastically. Especially for H - C\(^{6+}\) collisions, preliminary calculations are not showing a substantial improvement of the results.

A glimpse of the obtained results is given in Figs. 1 and 2. For clarity reasons only our results obtained with Coulomb potentials are given, together with a set of pertinent experimental results /10,11/. Although the obtained results are quite satisfactory, mainly in the restricted validity area of the CTMC method, it appears that an extension of the method has to be devised in order to obtain confident results in the entire energy region from 10 eV/u to 5 MeV/u which are potentially of interest to magnetic fusion research.

n-resolved partial cross sections for one electron capture into the various ions levels have also been computed. As an example, we are presenting in Fig. 3 the results for H - C\(^{6+}\) collisions.

In CTMC calculations for collisions with helium atoms more assumptions than in the hydrogen case are necessary. First the independent electron model has been used: each of the two helium electrons moves independently of the other in an effective potential \(V_{\text{eff}}(r)\) which has still to be determined. Next a statistical average of the correlated electrons is needed for processes involving two electrons: double charge transfer, double ionisation and transfer ionisation. In our group at Orsay, we have made a systematic study of the various parameters entering CTMC calculations namely the influence of the effective potential and the different approximations to take correlations into account. Some of our results are presented here.

Looking the well known Binary Encounter Approximation we can see that in the case of a bare incident projectile all the different cross sections depend only on the ionisation energy and on the velocity distribution of the target electron. For helium atom ionisation energy is 0.9a.u. and the mean kinetic energy is 1.43a.u. When using hydrogenic approximation for \(V_{\text{eff}}\) with an effective charge \(Z_{\text{eff}} = 1.69\) \((1.69^2/2 = 1.43)\) one can either have the good ionisation energy or the good kinetic energy. Our calculations show that the
two approximations give quite different results for projectile charges larger to three and then one needs a more appropriate potential. From the analytical effective potentials proposed /9/,/12/ we used either the potential of /1/ or the one given by the simpler formula:

$$V_e(r) = -(1 + 0.255/(0.255 + r)/r.$$ 

Using the last one, we have calculated the total charge transfer cross section $\sigma_{q=1}$ for the reactions C$^q+$ + He $\rightarrow$ C$^{(q+1)+}$ + He$^+$, and the ionisation cross section $\sigma_i$ for the reactions C$^q+$ + He $\rightarrow$ C$^{q+}$ + He$^+$ + $e^-$, $q=6,4$ which are shown on Figure 4 and 5 together with some published results/13/,/14/,/15/.

The influence of the statistical correlation can be quantified using the values of $\sigma_{tot}$ reported in Figures 4 and 5. As an example the ionisation cross section $\sigma_i$ is calculated summing on the impact parameter twice the probability that one electron is ionised and one electron is excited, whereas $\sigma_{tot}$ is calculated summing only twice the probability that one electron is ionised. The results show that the correlation greatly reduces the one electron cross section except for the total charge transfer cross section at high energy. Comparisons with experiment show that cross section for the two electrons reaction is too large. For example the ratio of the cross section for transfer ionisation to the cross section for single charge transfer is nearly ten times the experimental results given in /15/. This is not due to the CTMC calculation but mainly to the independent electron approximation. The same results were obtained in /12/ using a quantal calculation. One solution proposed in /2/ is to use different effective potential for the double electrons reaction and the best way is to suppose a two steps mechanism: The projectile interacts first with one electron and then with the second one which is bound to a hydrogenic potential with a charge $Z_{ef} = 2$. The results for C$^5+$ where the influence of correlation is the most important, reported in Figure 6, are satisfactory; it remains now to compare the two steps approximation with a full dynamic two electrons CTMC code. Work in this direction is under way in our group.
Figure 6 Same as Figure 4; x are the two steps CTMC results

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Investigation on the metastable fragmentation of mixed argon-oxygen and argon-krypton cluster ions: The observation of two new fragmentation decay channels - evidence for the existence of metastable excited dimer ions ArO+* and ArKr+* 

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Introduction
Recently we have discovered a new metastable fragmentation reaction channel of Arn+ and Ne2n+ cluster ions, initiated by the radiative decay and subsequent dissociation of an Ar2+(3Σ_u^+) or Ne2+(3Σ_u^+) excimer, respectively, embedded within the cluster [1,2]. This observation provided clear evidence for a possible coexistence of an ionized "chromophore" (e.g. Ar2+ or Ne2+) and a localized long-living electronically excited state (Ar2+(3Σ_u^+) or Ne2+(3Σ_u^+)) inside the cluster. The interesting question which still remains open is the type of bonding (or the type of interaction) between the ionized chromophore and the excited state.

In order to explain the red spectral shift between a new observed emission continuum attributed to the emission of an Rg2* excimer embedded within the Rgn+ cluster ion and the well known "main" continuum observed in free excimers and excimers embedded in neutral clusters, Bondarcenko and coworkers [3] have recently developed a simple model based on the polarization interaction between the Rg2+ ionic chromophore and the Rg2* excimer. Using this model, our preliminary molecular dynamics (MD) simulations on Arn+* cluster ions [4] have shown good agreement with the experimental values of the number of evaporated monomers for cluster sizes n>20, i.e. for the case when the first icosahedral shell of the parent ion Arn+* is filled completely and the second shell is filled at least partially. For smaller cluster sizes (n<20), however, agreement between the MD simulations and the experiment is poor, i.e. the number of evaporated monomers predicted by the MD simulation is too small. This disagreement indicates that it is important to examine the possibility of stronger (covalent) bonding between the Ar2* excimer and the ionic chromophore of the cluster ion, allowing the formation of an Ar4+* complex in the centre of the cluster ion, surrounded by the first shell of neutral Ar atoms, which might be boiled off when the complex dissociates. It is important to note that the ion core of the Ar2+(3Σ_u^+) excimer is equivalent to the Ar2+(2Σ_u^+) ion, i.e. the resonant electron exchange between the Ar2* excimer and the Ar2+ ionic chromophore in the cluster ion: Ar2* + Ar2+ → Ar2+ + Ar2* might promote formation of such a complex.

To give more insight into this phenomenon, we have studied in this work the metastable fragmentation in systems where the ionic chromophore of the cluster ion is different from the ion core of the excimer and where the above mentioned resonant electron exchange would be impossible.

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The metastable fragmentation of the stoichiometric and the nonstoichiometric argon-oxygen cluster ions

Electron impact ionization of \( \text{Ar}_n(\text{O}_2)_r \) clusters leads to production of both the stoichiometric \( \text{Ar}_n(\text{O}_2)_q^+ \) and the nonstoichiometric \( \text{Ar}_n(\text{O}_2)_{q-1}^+ \) cluster ions. Whereas all of those ions decay via the usual single monomer evaporation, only \( \text{Ar}_n^+ \) cluster ions are showing evaporation of more than one monomer in the metastable time regime. Fig. 1 shows the dependence of the metastable fraction (which is the ratio between the fragment ion current and the parent ion current measured at an electron energy of 70 eV) on the number of evaporated monomers \( p \) for \( \text{Ar}_n^O^+ \) and \( \text{Ar}_{10}^O^+ \) cluster ions decaying in the first field-free region of our mass spectrometer via the decay reaction

\[
\text{Ar}_n^O^+ \rightarrow \text{Ar}_n^{p^+} + p\text{Ar} \quad (p>1).
\]

For comparison, the same dependence is shown in Fig. 2 for the metastable fragmentation of \( \text{Ar}_5^+ \) and \( \text{Ar}_{11}^+ \) cluster ions, decaying by excimer-induced fragmentation. In both cases the average number of evaporated monomers from the parent ions of comparable size is similar, indicating that the excess energy which has to be released from intramolecular to intermolecular modes prior to the fragmentation is also similar (appr. 1 eV, see e.g. [1]).

![Figure 1](image1.png)

**Fig. 1.** \( \text{Ar}_n^O^+ \rightarrow \text{Ar}_n^{p^+} + p\text{Ar} \)

![Figure 2](image2.png)

**Fig. 2.** \( \text{Ar}_n^+ \rightarrow \text{Ar}_n^{p^+} + p\text{Ar} \)

In the case of pure Ar cluster ions the strongest evidence pointing to \( \text{Ar}_2^* \) as the origin of the metastable fragmentation was provided by the electron energy dependence of the fragment ion current, which is starting to appear at a threshold electron energy shifted by approx. 12 eV to higher energies with respect to the appearance energy of the \( \text{Ar}_n^+ \) parent ions. Since there exists no metastable excited state of the \( \text{Ar}^+ \) ion at these energies, it was concluded that a neutral excited state (\( \text{Ar}^+(1^3\text{P}) \)) is formed within the cluster, giving rise to production of the \( \text{Ar}_2^+(3\Sigma_u^+) \) excimer [1,2]. In this work the electron energy dependence was measured for both the parent and the fragment ion currents of the metastable decay reaction channel \( \text{Ar}_{10}^O^+ \rightarrow \text{Ar}_5^O^+ + 5\text{Ar} \) (see Fig. 3). For comparison, the electron energy dependence of the excimer-induced fragmentation \( \text{Ar}_{10}^+ \rightarrow \text{Ar}_5^+ + 5\text{Ar} \) is also shown (Fig. 4).
Interestingly, the appearance energy (AP) of the metastable fragmentation \( \text{Ar}_{10}^0 \rightarrow \text{Ar}_5^+ + 5.\text{Ar} \) (appr. 28 eV, see Fig. 3) is very similar to the AP of the excimer-induced fragmentation \( \text{Ar}_{10}^+ \rightarrow \text{Ar}_5^+ + 5.\text{Ar} \) (which is appr. 27 eV, see Fig. 4). Since the AP of the ground state \( \text{O}^+ \) is around 19 eV, one would expect that an excimer-induced fragmentation in the \( \text{Ar}_{10}^0 \) cluster ion should start at around 30 to 31 eV, but this value could be lowered somewhat due to the solvation (see the parent ion signal of \( \text{Ar}_{10}^0 \), which starts to appear at 17 eV instead of 19 eV, Fig. 3). However, in contrast to the pure argon, there exist ionic metastable excited states \( \text{O}^+ (2D) \) and \( \text{O}^* (2P) \) which are populated by the dissociative ionization of \( \text{O}_2 \) molecules just in the energy region of 22 to 30 eV.

The fact that these states are populated with a high probability also in \( \text{Ar}_{10}^0 \) cluster ions is proved by the electron energy dependence of the parent ion current. This leads to the conclusion that the metastable fragmentation (1) is not an excimer-induced decay, but (as follows from the electron energy dependence, Fig. 3) the metastable ions \( \text{O}^+ (2D) \) and \( \text{O}^* (2P) \) are involved in the fragmentation. The fact that this fragmentation is not observed for \( \text{Ar}_n(\text{O}_2)^q\text{O}^+ \) (\( q>0 \)) cluster ions can be explained by the quenching of the metastable \( \text{O}^+ \) ions in the fast charge-transfer reaction \( \text{O}^+ + \text{O}_2 \rightarrow \text{O}_2^+ + \text{O} \), which is observable in the gas phase with a reaction rate of more than \( 5 \times 10^{-10} \text{ cm}^3\text{s}^{-1} \).

It is known that the radiative lifetime of the \( \text{O}^+ (2D) \) and \( \text{O}^* (2P) \) ions is several seconds. In the gas phase under thermal conditions these ions can react with argon by the charge-transfer reaction \( \text{O}^+ + \text{Ar} \rightarrow \text{O} + \text{Ar}^+ \). The fact that during the metastable fragmentation (1) the oxygen atom (or atomic ion) remains within the cluster indicates that (in contrast to the \( \text{Ar}_2^* \) excimer-induced decay) the fragmentation is initiated by a bound-bound transition, followed (most presumably) by quenching of the vibrational excitation of the lower electronic state populated in this very transition. Thus we propose that a metastable dimer \( \text{ArO}^* \) (corresponding to \( \text{O}^* (2D, 2P) \) ion bound with the neighbouring \( \text{Ar} \) atom) is formed after the ionization within the cluster, which decays (radiatively or via curve-crossing) in the metastable \( \mu \)s time regime to a vibrationally excited lower bound state \( \text{Ar}^0 \text{O} \) or \( \text{ArO}^* \), which is then vibrationally quenched.
The metastable fragmentation of argon-krypton cluster ions

We found that in case of mixed $\text{Ar}_n\text{Kr}_q^+$ cluster ions ($n<41$, $q<4$) the rare gas excimer-induced metastable fragmentation channel is completely quenched. Since there is a spectral overlap between the $\text{Ar}_2^*$ emission and the Kr absorption, this effect can be explained by a fast energy transfer from the $\text{Ar}_2^*$ excimer to the Kr impurity, proceeding either via the Förster-Dexter long-range electronic energy transfer mechanism (known in solids) or by direct energy transfer during bimolecular collision of Kr and $\text{Ar}_2^*$ inside the cluster. Surprisingly, when only one Kr atom is present in an argon cluster ion, a new metastable fragmentation channel is appearing, very much different from the excimer-induced fragmentation (both in the appearance energy, the time dependence and the number of evaporated monomers) and not observed up to now.

The number of evaporated monomers $p$ in this new decay channel increases from $p=2.3$ for $n=11$ to $p=4.6$ for $n=40$. For $n<11$ this fragmentation channel cannot be resolved from the usual single-monomer evaporation. For $n>24$ the number of evaporated monomers is reaching an asymptotic value of $p_{\text{max}}=4.6$. When comparing this value with the number of monomers evaporated during the photofragmentation of argon cluster ions, it is possible to estimate the energy needed for this evaporation to be approx. 0.45 to 0.55 eV.

The electron energy dependence of the fragment ion current for this metastable decay reaction of $\text{Ar}_n\text{Kr}^+$ cluster ions follows the electron energy dependence of the parent ion current and the difference in appearance energies is smaller than 1 eV. It was found that this metastable decay reaction is much slower than both the excimer-induced fragmentation of pure $\text{Ar}_n^+$ cluster ions and the new metastable fragmentation (1) of $\text{Ar}_n\text{O}^+$ cluster ions.

This new metastable fragmentation channel is quenched when two Kr atoms are present in the cluster. This fact and the appearance energy indicate that a metastable state of the ArKr+ chromophore (e.g. a $\text{Ar}(1\text{S}_0)+\text{Kr}^+(2\text{P}_{1/2})$ or $\text{Ar}^+(2\text{P}_{1/2,3/2})+\text{Kr}(1\text{S}_0)$ bound state) is responsible for this fragmentation. When more than one Kr atoms are present inside the cluster, this state is apparently quenched by a $\text{ArKr}^++\text{Kr} \rightarrow \text{Ar}+\text{Kr}_2^+$ switching reaction.

Acknowledgement

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References
ION ATTACHMENT TO VAN DER WAALS CLUSTERS:  
REACTIONS OF NO$^+$ WITH Ar$_m$

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1. Introduction

Besides electron impact ionization /1/ as the major tool in producing cluster ions from expanded cluster jets, photon ionization /2/, Penning and associative ionization /2,3/ and electron attachment /4/ have been used to prepare in many ways and to study in detail atomic and molecular cluster ions /5/. In the present communication we present for the first time (see also Ref. /6/) an alternative ionization method based on the interaction of ions with neutral van der Waals clusters, i.e. the attachment of NO$^+$ ions to Ar$_m$ clusters.

2. Experimental technique

The supersonic expansion type cluster source, the electron impact ion source and the double focussing sector field mass spectrometer analyzer used in the present studies have been described in detail previously /1/. The present use of this apparatus differs from this earlier configuration in that a second reagent port has been added to the ion source and the operating conditions of the ion source have been changed. In operation, a neutral argon cluster beam is produced by expanding pure Ar gas at a constant temperature (typically -160°C) and a constant pressure (typically 1.5 bar) through a 20 μm nozzle into vacuum. The ensuing supersonic beam proceeds through a skimmer into a differentially pumped Nier type ion source. In this ion source stagnant NO gas (introduced into the ion source via a supplementary gas port) is ionized by an electron beam (directed at right angles to the cluster beam) of variable energy thereby producing thermal NO$^+$ ions. Primary NO$^+$ ions and NO$^+$ ions reacted with Ar$_m$ clusters (yielding associates of
the form \( \text{Ar}_n \cdot \text{NO}^+ \) are then (i) extracted at right angles from this ion reactor, (ii) accelerated by an acceleration voltage \( U \) (up to 3 keV), (iii) analyzed in a 48.5° magnetic sector field (with a radius of 60 cm) followed by a 90° electric sector field (with a radius of 21 cm), and (iv) detected after postacceleration with a Faraday cup or an electron multiplier. This two sector configuration has several uses. Its combined action permits mass analysis with high resolution and sensitivity, whereas separate use allows quantitative studies on metastable decay reactions.

Because the reacting \( \text{NO}^+ \) ions are produced by electron impact ionization at the very position where the \( \text{Ar}_m \) clusters are to react with the \( \text{NO}^+ \) ions, it would appear impossible to distinguish \( \text{Ar}_n \cdot \text{NO}^+ \) ions produced by \( \text{NO}^+ \) ion attachment to \( \text{Ar}_m \) clusters from other growth processes, such as a possible two step process where the \( \text{Ar}_m \) cluster are first ionized by the electron beam and the \( \text{Ar}_m^+ \) cluster ions formed are then reacting with the stagnant \( \text{NO} \) gas. Using however, electrons with energies below the appearance energies of argon and argon cluster ions (i.e. 15.8 eV and 14.6 eV, respectively /1/), it is possible to only ionize \( \text{NO} \) molecules. Varying the electron energy and the \( \text{NO} \) gas pressure, we are able to demonstrate that these experimental conditions actually do allow the \( \text{NO}^+ \) attachment reactivity to be readily isolated from other processes.

3. Results and Discussion

Fig. 1a shows a mass spectrum of an argon cluster beam obtained by \( \text{NO}^+ \) attachment. The resulting mixed cluster ions \( (\text{Ar}_{n-1} \cdot \text{NO})^+ \) show strong peaks at cluster sizes 13, 19, 23, 26, 29, 49 and 55 (where for instance cluster size 13 corresponds to 12 \( \text{Ar} \) plus one \( \text{NO} \) constituent). It is interesting to compare these results with those found for direct electron impact ionization of neutral \( \text{Ar}_m \) or \( \text{Ar}_m \cdot \text{NO} \) clusters produced by jet expansion. Whereas the mass spectrum for the mixed clusters \( \text{Ar}_m \cdot \text{NO} \) is rather similar to the one shown in Fig. 1a, the mass spectrum for pure \( \text{Ar}_m \) clusters (see Fig. 1b) is
significantly different in general shape and in the sequence of maxima (i.e. strong peaks at 14, 16, 19, 21 and 27).

From previous investigations on the electron impact ionization of vDW clusters /1,5/ it can be concluded that the different shape (i.e. shift of the maximum of the distribution and the center of gravity of the distribution to lower n in case of electron ionization) is due to stronger fragmentation during the ionization events. That is, NO\(^+\) attachment is a softer ionization technique preserving more of the neutral distribution than electron impact ionization. Similar effects have been observed when comparing electron impact ionization with electron attachment spectra /4/.

This point also leads to an interpretation of the difference in the structure of the well defined anomalies (magic numbers). Very likely, in case of electron impact ionization, in addition to structure in the spectrum arising from the initial neutral cluster distribution, additional anomalies may be introduced by a size dependent ionization and fragmentation cross section, by size dependent energy deposition and disposal, and by variation in the stability of the resulting charged clusters.

4. Acknowledgements
Work partially supported by the Österreichischer Fonds zur Förderung der Wissenschaftlichen Forschung and the Österreichische Nationalbank, Wien.

5. References

/2/ C.Y. Ng, Vacuum Ultraviolet Photoionisation and Photodissociation of Molecules and Clusters, World Scientific, Singapore (1991)
Fig. 1. Mass spectra of an argon cluster beam produced by nozzle expansion.
Upper part: NO⁺ attachment ionization
Lower part: Electron impact ionization (electron energy 17 eV)
On the Interaction of Low Energy Electrons with vdW-Clusters:  
Electron Attachment Reactions and Auto-Scaevenging


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1. Introduction:

Besides investigations on electron impact ionization, studies on electron attachment reactions and on electron scavenging helps to elucidate elementary processes occurring in various plasmas. In this respect it is not only interesting to study interactions with atoms or molecules, but also with clusters due to their unique properties and the possibility to study ions not being easily accessed otherwise, e.g. CO₂⁻ or SF₇⁻.

We are presenting here relative electron attachment cross sections, called electron attachment spectra (EAS), of (i) the non-stoichiometric cluster anions (CO₂)ₙ-O⁻, (CO₂)ₙ-O₂⁻, (SF₆)ₙ-SF₅⁻, (SF₆)ₙ-SF₄⁻, (SF₆)ₙ-SF₃⁻, and the superhalogenic (SF₆)ₙ-SF₇⁻ formed by direct and indirect electron attachment processes and of (ii) mixed rare gas/molecule cluster anions, i.e. Xe-SF₅⁻, Ar-O₂⁻, and Ar-O₃⁻, giving clear evidence on sequential inelastic scattering processes of the incoming electron within these clusters (auto-scaevenging).

The interaction of low energy electrons (with energies below the ionization potential) with neutral clusters produces negatively charged cluster ions according to the following three schemes:

1. direct attachment processes (associative and dissoziative) [1]
2. indirect mechanisms (ion molecule and charge transfer reactions within clusters, quenching of dissoziative attachment channels)
3. scavenging processes (within clusters we term them "auto-scaevenging")

2. CO₂-cluster anions:

The interaction of near thermal electrons with clusters gives rise to the well known zero-resonance in almost all systems [2]. It is therefore possible to produce the CO₂ anion and higher homologues via the interaction of thermal electrons with a CO₂ cluster beam despite the fact that CO₂ has a negative electron affinity of EA = -0.6 eV.

Rising the energy of the electrons interacting with the CO₂ cluster beam we are also able to detect non-stoichiometric compounds (CO₂)ₙ-O⁻ and, for the first time, (CO₂)ₙ-O₂⁻. Both complexes are formed above an electron energy of 8 eV by direct dissoziative electron attachment (1). The (CO₂)ₙ-O₂⁻ compounds can also be produced via a second, indirect process (2): i.e., by an ion molecule reaction of a primary O⁻ ion with a neighbouring CO₂ molecule within the cluster in the following steps:
The second step of the reaction has an exothermicity of about 1.35 eV. The resulting O$_2$ molecule will probably be stabilized into a CO$_4$ molecule in a reaction, which is found to be very fast in the gas phase [3]. Fig.1 shows the EAS (a) of (CO$_2$)$_n$O' (n = 2, 10) peaking at appr. 4, 9, and 12 eV and (b) of (CO$_2$)$_{10}$O$_2$' peaking at about 13 eV and of (CO$_2$)$_5$' peaking at appr. 3, 9, and 12 eV (for more details see [4]).

3. SF$_6$ cluster anions:

In the case of SF$_6$ cluster anions the situation is totally different. Besides the well known zero-resonance of SF$_6$ clusters there are no other resonant structures in the EAS and it was well accepted, that non-stoichiometric SF$_6$ cluster anions do not exist in the mass spectrum of SF$_6$ clusters. The formation of e.g. (SF$_6$)$_n$SF$_5^-$ via direct dissoziative electron attachment was reported to be effectively quenched within the cluster [5]. Expanding our electron energy scale up to 20 eV we have for the first time been able to detect anions of the structure (SF$_6$)$_n$SF$_5^-$ (5.5 and 10 eV), (SF$_6$)$_n$SF$_4^-$ (6 and 13 eV), (SF$_6$)$_n$SF$_3^-$ (14.5 eV), and of the superhalogenic structure (SF$_6$)$_n$SF$_7^-$ (6 and 13 eV) - in brackets are given the peak positions of the resonances shown in the EAS of Fig.2. Comparing the positions of the observed resonance maxima of the cluster beam with those of the various fragments of gas phase molecules [6] it is clear, that the non-stoichiometric SF$_6$ cluster anions cannot be produced via direct (1) but must be generated via indirect processes (2).

Considering the energetics of reactions between primary fragments with a neighbouring cluster constituent we have to conclude, that this kind of indirect anion production is
endothenmic in almost all cases. Nevertheless the position of the resonance peaks in the EAS of the various complexes gives clear hints as to the energetics of possible primary fragment ions. Taking into account this information we propose that non-stoichiometric SF₆ cluster ions are produced via a two step mechanism, where a primary fragment ion reacts with one of its neutral counterparts caged inside the cluster (and not with a neighbouring molecule).

One example is the production of (SF₆)ₙ-SF₄⁻ at about 12 eV:

\[ \text{SF}_6 + e^- \rightarrow \text{SF}_2^+ + \text{F}_2 + \text{F}_2 \]

i.e. within the cluster complex

\[ (\text{SF}_6)_n + e^- \rightarrow (\text{SF}_6)_n-p^- \rightarrow (\text{SF}_2^+ + \text{F}_2) + \text{F}_2 + p^-\text{SF}_6 \]

\[ \rightarrow (\text{SF}_6)_{n-p}^- \rightarrow \text{SF}_4^+ + p^-\text{SF}_6 \]

It is interesting to note that we have some experimental evidence, these caging reactions give rise to more excess energy in the energy region below 8 eV than in the region above 8 eV (see [7]).

4. Auto-scavenging:

SF₆ is widely used in high voltage devices and in plasma etching processes due to its unique scavenging property. We tried successfully to use this specific property to monitor excited states of the neutral Xe atom in mass spectrometric investigations studying the EAS of mixed Xe/SF₆ cluster anions shown in Fig. 3(a) [8]. The first maximum corresponds to the zero-resonance of SF₆ (the only one in the EAS of neat SF₆ cluster anions); the second resonance rises at about 8.5 eV and shows two maxima appr. 1 eV apart. We attribute the second structure to the first metastable (8.315 and 9.447 eV) and the first radiative (8.44,
9.57, and 10.03 eV) states of the Xe atom [9], respectively. These states are populated via an inelastically scattered electron exciting the Xe atom, loosing all of its energy, and attaching to the SF$_6$ molecule (3).

Is this third class of anion production mechanisms also observable in other systems, not known as scavenger-gas? Yes, it is. Producing mixed Ar/O$_2$ cluster anions the EAS shows two additional structures compared to neat O$_2$ cluster anions. In Fig. 3(b) the EAS of Ar-O$_4^-$ is compared with that of O$_4^-/(O_2)_n$. The resonant structure at 11.5 eV may be ascribed to excitation of the first excited states of the neutral Ar atom within the mixed cluster prior to zero energy attachment to an O$_2$ compound. For more details see Ref. 10.

![Diagram](image)

**Fig. 3:** (a) EAS of $^{131}$Xe-$^{32}$SF$_6^-$; (b) EAS of Ar-O$_4^-$ and O$_4^-/(O_2)_n$.

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Particle Exchange Reactions at Low Kinetic Energies

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A Selected Ion Flow Drift Tube (SIFDT)\textsuperscript{1} was used to investigate particle exchange reactions of \( \text{N}_4^+\), \( \text{Ar}_2^+\), \( \text{Kr}_2^+\) and \( \text{ArN}_2^+\) dimer or dimer like ions colliding with various neutrals at mean center of mass energies, \( \text{KE}_{\text{cm}} \), ranging from 0.05 to about 2 eV. The \( \text{KE}_{\text{cm}} \) is changed by variation of \( E/N \) (\( E \) is the axial electric field strength and \( N \) the buffer gas density) and is calculated in the usual way\textsuperscript{2} using the relation

\[
\text{KE}_{\text{cm}} = \frac{M}{m+M} \left( \text{KE}_{\text{ion}} - \frac{3}{2} k_B T \right) + \frac{3}{2} k_B T,
\]

where \( \text{KE}_{\text{ion}} \) is the mean kinetic lab energy of the reactant ions according to Wannier\textsuperscript{3},

\[
\text{KE}_{\text{ion}} = \frac{3}{2} k_B T + \frac{1}{2} m v_d^2 + \frac{1}{2} M v_d^2.
\]

Here, \( v_d \) is the drift velocity of the reactant ions, \( T \) is the buffer gas temperature, \( k_B \) is the Boltzmann constant and \( m, M \) and \( M_{\text{ion}} \) are the masses of the buffer gas, the reactant gas and the ions, respectively. As helium was used as a buffer gas, the relative kinetic energy between the ions and neutral reactants (all having much larger masses than the buffer gas atoms) is always larger at elevated \( E/N \) than the relative energy between the ions and the buffer gas atoms. The latter one determines the internal excitation of the ions\textsuperscript{4}, therefore any changes observed (as dependent on \( E/N \)) in the rate coefficients or product channels are mainly due to the variation in \( \text{KE}_{\text{cm}} \) and do not primarily reflect an influence of the internal energy (vibrational excitation) of the ions. The present results indicate, that particle exchange (switching) is competing with charge transfer, but is usually not dominant, when the recombination energy \( \text{RE}(I) \) of the dimer ions is larger or at least close to the ionization potential \( \text{IP}(R) \) of the neutral reactant. Switching is the only possible reaction mechanism if charge transfer is endothermic. The overall rate coefficients
Observed rates are close to the collisional or Langevin limit in most cases and independent of KE$_{cm}$ (see Fig. 1), whereas the product channels resulting in particle exchange decline at elevated KE$_{cm}$ (see e.g. Fig. 2). As Fig. 2 shows, there is a qualitative correlation between the bond strengths of the product ions and the KE$_{cm}$ values, where a significant decline of the fraction of the switching channel occurs.

Several reactions of N$_2$Ar$^+$ ions show additionally interesting details. The main reactions are as follows:

- $\text{Kr}_2^+ + \text{CH}_4 \rightarrow \text{KrCH}_4^+$
- $\text{Kr}_2^+ + \text{H}_2\text{O} \rightarrow \text{KrH}_2\text{O}^+$
- $\text{Ar}_2^+ + \text{N}_2\text{O} \rightarrow \text{ArN}_2^+$
- $\text{Kr}_2^+ + \text{CO} \rightarrow \text{KrCO}^+$
- $\text{Kr}_2^+ + \text{H}_2\text{O} \rightarrow \text{KrH}_2\text{O}^+$
- $\text{Kr}_2^+ + \text{C}_2\text{H}_4 \rightarrow \text{KrC}_2\text{H}_4^+$

Fig. 1: Rate coefficients of the overall reactions indicated in the figure as dependent on KE$_{cm}$. (——) indicates the collisional limiting values for the reactions of H$_2^+$, N$_2$Ar$^+$, Ar$_2^+$ and Kr$_2^+$ with H$_2$O, and for Kr$_2^+$ with CO (——) and C$_2$H$_4$ (——) respectively.

Fig. 2: Rate coefficients for the switching channels only of the reactions indicated in the figure.
products in the reaction with H₂ are ArH⁺ and ArH₂⁺ (Fig.3), while hardly any N₂H⁺ (or N₂H₂⁺) is observed. This infers, that the charge in the N₂Ar⁺ ion is connected with the Ar-atom rather than with the N₂ molecule, despite of the slightly larger ionization potential of Ar as compared to the one of N₂. Probably the internuclear distance R in the N₂, bound within the N₂Ar⁺ ion is slightly shifted with respect to R₀ of the isolated neutral N₂, so that the position of the charge on the Ar-atom is energetically favoured. This is also supported by recent calculations of K. Hiraoka et al. Additional support for that arises from data on the reaction of N₂Ar⁺ with Kr, where ArKr⁺ is the only switching product observed.

**Fig. 3:**
Energy dependencies for the reactions of N₂Ar⁺ with H₂.

**Fig. 4:**
Energy dependencies for the reactions of N₂Ar⁺ with Ar.
No charge transfer product $H_2^+$ was observed as the recombination energy of $N_2Ar^+$ ($RE=14.5\text{eV}$) is smaller than the ionisation potential of $H_2$ ($IP=15.43\text{eV}$). At elevated $KE_{cm}$ the loosely bound $ArH_2^+$ ion becomes unstable and presumably a successive proton transfer $H_2^++Ar\rightarrow ArH^++H$ occurs before the $H_2^+$ could leave the ($Ar...H_2^+$) collision complex.

In the case of $N_2Ar^+$ colliding with Ar an increase of the overall rate coefficient from $k=2\times10^{-11}\text{ cm}^3\text{sec}^{-1}$ at $KE_{cm}=0.04\text{ eV}$ to $k=5\times10^{-10}\text{ cm}^3\text{sec}^{-1}$ at $1.2\text{ eV}$ is observed (Fig.4). At low energies obviously an exchange of Ar-Ar is favoured due to the better mass-match as compared to the Ar-$N_2$ exchange. The Ar-Ar exchange however cannot be seen as long as no isotopically labelled Ar is used, thus the observed rate coefficient reflects only the ion-mass changing Ar-$N_2$ exchange channel, resulting in the product $Ar_2^+$. Also an increase of $k$ is observed in the analogous reaction of $N_2Ar^+$ with $N_2$, where only the exchange Ar-$N_2$ results in a product ion of different mass ($N_4^+$) but not the $N_2-N_2$ exchange, which again due to the better mass-match is favoured at low collision-energies.

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References
INTRODUCTION

Static drift tubes and flow drift tubes have been used for many years to study ion-molecule reactions in the energy regime from thermal to a few electron volts [1]. Thus the rate coefficient, $k$, and product ions have been determined for many ion-molecule reactions, and it has been the usual procedure to present the data as plots of $k$ versus $E_r$ (the reactant ion/reactant neutral center-of-mass energy) and to describe the variation of $k$ with $E_r$ as due to the variation of kinetic energy of the reactants. When the reactant ions are atomic this is clearly a sensible procedure, since no significant internal (electronic) excitation of the ions can occur in collisions with the buffer (or reactant) gas atoms at the ion energies involved. Obviously this is not the case when molecular ions are involved, because of the high probability that rotational/vibrational excitation will occur, and which can immediately influence the reactivity of the ion. Several drift tube studies of this phenomena have been reported and it is with this that this paper is largely concerned. Clearly in this case the relevant energy to consider is the reactant ion/carrier gas atom center-of-mass energy which here we designate as $E_{C}$. Both internal excitation and de-excitation will occur in collisions of the ion with the carrier gas, and under steady state conditions (i.e. excitation rate = de-excitation rate) then the internal energy state distribution of the ion will be characterised by $E_{C}$. This has been demonstrated by Lindinger and his colleagues for the vibrational excitation of $N_2^+$ and $O_2^+$ in helium carrier gas in a selected ion flow drift tube (SIFDT) experiment, who have also shown how vibrational excitation (and kinetic excitation) influences the course of ion/molecule reactions in SIFDT experiments in He and Ar carrier gases (these studies have been summarised by Lindinger [2]). The data are in agreement with theoretical calculations of Viehland et al. [3] predicting that molecular ions drifting in an atomic buffer gas will reach an effective internal temperature, $T_{\text{eff}}$ given by

$$3/2k_BT_{\text{eff}} = [1 + (M_b/M_i)\xi]^{-1} \times (3/2k_BT + 1/2M_bv_d^2) \times (1+\beta),$$

where $k_B$ is Boltzmann's constant, $M_b$ and $M_i$ are the masses of the buffer gas atom and the ion, $\xi$ is a dimensionless ratio of collision integrals that characterizes the fractional energy loss due to inelastic collisions, $T$ is the temperature of the buffer gas, $v_d$ is the drift velocity and $\beta$ is a minor correction term.

On the addition of a reactant gas to the drift tube (which almost invariably has a larger atomic (molecular) mass than the carrier gas which usually is helium) then reactant ions which are already vibrationally excited (according to $E_{C}$) collide with the reactant gas, the relevant collisional energy then being $E_{r}$. The total energy brought into the reaction is then in some way determined by both $E_C$ and $E_r$, and not by $E_r$ alone. In order to gain further understanding of the relative influence of $E_C$ and $E_r$ we have chosen to study perhaps the simplest of reactive systems involving molecular ions which is the collisional induced dissociation (CID) of...
some molecular ions, specifically Kr$_2^+$, N$_4^+$, (CO)$_2^+$, and N$_2$Ar$^+$ dimer and dimer-like ions, firstly in pure helium carrier gas to properly appraise the dependence of the dissociation rates on $E_C$ alone and then secondly by adding chemically inert 'heavy' gases (i.e. variously Ne, Ar and N$_2$) to investigate how the combination of $E_C$ and $E_r$ influences the dissociation rates of these ions.

**EXPERIMENTAL**

The experiments were carried out using the Innsbruck SIFDT [4]. The experimental approach used to the study of the collisional dissociation has been described previously by Smith et al [5]. The mobilities of Kr$_2^+$ and (CO)$_2^+$ were determined in our laboratory using the SIFDT; the mobilities of N$_4^+$ and N$_2$Ar$^+$ were taken from [6].

In order to determine $k$ values for the dissociation of these ions in collisions with other gases, conventional SIFDT experiments were carried out in which the length of the drift field is fixed and the flow of the "reactant gas" is varied. Thus $k$ is obtained as a function of $E_r$ (remembering that $E_C$ also varies with $E/N$). It is important to note that no variations of $k$ were observed with helium pressure over the limited range over which meaningful measurements could be made (i.e. 0.13 to 0.23 torr).

Arrhenius-type plots for the rate coefficients for the dissociation of Kr$_2^+$, (CO)$_2^+$, N$_4^+$ and N$_2$Ar$^+$ drifting in pure Helium.

**RESULTS and DISCUSSION**

Typical data are presented in Fig.1 as plots of $k$ versus $E_C^{-1}$. The good linearity of these plots indicates that $k$ is related to $E_c$ according to an expression of the form $k= A\exp(-3E_a/2E_c)$. This is of the form of the well-known Arrhenius equation ($k= A\exp(-E_a/kT)$) in which $E_a$ is equated to an "activation energy" for the reaction in this case the reaction is dissociation. Note: The 2/3 is introduced since $E_c$ can be equated to $3/2kT$. The constant $A$ can be related (with caution !) to the collisional rate coefficient, $k_L$, for the reactant ion/He atom collisions. Thus, values of $E_a$ and $A$ have been obtained for the reactions studied and they are given in Table 1 together with the experimentally-determined dissociation energies of the ions, and the calculated $k_L$ values. The similarity of the $E_a$ and $D$ values is striking and indicates that, at least, for these ions the nonthermal drift tube experiment
approximates to a thermal system. It also indicates that the Viehland expression mentioned above describes quite well the internal excitation of the ions and that the factor including $\xi$ represents only a minor correction.

<table>
<thead>
<tr>
<th>Ion</th>
<th>$E_a$(eV)</th>
<th>$D$(eV)</th>
<th>$A$(*$10^9$cm$^3$s$^{-1}$)</th>
<th>$k_L(*10^9$cm$^3$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kr$^{2+}$</td>
<td>1.1</td>
<td>1.1-1.5$^a$</td>
<td>0.2</td>
<td>0.54</td>
</tr>
<tr>
<td>(CO)$_2^+$</td>
<td>1.0-1.3</td>
<td>0.97-1.21$^b$</td>
<td>1.0</td>
<td>0.55</td>
</tr>
<tr>
<td>N$_4^+$</td>
<td>0.9</td>
<td>0.9-1.05$^c$</td>
<td>2.0</td>
<td>0.55</td>
</tr>
<tr>
<td>N$_2$Ar$^+$</td>
<td>0.9</td>
<td>0.94-1.09$^d$</td>
<td>5.0</td>
<td>0.55</td>
</tr>
</tbody>
</table>

$^a$ see ref. [7], $^b$ see ref. [8], $^c$ see ref. [9], $^d$ see ref. [6]

The derived values of $A$ are within a factor of four of the $k_L$ values. Intuitively, it is expected that the $A$ value should relate to the efficiency of the reaction, (i.e. the probability that dissociation will occur above the threshold energy for dissociation). Thus for efficient reactions $A$ should approximate to $k_L$ as is observed. However, it is difficult to see why $A$ should exceed $k_L$. Clearly, much more work is required before a complete understanding of this is obtained. Fig. 2 shows additional data on the dissociation of N$_2$Ar$^+$ due to collisions with Ne and N$_2$.

Fig. 2a,b: Arrhenius-type plots for the dissociation rate coefficients of N$_2$Ar$^+$ drifting in pure Helium and similar plots for the dissociation of this ion in collisions with a) N$_2$ and b) Ne, added in small amounts to the helium buffer gas. The numbers in the figures correspond to different $f$ factors.

On the addition of 'reactant gas' to the SIFDT, the drifting ions, with internal energy characterised by $E_c$, established by multiple collisions with the helium buffer gas, undergo collisions with the reactant gas atoms or molecules, the center-of-mass energy being $E_r$. Such collisions are relatively infrequent, the collision frequency obviously depending on the amount of reactant gas introduced into the helium. In effect, these are single collisions, the total energy in the collision being $E_c + E_r$. This results in an increase of the internal energy of the ion and hence an increase in the probability of and the rate coefficient for dissociation. Obviously in these 'single' collisions the internal energy state distribution cannot be equalibrated to $E_r$ (as it is
to $E_C$), and thus to use $E_T$ alone in an Arrhenius-type plot with the expectation of deriving $E_a$ values for dissociation is unsound. As can be seen in Fig. 2, the plots involving $E_T$ alone result both in a large displacement along the $E_T$ axis (and hence to high $E$) from the equilibrium plots involving $E_C$ alone and non-linear plots from which $E_a$ values cannot be obtained. Clearly $E_T$ alone is not the appropriate energy to characterise the dissociation rate; the following is a more appropriate (albeit crude) description of the interaction.

Immediately following the collision with the impurity gas (i.e. Ne, Ar or N$_2$), a fraction of $E_T$ is converted into internal energy of the ion. Assume that all the $E_T$ is converted to vibrational energy of the ion (i.e. rotational and translational excitation are relatively unimportant). However, at the $E_T$ involved it is unlikely that all the modes of vibration of the polyatomic ions are accessible. Now assume that $f$ modes are accessible and the $E_T$ is shared equally amongst them in some quasi-equilibrium fashion. Then a more appropriate characteristic energy in the interaction is $(E_C + E_T/f)$. Thus Arrhenius-type plots involving $(E_C + E_T/f)^{-1}$ for various $f$ are included in Fig. 2. As one might expect, a higher fraction of the kinetic energy is converted into "dissociating energy" for N$_2$ than for Ne. In the combination of N$_2$Ar$^+$-N$_2$ more modes for energy storage are accessible than the in N$_2$Ar$^+$-Ne-complex.

The above represents only a first attempt to explain the respective importance of $E_C$ and $E_T$ in drift tube studies. The phenomenon of collisional dissociation offers the opportunity to gain some understanding on this, but these considerations must be applied also to the study of ion-molecule reactions, for the study of which SIFDT experiments are widely used.

Acknowledgement
This work was supported by the Fonds zur Förderung der Wissenschaftlichen Forschung.

References
Theoretical studies of the vibrational relaxation of $\text{N}_2^+(v=1)$ in collisions with He using the semiclassical method

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Introduction
Rigorous theoretical studies of vibrational relaxation of diatomic ions in collisions with neutrals are, in contrast to their neutral-neutral analogues, relatively scarce. At the same time the vibrationally inelastic scattering processes in ion-neutral collision systems are of great interest for dynamical investigations in connection with their pronounced long-range electrostatic interaction potential (see for details, see Ref. 1). A number of recent experimental studies in this field (for a review see /1-3/) form a sound experimental background to begin with systematic theoretical investigations.

In this contribution we present the theoretical studies of vibrational relaxation of $\text{N}_2^+(v=1)$ in collision with He atoms, a process which was investigated experimentally in our laboratory /4/ using the SIFTD-technique in the mean kinetic centre-of-mass energy ($KE_{cm}$) range from thermal to 0.8 eV. The semiclassical (SC) method /5/, which has proven to give adequate descriptions of vibrationally inelastic processes for a number of neutral and charged collision systems, was used to perform dynamical calculations.

Modelling the ab initio potential energy surface
The analytic potential energy surfaces (PES) used in the framework of the SC model were obtained as fits to recent 2D quantum-chemical data /6/. The missing vibrational coordinate dependence in the ab initio data /6/ was modelled using the following dumb-bell-like expressions:

\[
\begin{align*}
V_A &= -\frac{P}{R^4} + A e^{-\beta R} + \sum_{i=1}^{3} \left[\sum_{j=1}^{L} e^{-\gamma_{ij} R_j} - 2 \sum_{j=1}^{L} e^{-\gamma_{ij} R_j} \right] \\
V_B &= \sum_{i=1}^{3} \left[ C_i e^{-\alpha_i R_i} - a_i R_i^{-4} \right] \\
V_C &= -\frac{P}{R^4} + \sum_{i=1}^{3} C_i e^{-\beta_i R_i}
\end{align*}
\]
where R is the distance between the He atom and the centre-of-mass of the N$_2^+$ ion, R$_i$ are the distances between the He atom and the i-th atom of the N$_2^+$ ion. The values of parameters in all three forms (1) - (3) were obtained using a non-linear constrained minimization algorithm. The following minimization function (weighted rms-fitting) was adopted:

$$G = \left[ \frac{1}{N} \sum_{i=1}^{N} \omega_i (V_i - E_i)^2 \right]^{1/2}$$

where N is the number of ab initio points (N= 39), E$_i$ - ab initio energies, V$_i$ - values of the analytic PES, $\omega_i$ weighting coefficients. Different weighting conditions, more or less emphasizing the ab initio points in the vicinity of the potential well, were analyzed.

**Results and discussion**

The sets of potential parameters for A- and B- type of the interaction potential and different weighting conditions are given in Tables 1 and 2, respectively. The dynamical results obtained using the abovementioned analytic PES are shown in Fig. 1. Similar results are obtained for a PES of the C-type (eq. (3)). It is to see that the dynamical results satisfactorily reproduce the experimental data /4/ in the high and intermediate energy/temperature range. A somewhat poorer agreement is achieved in the near-thermal range (300-500 K). Two reasons can be given to explain the observed difference.

First, though our analytic PES are the fits to the 2D ab initio data, the vibrational coordinate dependence - a feature of crucial importance in the vibrational energy exchange processes - is a model construction in the present study. It is not possible to exclude that for this particular ion-neutral interaction potential the dumb-bell-like fitting forms used are not capable to predict the vibrational coordinate dependence accurately. A more detailed discussion of this subject is given in /8/.

Secondly, in spite of the large basis set and the powerful MCSCF-CI method used, the quantum-chemical data /6/ probably, as it is sometimes the case for other systems, to some extent underestimate the dispersion contribution and, accordingly, the potential well depth. But just this feature of the interaction potential strongly influences the low-temperature reaction attributes, as it was discussed earlier /9/.

Thus, it seems that the observed difference between experimental and theoretical results in the near-thermal range is mainly caused by probably not quite adequate vibrational coordinate dependence in the dumb-bell potential representation, though also some influence of the possible underestimation of the potential well depth in the ab initio data /6/ can not be excluded.

From the experimental point of view, more complex reaction mechanisms like clustering processes can eventually take place to some extent in the near-thermal range and thus influence the experimental results. These processes certainly can not be taken into account in terms of the direct, adiabatic binary collision reaction mechanism, assumed in the present theoretical investigation.
Thus, to explore more completely the dynamics of vibrational relaxation in collisions between N$_2^+$ and He, additional theoretical and experimental work is needed. Such investigations are under way.

Table 1: Potential parameter sets for different weighting conditions for the A-type potential eq.(1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A$_1$</th>
<th>A$_2$</th>
<th>A$_3$</th>
</tr>
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<tbody>
<tr>
<td>P [eVÅ$^4$]</td>
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<td>1.494</td>
<td>1.494</td>
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<tr>
<td>A [eV]</td>
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<td>3.292</td>
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<tr>
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<tr>
<td>$\beta_i$ [Å$^{-1}$]</td>
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<tr>
<td>R$_i$ [Å]</td>
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Table 2: Potential parameter sets for different weighting conditions for B-type potential eq. (2).

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<td>$C_i$ [eV]</td>
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<td>256.7</td>
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<td>$\alpha_i$ [Å$^{-1}$]</td>
<td>3.397</td>
<td>3.646</td>
<td>3.921</td>
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</table>

Acknowledgements

This work was done in the frame of a bilateral agreement between the Institut für Ionenphysik der Universität Innsbruck and the Heat and Mass Transfer Institute of the Byelorussian Academy of Sciences. V.A.Z. thanks the Bundesministerium für Wissenschaft und Forschung for financial support. It is a pleasure to thank Prof. G.D. Billing for useful discussions. The work was partly supported by the Fonds zur Förderung der wissenschaftlichen Forschung.

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8. V.A. Zenevich, W. Freysinger and W. Lindinger (submitted to publication)
Ionization and Recombination Processes in Dense Nonideal Plasmas

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In this paper, we consider the nonequilibrium properties of nonideal plasmas which are of interest from the theoretical point of view as well as from that of experimental investigations e.g. in shock waves or laser compression. Especially, we are interested in the influence of nonideality effects on the ionization kinetics and on diffusion. The behaviour and the properties of a nonideal plasma are determined by the Coulomb interaction and degeneracy. The Coulomb interaction is important in the "corner of correlation" which is enclosed in the density–temperature–plane by the parameter lines

\[ n_e l^3 = 1 \quad \tau_s = 1 \]

Here \( l = e^2/(k_B T) \) is the Landau length and \( \tau_s = d/a_B \) is the Brueckner parameter. In this area, many-particle effects are important, such as dynamical screening of the Coulomb interaction, dynamical self-energy, Pauli blocking, formation of bound states and pressure ionization (Mott-effect) [1].

Reactions–diffusion processes are described phenomenologically by coupled reaction–diffusion equations:

\[ \frac{\partial n_a}{\partial t} + \nabla j_a = W_a(n_1, \ldots, n_f), \quad a = 1, 2, \ldots, f \]  

with \( j_a = -D_a \nabla n_a \) being the diffusion current and \( W_a(n_1, \ldots, n_f) \) the source function describing the ionization kinetics. An essential problem is the foundation of such reaction–diffusion equations from the fundamental equations of non-equilibrium statistical mechanics.

In order to describe the nonequilibrium properties of nonideal plasmas we have to start from generalized kinetic equations. Then, reaction-diffusion equations can be derived with generalized expressions for the reaction rate and diffusion coefficients which become density-dependent in a complicated nonlinear way due to the nonideality. Therefore, nonlinear macroscopic phenomena like ionization fronts, phase separation etc. can be expected.

Kinetic equations for nonideal reactive systems in which many-body effects must be taken into account were derived in [7] using the method of nonequilibrium real-time Green’s functions with the following result for the Wigner distribution function \( f_a \) of the free quasiparticles of species \( a \)

\[ \left\{ \frac{\partial}{\partial t} + \frac{\partial E_a}{\partial p} \frac{\partial}{\partial r} - \frac{\partial E_a}{\partial r} \frac{\partial}{\partial p} \right\} f_a(p, r, t) = \sum_b I_{ab}(p, r, t) + \sum_{bc} I_{abc}(p, r, t) \]

The single-particle energies on the l.h.s. are given by the dispersion relation

\[ E_a(p, r, t) = \frac{p^2}{2m_a} + \text{Re} \Sigma^R_a(p, \omega t)|_{\omega = E_a(p, r, t)} \]

where \( \Sigma^R_a(p, \omega t) \) is the retarded self-energy function.

On the r.h.s. of the kinetic equation (2) the collisions between quasiparticles are taken into account. The integrals \( I_{ab} \) contain all the possible three-body scattering processes with free and bound particles. Especially, break-up and formation reactions are taken into account. The dynamics of the scattering processes enter the collision integrals via two- and three-particle interactions.
T-matrices. Additionally to the kinetic equation (2), a similar equation for the distribution functions $f_j$ of the bound particles can be derived [2].

In the following we consider a plasma consisting of electrons, one-fold charged ions and atoms in the state $|j>$, where $j$ denotes the set of internal quantum numbers. The number densities are $n_e$, $n_i$ and $n_j$, respectively.

There are different many-particle effects which have to be taken into account:

1. It is well-known that the properties of strongly coupled plasmas are essentially determined by the screening of the long-ranged Coulomb interaction, i.e. the Coulomb potential has to be replaced by a screened one.

2. The quasiparticle approximation leads to momentum dependent energy shifts to work with is rather difficult. Instead, we used the "rigid shift" approximation [3] in which the energy shift is replaced by a momentum-independent shift reproducing the correct normalization (the correct density $n_a$). It can be shown that the thermally averaged shift is related to the chemical potential by

$$\mu_a = \mu_a^{id} + \Delta_a$$  (4)

3. The two-particle bound states have to be determined from an effective wave equation which was determined on the basis of Green's function techniques [1].

$$\left( \varepsilon_\alpha(p_a) + \varepsilon_\beta(p_b) + \Delta_{ij}^{eff}(p_ap_b,z) - z \right) \psi(p_a p_b)$$

$$- \left( 1 - f_\alpha(p_a) - f_\beta(p_b) \right) \int v_{ij}^{eff}(p_a p_b q z) \psi(p_a + q, p_b - q) dq = 0$$  (5)

In comparison to the Schrödinger equation of an isolated pair of particles there are some differences arising from the self energy corrections $\Delta_{ij}^{eff}$, Pauli-blocking and the effective potential. Dynamical self energy and effective potential are not independent: thus there is a compensation between self energy and Pauli-blocking effects for bound states whereas the continuum edge is determined by the self energies only. This results in a lowering of the ionization energy with increasing plasma density. The cross over of the continuum edge and the ground state energy defines the so-called Mott density. Above this density, bound states do not exist.

Equations governing the evolution of the number densities in space and time can be obtained by integrating the kinetic equations with respect to the corresponding momentum. The result is an equation similar to (1). The explicit form of the source function $W_a$ follows from the r.h.s. of the kinetic equations, especially from the reaction terms in the three-particle collision integrals. If in the equation for the electron density the source function is written in the form

$$W_a = \sum_{c=1}^{\infty} \sum_{j} \left( \alpha_j^{ij} n_c n_j - \beta_j^{ij} n_c n_j n_i \right)$$  (6)

defining the coefficients of impact ionization and three body recombination of the atomic level $j$, these rate coefficients are given in terms of scattering quantities (T-matrices) and distribution functions [5]. The calculation of the rate coefficients is considerably simplified if we take into account that there are different time scales during the approach to thermodynamic equilibrium. In the following we will assume that with respect to the translational degrees of freedom local equilibrium has been reached already. Thus the distribution functions are Maxwellian.

In this approximation, the following simple relation between the rate coefficients holds

$$\beta_j = \alpha_j \Lambda_j^2 \exp(I_j^{eff}/k_BT), \quad I_j^{eff} = |E_j| - (\Delta_e + \Delta_i - \Delta_j)$$  (7)
The ionization coefficient for electron impact can be written in the form

$$\alpha_j = \frac{8\pi m_e}{(2\pi m_e k_B T)^{3/2}} \int_{E_j}^{\infty} dE \varepsilon \sigma_j^{\text{ion}}(\varepsilon) e^{-\beta E} \, ; \quad \varepsilon = \frac{p^2}{2m_e}$$

(8)

Here $\sigma_j^{\text{ion}}$ is the impact ionization cross section from the atomic state $|j\rangle$ which in statically screened first Born approximation reads

$$\sigma_j^{\text{ion}} = \frac{8\pi \hbar^2}{p^2 m_B} \left(\frac{\pi^2 - 2m_e r_{1f}}{r_{1f}}\right)^{1/2} \int_{0}^{p_{\text{max}}} dp_{\beta}^2 d\Omega_{\beta} \int_{q_{\text{min}}}^{q_{\text{max}}} dq d\Phi \left[ V_{\text{ee}}^{1f}(q) \right]^2 \left| \int d^3 r \Psi_j^{1f}(r) \Psi_{1f}^{1f}(r) e^{ik_{q}r} \right|^2$$

(9)

The two-particle wave functions $\Psi_j(r)$ for the atomic bound states and $\Psi_{1f}(r)$ for the scattering states were determined from the effective wave equation (5) in static approximation.

Results for the ionization coefficient from the ground state and the respective recombination coefficient are given in Figs. 1 and 2 [2]. The most important feature is the strong nonlinear density dependence of the ionization coefficient which is mainly caused by the lowering of the ionization energy. The results are compared with the simple formula $\alpha = \alpha_0 \exp((\Delta_1 - \Delta_e - \Delta_p)/k_B T)$ (dashed lines) given in [5].

Fig. 1: Ionization coefficient versus free electron density ($T = 10,000\, K$ (1), $15,000\, K$ (2), $20,000\, K$ (3) and $30,000\, K$ (4)).

Fig. 2: Recombination coefficient versus free electron density. Curves are shown for the cases where degeneracy was taken into account (solid lines) and where not (dashed).

An expression for the diffusion current density can be obtained from the equation of motion for the mean velocity [8, 6]. Results shall be given here for the case of the ambipolar diffusion stage with $n_e(r,t) = n_i(r,t)$ and $j_{\text{D}}^{e}(r,t) = j_{\text{D}}^{e}(r,t)$. Then, the system of reaction-diffusion equations reduces to only one equation. The electron diffusion current density can be written as

$$j_e^D = -D_{\text{AMB}} \nabla n_e = -D_{\text{AMB}}^{0} \frac{n_e}{k_B T} \left( \frac{\partial}{\partial n_e} + \frac{\partial}{\partial n_i} \right) (\mu_e + \mu_i) \nabla n_e$$

(10)
where $D_{AMB}^0$ is the usual ambipolar diffusion coefficient. Results for the nonideal ambipolar diffusion coefficient are given for hydrogen (Fig. 3). The classical result $D_{AMB} = 2D_{pH}$ occurs only for electron densities below $10^{15} \text{cm}^{-3}$. For higher densities the many-body effects cause a minimum and below $T=17000 \text{K}$ even negative values of $D_{AMB}$ being a direct consequence of the "Van der Waals-loop" in the plasma chemical potential (for which Padé-approximations were used including the Hartree-Fock and Montroll-Ward contributions [4]) and indicating mechanical instability and a phase transition. Obviously, the diffusion of particles in a region of high electron density will differ from that at low concentration.

Due to the nonideality, the source function in the reaction-diffusion equation depends on the density in a complicated nonlinear way. A numerical analysis of the r.d.e shows the possibility of ionization fronts, nonequilibrium phase separation and droplet growth in such a dense nonideal plasma.

References


Experimental and theoretical determination of electron impact ionization cross sections of atoms and molecules


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INTRODUCTION

The data of total and partial electron impact ionization cross sections of molecules are of great interest in different fields of applied physics, like plasma etching or mass spectrometry. But the experimental determination of these cross sections is rather difficult, especially for polyatomic molecules, as their fragment ions suffer discrimination during extraction from the ion source and flight through the mass spectrometer. This discrimination is due to the excessive kinetic energy of the ions, and therefore the discrimination effects are biggest for high-energetic fragment ions. In the following a method will be presented, which enables to correct the measured partial cross sections by taking into account these effects. Moreover, to be able to predict absolute cross sections for atoms and molecules theoretically, we have developed a semiclassical formula.

EXPERIMENTAL SETUP

A detailed description of the apparatus, the experimental procedure and the method of detecting ions with high translational kinetic energies has been given previously /1/. Briefly the experimental set-up used consists of a conventional three-electrode type electron impact ion source, and a high-
resolution double focussing (reversed geometry) sector field mass spectrometer. The properties of the ion source and the mass spectrometer have been studied in detail previously and essential improvements in the performance could be achieved /2/. With the help of these improved operating conditions, like penetrating field extraction or the ion-beam deflection method /1/, it was possible to measure with high accuracy absolute partial ionization cross sections for atomic or molecular parent ions. But it was found that smaller fragment ions of polyatomic molecules, which have larger excess kinetic energies, are not extracted from the ion source with the same efficiency as the larger fragment ions. In order to account for this discrimination an additional correction procedure has been developed in our laboratory.

CORRECTION PROCEDURE

This correction procedure is based on a study of the discrimination in our ion source and mass spectrometer by means of computer simulations of the electric field distribution and the corresponding ion trajectories. With the help of these simulations we were able to determine a discrimination factor D as a function of the kinetic energies of the ions. This relationship allows to correct fragment ion currents to the same extraction efficiency as the parent ion.

In order to apply this correction procedure the respective kinetic energies of the fragment ions under study must be known. A simple method to determine these properties using the same experimental set-up has been described previously /3/. According to this work and references therein the kinetic energies can be determined from the relation

$$E_{\text{kin}} = 10^{-3} \cdot U_z(HWFM)^2,$$

where $U_z(HWFM)$ is the half width at full maximum of the measured ion beam profile in $z$-direction (direction of the entrance slit of the mass spec-
The agreement of the kinetic energies calculated with previously reported values \( /4/ \) was satisfactory. We have applied and tested this correction method to several gases, like \( \text{CF}_4, \text{SF}_6, \text{C}_3\text{H}_8, \text{C}_2\text{H}_6, \text{H}_2, \text{N}_2 \) and \( \text{C}_2\text{H}_5\text{OH} \), and have found good agreement with cross section data of other groups. For example the absolute cross section for propane as shown in the following figure in comparison with the data of Kurepa et al. \( /5/ \) and those of Schram et al. \( /6/ \) has been obtained with the described correction method.

![Graph showing the total ionisation cross section for propane](image)

**FIG.1** Total ionisation cross section for propane: our results (solid line) in comparison with the absolute data of Kurepa et al. \( /5/ \) (○) and Schram et al. \( /6/ \) (○).

**THEORETICAL WORK**

For the prediction of absolute electron ionization cross section functions we have developed \( /1/ \) a semiclassical formula. This formula consists of an
energy dependent part (classical binary encounter approximation) and an additional factor equal to the weighted sum of the squared electron radii (Born-Bethe approximation).

\[ \sigma = \sum_{n,l} g_{nl} r_{nl}^2 \cdot \xi_{nl} \cdot u^{-1} \left[ \frac{u-1}{u+1} \right]^{1.5} \cdot \left[ 1 + \frac{2}{3} \left( 1 - \frac{1}{2 \cdot u} \right) \cdot \ln \left( 2.7 + (u-1)^{0.5} \right) \right] \]

with \( g_{nl} \) = weighting factors (see also Bethe /7/, who calculated these Ionisierungsfaktoren as a function of the quantum numbers \( n \) and \( l \) by using hydrogen wave functions). Here these factors were determined via a fitting procedure:

- \( r_{nl}^2 \) = mean square radius of the \( nl \) shell;
- \( \xi_{nl} \) = number of electrons in the \( nl \) subshell;
- \( E \) = energy of the incident electron;
- \( E_{in} \) = ionization energy in the \( n \)-th subshell;
- \( u = \frac{E}{E_{in}} \)

REFERENCES

MICROWAVE DETERMINATION OF THE COEFFICIENT OF DISSOCIATIVE RECOMBINATION OF MOLECULAR IONS OF ARGON

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1. Introduction

In several experiments of determining the coefficient of dissociative recombination $\alpha$, there have been wide-spreadting values ranging from $3 \times 10^{-7}$ to $1.1 \times 10^{-6}$ cm$^3$s$^{-1}$. These discrepancies may be due to the ignorance of microwave power dissipation throughout the glass discharge tube immersed in the microwave cavity. Therefore, this study is to take into account the power dissipation in addition to the other loss and the form factor and to calibrate the cavity.

Using a calibrated cavity taken into consideration electric field loss from the end holes and the presence of a glass discharge tube, we have determined the value of $\alpha$ of molecular ion of argon at 300 K by means of the computer fitting of the rate equation of electrons to the measured decay of electron density.

2. Calibration of the cavity technique

The electron density on the axis $n_+(0)$ is deduced from the equation given by reference/1/ as

$$n_+(0) = \frac{C}{K_4} \cdot \frac{\int_{\text{plasma}} E_2^2 dv}{\int_{\text{plasma}} g(r) E_2^2 dv} \cdot f \cdot \Delta f,$$

where $K_4 = \int_{\text{plasma}} E_2^2 dv / (2 \int_{\text{cavity}} E_2^2 dv)$; $C = (4 \pi^2 m \varepsilon_0) / e^2$; $f, f_0$ are the resonant frequencies of the cavity with and without the plasma; $\Delta f = f - f_0$; $e, m, n$ are electron charge, mass and density, respectively; $E$ is the electric field strength; $g(r)$ is radial distribution of electrons; $\varepsilon_0$ is the permittivity of free space. In order to determine absolute electron density, $K_4$, $E^2$ and $g(r)$ have to be measured.

A TM$\omega_{10}$ mode cavity was used. The resonant frequency of the cavity with the empty discharge tube is 2.7253 GHz. When the inside of the discharge tube is filled with a uniform dielectric rod, frequency shift is given by/2/

$$\Delta f_4 / f = -(\varepsilon - 1) K_4,$$
where $\varepsilon_r$ is the relative permittivity of the rod. We have measured the frequency shifts $\Delta f_4$ by immersing rods of different $\varepsilon_r$ ranging from 1.029 to 2.9. The value of $\Delta f_4/f'$ is plotted against $-(\varepsilon_r - 1)$ in Fig. 1. From the slope of the line $K_d=0.140$ is determined.

A small probing dielectric ball ($\varepsilon_r=4.5, 2.0$) with radius $a=1$ mm was used and moved in radial and axial direction inside the empty discharge tube. Since the frequency shift $\Delta f_4$ by the presence of the ball is proportional to $E^2/3$, we have determined the spatial distribution of $E^2$ at $f_0=2.7253$ GHz by measuring the change of $\Delta f_4$. The result is shown in Fig. 2.

The profile of $g(r)$ was measured with a movable Langmuir probe in dc argon discharges. Substituting values of $K_d$, $E^2$ and $g(r)$ into equation (1), we have calculated $n_a(0)$. Values of $n_a(0)$ obtained above were compared with data obtained by Langmuir probe technique.

![Fig. 1: $\Delta f_4/f$ against $-(\varepsilon_r - 1)$](image1)

![Fig. 2: Spatial distribution of $E^2$ inside the discharge tube](image2)

3. Determination of the coefficient of dissociative recombination of molecular ions of argon

Taking into consideration the ionization as the result of collisions between two metastable atoms, we have

$$\frac{\partial M}{\partial t} = D_m V^2 M - (\sigma <v_n^+ n_2^+ \gamma n_e^2) M - \beta M^2,$$

(3)

$$\frac{\partial n_1}{\partial t} = D_{*1} V^2 n_1 - k n_k^2 n_1 + \beta M^2,$$

(4)

$$\frac{\partial n_2}{\partial t} = D_{*2} V^2 n_2 + k n_e^2 n_2 - \alpha n_e n_2.$$

(5)

Here, no distinction is made between $6^3P_2$ and $6^3P_0$ metastable atoms; $M$, $n_1$, $n_2$, $n_k$, and $n_e$ denote the densities of metastable atoms, atomic ions, molecular ions, ground-state atoms of argon and electrons which are functions of space and time; $D_m$, $D_{*1}$ and $D_{*2}$ are the diffusion
coefficient of the metastable atoms and the ambipolar diffusion coefficients of atomic ions and molecular ions, respectively; $\sigma$ and $\gamma$ are the de-excitation cross section of two-body and the de-excitation coefficient of three-body of metastable atoms; $\langle v \rangle$ is the mean relative speed of the colliding atoms; $k$ is the rate coefficient of conversion of atomic ions into molecular ions; $\beta$ is the rate coefficient of ionizing collisions between two metastable atoms.

Assuming quasi-neutrality, we put electrons as $n_+=n_1+n_2$. Since the electron temperature dependence of $\alpha$ was introduced to be proportional to $T_e^{-0.5}/4$, we put $\alpha(T_e)=\alpha_0(T_e/300)^{-0.5}$, where $T_e$ and $\alpha_0$ denote electron temperature and the value of $\alpha$ at 300 K, respectively.

Experiments were carried out in a Pyrex glass discharge tube with 3.1 cm in inner diameter and 56 cm in electrode separation. The discharge current was conducted in dc pulse operation of time interval of 9 ms, duration of 1 ms and peak current of 0.35 A. The operating pressure was 5 Torr. The discharge tube was baked out at 200 °C for several days before the experiments were carried out. The ultimate vacuum attained was $2 \times 10^{-7}$ Torr. The purity of 99.999 % of argon gas was used. Electron temperature decay was measured with a time-resolving 2.8 GHz radiometer with the time resolution of 5 $\mu$s/5. Probing microwave power into the cavity was adjusted below several $\mu$W.

The frequency shifts at moments of the afterglow period were measured. The radial distributions of electrons were also measured with an asymmetric double probe. By substituting these data into equation (1) $n_e(0)$ was obtained. On the other hand, by solving equations (3), (4) and (5) we have calculated $n_+(0)$.

Coefficients and initial values used are listed in Table 1.

Table 1: Parameters and initial values.

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<th>Items</th>
<th>Parameters</th>
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<tr>
<td>Ar$^+$</td>
<td>$D_{e1} n_e = 2.34 \times 10^{18}$ cm$^{-1}$ s$^{-1}$</td>
<td>/5/ (a)</td>
</tr>
<tr>
<td></td>
<td>$k = 2.26 \times 10^{-31}$ cm$^2$ s$^{-1}$</td>
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<td></td>
<td>$\gamma = 1.14 \times 10^{-32}$ cm$^6$ s$^{-1}$</td>
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<td></td>
<td>$n_1(0,0) = 2.8 \times 10^{11}$ cm$^{-3}$</td>
<td>present (b)</td>
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<td></td>
<td>$n_2(0,0) = 6.2 \times 10^{10}$ cm$^{-3}$</td>
<td>present (b)</td>
</tr>
<tr>
<td></td>
<td>$M(0,0) = 1.56 \times 10^{12}$ cm$^{-3}$</td>
<td>present (c)</td>
</tr>
</tbody>
</table>

(a): room temperature, (b): measurement, (c): calculated followed the same manner as reference 10/
The values of $\alpha_0$ and $\beta$ were determined so that $n_e(0)_t$ fitted well $n_e(0)_E$. We have consequently determined $\alpha_0 = (8.2 \pm 0.5) \times 10^{-7} \text{ cm}^3\text{s}^{-1}$ and $\beta = (1.5 \pm 0.1) \times 10^{-9} \text{ cm}^3\text{s}^{-1}$. The calculated values of temporal behaviors of electrons, atomic and molecular ions and metastable atoms of argon are shown in figure 3. The present value of $\alpha_0$ is compared well with previous values of $(8.5 \pm 0.8) \times 10^{-7} \text{ cm}^3\text{s}^{-1}/6$ and $(9.1 \pm 0.9) \times 10^{-7} \text{ cm}^3\text{s}^{-1}/7$ at 300 K. No comparison of the value of $\beta$ is made because other data are not available as far as we know.

![Graph](image)

Fig.3: Temporal behaviors of electrons, atomic and molecular ions and metastable atoms of argon.

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6/6 F.J.Mehr and M.A.Biondi, Phys.Rev.176 322(1968)
On the Laser Cooling of Ions in Oscillating Traps

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Abstract:
The probability distribution of a two-level ion trapped in an oscillating potential well is calculated. The ion absorbs photons from an incident laser beam and relaxes back to the ground state by either induced or spontaneous emissions. The present calculations reproduce the quantum results for heavy particles in static traps, and the classical results for radio-frequency confining wells.

1. Introduction

The development of techniques to confine clusters of a small number of ions [1], and even individual ions [2], has largely improved the understanding of the interaction of atoms and light. Ion clusters may now be confined combining static and oscillating potential wells, or traps, and cooled to middegree Kelvin temperatures with a light resonance technique termed laser cooling [3][4].

The cooling is produced by a laser beam with a frequency slightly lower than the resonance frequency of an atomic transition of the ion. When the ion moves towards the laser beam, the frequency of the laser is Doppler shifted towards resonance with a resulting increase in the photon absorption rate. The opposite effect takes place when the ion moves away from the laser beam. When the ion absorbs a photon, it goes into an excited state with a change of momentum $\hbar q$. The transition back to the ground state can happen either through induced emission, in which case the ion looses the momentum gained during the photon absorption, or through spontaneous emission. During an spontaneous emission, the ion emits a photon of total momentum $\hbar q$ but in a randomly distributed direction, thus the combination of an absorption followed by an spontaneous emission results in a total momentum loss [4].

The ion confinement and cooling in oscillating traps is characterized by parametric instabilities resulting from the time dependence of the trapping potential: ions can only be confined if the voltage of the trap is restricted to stable regions [5].

2. Ions driven by laser beams

In this work we present a one dimensional model for the confinement of ions in a Paul trap. We describe the dynamics of the trapped ion in terms of a system of stochastic differential equations accounting for the main features of the laser cooling process. We analyze the laser cooling of ions in static potential traps and in radio-frequency traps.

The absorption and emission times of photons are assumed to be randomly distributed according to Poisson statistics. For simplicity here we assume that the absorption is immediately followed by a spontaneous emission, resulting in a maximum ion velocity change $k = 2q/m$, where $q$ is the wavenumber of the laser and $m$ the ion mass. The Doppler shift effect responsible for the cooling results in a velocity dependent rate for
the distribution of the absorption/emission events:

\[ \tilde{\gamma}(u) = \frac{\gamma r(u)}{2r(u) + \gamma}, \quad \text{where} \quad r(u) = \frac{\chi^2}{4 (\Delta - qu)^2 + \gamma^2/4}. \tag{1} \]

Here \( u = dx/dt \) is the ion velocity and \( \gamma \) is the spontaneous emission rate, which sets up the linewidth of the spectral line. Other parameters: \( \Delta \), the detuning between the laser frequency and the atomic transition frequency, and \( \chi \), the coupling constant between the light beam and the ion \[3\text{-}4\]. Note that in this model quantum effects are only taken into account in the discrete nature of the applied radiation field.

The confining potential is assumed harmonic: \( \Phi(x, t) = m(a - 2b \cos(2\omega t))\omega^2 x^2/2 \), where \( a \) and \( b \) are independent parameters describing the potential. The equation of motions result in

\[ \frac{d^2x}{dt^2} + \omega^2(u - 2b \cos(2\omega t))x = k \sum_{t_n} \delta(t - t_n) R_n, \tag{2} \]

where the noise is applied at times \( t_n \) distributed according to Poisson statistics with rate given by Eqn.\( (1) \). \( R_n \) is a random number uniformly distributed in the interval \([0, 1]\). This is the one dimensional projection of the resulting change in velocity of the fully three dimensional laser cooling process. Discussions on the limitations of this simplification as well as results for the static trap may found in \([4]\).

The interaction between the light beam and the ion is characterized by two very different time-scales: the lifetime of the excited state and the time scale of the cooling process. The lifetime of the excited state is determined by the spontaneous emission frequency \( \gamma \) while the cooling is given by the rate of change in the spontaneous emission: \( k \, d\gamma(u)/du \). The time scale of the ion cooling, induced by the laser beam, is very slow compared to \( \gamma^{-1} \). We restrict the present analysis to the limiting situation \( \omega a^{1/2}, \omega b^{1/2} \ll \gamma \), termed the heavy particle limit because the ion velocity changes slowly during the absorption-emission processes \([3]\text{-}[4]\).

An equation for the probability density \( \rho(x, u, t) \) may be found by studying the changes in velocity and position over a small time interval \( \Delta t \). If \( X \) and \( V \) are the position and velocity after the time interval \( \Delta t \); we have that

\[ \langle X - x \rangle = u \Delta t, \quad \langle U - u \rangle = \left( -\omega^2(u - 2b \cos(2\omega t))x + \frac{k \tilde{\gamma}(u)}{2} \right) \Delta t, \]

\[ \langle (X - x)^2 \rangle = O(\Delta t^2), \quad \langle (X - x)(U - u) \rangle = O(\Delta t^2), \]

\[ \langle (U - u)^2 \rangle = \frac{\tilde{\gamma}(u)k^n}{n + 1} \Delta t, \quad n \geq 2, \tag{3} \]

to the leading order in \( \Delta t \). Here \( < \rho > \) is the expected values of any of the considered quantities. Thus,

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} - \omega^2(u - 2b \cos(2\omega t))x \frac{\partial \rho}{\partial u} = \sum_{n=1}^{\infty} \frac{(-k)^n}{(n + 1)!} \frac{\partial}{\partial u} \left( \tilde{\gamma}(u)k^n \rho \right). \tag{4} \]

Expanding this equation to order \( O(k^3) \) yields the Fokker-Planck equation derived in \([4]\).

Since the change in momentum is small compared to a typical momentum of the ion, the probability distribution should be peaked around the deterministic limit, i.e. \( k = 0 \). Thus, we introduce the following singular expansion based on the smallness of the driving noise amplitude:

\[ P(x, u, t, r) \sim e^{y(x, u, t, r)/k} \left( z_0(x, u, t, r) + O(k) \right), \quad \tau = kt, \tag{5} \]

which combined with Eqn.\( (3) \) results in

\[ \frac{dx}{dt} = u, \quad \frac{du}{dt} = -\omega^2(u - 2b \cos(2\omega t))x, \]

\[ \frac{d\phi}{dt} = 0, \quad \frac{d\phi_0}{dt} = -\left( \gamma(u) e^{-\phi_0} + \phi_a - 1 + \frac{\partial \phi}{\partial t} \right) z_0. \tag{6} \]
Thus
\[ x = A \frac{dM_A}{dt}(t) + B \frac{dM_B}{dt}(t), \quad u = A \frac{dM_A}{dt}(t) + B \frac{dM_B}{dt}(t) \] (7)
where the functions \( M_A(t) \) and \( M_B(t) \) are two independent solutions of the Mathieu oscillator Eqn.(6) given by
\[ M_A = \frac{1}{2}(M^+ + M^-); \quad M_A(0) = 1, \quad \frac{dM_A}{dt}(0) = 0 \]
\[ M_B = \frac{1}{2}(M^+ - M^-); \quad M_B(0) = 0, \quad \frac{dM_B}{dt}(0) = \mu + 2\omega \sum_{k = -\infty}^{\infty} c_{2k} \] (8)
\[ M^+ = e^{i\mu t} \sum_{k = -\infty}^{\infty} c_{2k} e^{2i\omega kt}, \quad M^- = e^{-i\mu t} \sum_{k = -\infty}^{\infty} c_{2k} e^{-2i\omega kt}. \]
The coefficients \( c_{2k} \) satisfy a three point recursion formula and \( \mu \) is called the Floquet exponent [6]. The secular frequency of the ion in the trap corresponds to the Floquet exponent \( \mu \).
Solution to Eqn. (6) for \( z_0 \) must be bounded if the ion is assumed to be confined, thus
\[ \lim_{T \to -\infty} \frac{1}{T} \int_0^T \left\{ \tilde{\gamma}(u) \frac{e^{-\mu t} + \phi - 1}{\phi} \right\} dt = 0, \] (9a)
where
\[ \phi = \frac{1}{W} \left( M_A(t) \frac{d\phi}{dA} - M_B(t) \frac{d\phi}{dA} \right) \quad \text{and} \quad W = M_A(t) \frac{dM_B}{dt}(t) - M_A(t) \frac{dM_B}{dt}(t) = \frac{dM_B}{dt}(0). \] (9b)
After the dynamical initial stage, the ion velocity becomes smaller than the resonant value \( \Delta \sqrt{q} \) and photon absorption is largely reduced. At this stage we may assume the approximation
\[ \tilde{\gamma}(u) = \tilde{\gamma}_0(1 - \beta u + \ldots) \quad \text{where} \quad \tilde{\gamma}_0 = \frac{\lambda^2\gamma}{2\gamma^2 + 4\Delta^2 + \gamma^2} \quad \text{and} \quad \beta = \frac{8\Delta^2}{2\gamma^2 + 4\Delta^2 + \gamma^2}, \] (10)
which reduces Eqs. (9) to
\[ \frac{\partial \phi}{\partial r} = \frac{\tilde{\gamma}_0}{12} \left\{ 3\beta \left( A \frac{d\phi}{dA} + B \frac{d\phi}{dB} \right) + \frac{2\|M\|^2}{W^2} \left[ \left( \frac{d\phi}{dA} \right)^2 + \left( \frac{d\phi}{dB} \right)^2 \right] \right\}. \] (11)
Here \( \|M\| = M_A = M_B = \frac{\omega}{2\pi} \int_0^{T/2} |M|^2 dt, \quad M_A M_B = 0, \) and \( M = \lim_{T \to -\infty} \frac{1}{T} \int_0^T M(t) dt. \)
The shift of the ion in the trap, \( z_0 \), can be found from Eqs. (6) and (11)
\[ \log(z_0) = \frac{\tilde{\gamma}_0}{2W} \int_0^t \left( M_B(s) \frac{d\phi}{dA} - M_A(s) \frac{d\phi}{dB} \right) ds. \] (12)
Assuming an initial distribution \( \phi(A, B, \tau) = -(A^2 + B^2)/\psi \) and an initial uncertainty \( \psi_0 \), a particular solution to Eqn. (11) is
\[ \psi(\tau) = \left( \psi_0 - \frac{4\|M\|}{3\beta W^2} \right) e^{-\frac{\lambda^2}{\beta W^2} \tau^2} + \frac{4\|M\|}{3\beta W^2}, \] (13)
leading to a cooling time scale \( t_c = 2/\tilde{\gamma}_0 \beta \), and final variances of the probability distribution \( \langle (A - \langle A \rangle)^2 \rangle \)
\[ = \langle (B - \langle B \rangle)^2 \rangle = \frac{4\|M\|}{3\beta W^2}. \]
The variances of the probability distribution of the dynamical variables \( x \) and \( u \) can be obtained using the change of variables (7) and the result provided in Eqn. (13), which yields

\[
\langle (x - <x>)^2 \rangle = \frac{4k||M||^2}{3\beta W^2} \quad \text{and} \quad \langle (u - <u>)^2 \rangle = \frac{2k||M||}{3\beta W^2} \left\| \frac{dM}{dt} \right\|
\]

where

\[
\left\| \frac{dM}{dt} \right\|^2 = \left( \frac{dM_A}{dt} \right)^2 + \left( \frac{dM_B}{dt} \right)^2 = \frac{\omega}{2\pi} \int_0^\infty \left| \frac{dM}{dt} \right|^2 dt.
\]

4. Discussion

The above Eqn.(14), agrees with the quantum calculations derived by Javanainen and S"{a}nneck for heavy atoms confined in static harmonic traps \((a = 1, b = 0)\) [3][4]. The lowest energy that can be achieved using laser cooling technique is of order \( \hbar \gamma \), and is basically limited by the spontaneous emission line width, which is the energy uncertainty of the emitted radiation [3][4]. The final energy is independent of the photon momentum provided \( \hbar^2 q^2/m \ll \hbar \gamma \).

Our analysis indicates the existence of stable and unstable ion confinement resulting from the relation between the potential parameters \( a \) and \( b \) [7]. A general discussion about Mathieu oscillator instabilities can be found elsewhere [6][7]. The current results are in agreement with those previously derived by Blatt et al. in terms of continued fractions expansions if we assume the limit of negligible small damping in their results [5].

We stress that pseudopotential models [4], usually assumed to describe ion traps, fail to predict the correct laser cooling dynamics close to a parametric instability of the radio-frequency trap.

References

Topic 13:

Plasma Applications
ONE-DIMENSIONAL MODELING FOR MAGNETO-MICROWAVE PLASMA USING THE MONTE CARLO METHOD

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1. Introduction

Integrated circuit (IC) chips are advancing towards high integration and small features, and the most recent development is VLSI (very large scale integration). VLSI devices have at least $10^5$ components per chip and their minimum device dimensions are scaled down to submicron dimensions. Magneto-microwave plasma, or ECR (electron cyclotron resonance) plasma, is often used in etching processes for VLSI fabrication/1/2/. This is because this plasma reactor has several advantages compared with conventional RF plasma reactors, such as anisotropic etching (due to a low working gas pressure) and less contamination (due to the absence of electrodes). However, our understanding of the magneto-microwave plasma production mechanism used for the etching process is far from complete. A self-consistent plasma simulator has to be built to analyze the plasma distribution in the reactor. Fewer simulations have been reported for the magneto-microwave plasma process than for RF or DC plasma processes. Such simulators have recently been reported /3//4//5/. However, these do not seem to be sufficient for modeling the ECR phenomenon.

We have developed a one-dimensional plasma simulator which combines a particle plasma model and a microwave damping model based on a "cold plasma" approximation. This paper demonstrates this model and some approximate results obtained from it. In particular, the effects of gas pressure on the plasma distribution in the direction of microwave propagation are described.

2. Modeling

In this simulator, the microwave and the plasma are solved in different ways. The plasma is simulated using the Monte Carlo particle model, while the microwave is simulated using a dispersion equation based on a wave equation and a fluid (cold) plasma model. Figure 2.1 shows a schematic view of these two models combined. A discharge occurs in the space between two infinite parallel plates (solid walls). The $x$, $y$ axes are set on the left wall. The $z$ axis is set perpendicular to the wall. The magnetic field is formed in the $-z$ direction. The intensity of this field increases linearly in the $z$ direction and has a strength of 0.0875 T (875 Gauss), corresponding to the ECR condition, at the mid-point between the two walls. The microwave (transverse electromagnetic wave, 2.45 GHz) propagates in the same direction as the magnetic field (from the higher magnetic field). The discharge space is divided into many cells for the Monte Carlo particle plasma method.

2.1 Plasma analysis

Several particle plasma models have been reported to date /6//7//8//9/. Our particle plasma simulator was derived from a rarefied gas flow simulator that had been developed using the DSMC method /10/. The weakly ionized plasma used in the semiconductor manufacturing processes consists of an overwhelmingly large amount of neutral species and a very small amount (about 0.01%) of charged species. Consequently, in the simulation, only the motion of charged particles (ions and electrons) is calculated in the group of neutral molecules having a Maxwellian velocity distribution. The plasma is calculated in this particle simulation with Poisson’s equation:

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

(2-1)
where $E$ is the electric field intensity, $\rho$ is the charge density and $\varepsilon_0$ is the dielectric permittivity of free space.

Figure 2.2 shows the calculation sequence for our simulator in time step $\Delta t$. The flight paths and collisions of molecules, charge density $\rho$, electric field intensity of charged particles $E_x$, and the microwave electric field intensity $E_x, E_y$ are calculated in order during this timestep. The timestep is $1.02 \times 10^{-11}$ s in this paper. The equation of motion is integrated using the leapfrog scheme with the frequency correction factor $/9/$. The collisions are calculated using a simplified time-counter method that has been modified from Bird's method $/11/$ as a multi-test particle method. Elastic collisions and ionization collisions are calculated for electrons. Hence, one electron and one ion is produced if ionization occurs during the electron-neutral molecule collisions. Only elastic collisions are considered for ions. A model gas like argon is used in this simulation. The collision frequencies for electron (0.67 Pa) are given by $\nu_{en}=1.8 \times 10^8$ (s$^{-1}$) (elastic), $\nu_{i0}=\nu_{i0}\exp[-(E-E_i)/100]$ (s$^{-1}$) (ionization), $\nu_{i0}=1.1 \times 10^7$ (s$^{-1}$) ($\nu_{i}=0$ if $E < E_i$), where $E$ (eV) is the electron energy, $E_i(=15.8)$ eV is the ionization potential. The collision frequency for ion (0.67 Pa) is given by $\nu_{in}=1.6 \times 10^5$ (s$^{-1}$) (elastic).

Charged particles bombarding the solid walls are assumed to be absorbed by the walls. Secondary emissive electrons, with probability $\gamma_e=0.046$, are produced by the ion bombardment on the walls.

The Poisson's equation is integrated using the following equation $/8/$:

$$E_x = -\frac{1}{\varepsilon_0} \int \rho(z) dz + \frac{1}{\varepsilon_0} \int z \rho(z) dz$$

(2-2)

where $H$ is the distance between the two parallel walls.

2.2 Microwave analysis

The microwave propagating in the $z$ direction is assumed to be given by the following plane wave equation:

$$E = E_0 \exp[f(z) \omega t] = E_0 \exp[f(z) \alpha z] \exp[f(\alpha \omega t)] \gamma = \alpha + j \kappa$$

(2-3)

where $E$ is the electric field intensity, $\omega = 2\pi f$ is the angular frequency ($f=2.45$ GHz), $\gamma$ is the propagation constant, $\kappa$ is the attenuation constant, and $\kappa$ is the wave number.

The microwave dispersion equation with the "cold plasma" model, including electron and neutral molecule collisions, is given by the following equation $/12/$.

$$\frac{\sigma_0^2}{\omega \varepsilon_0} + \frac{j \sigma_2^*}{\sigma_1^*} = \gamma$$

(2-4)

where $\sigma_1^*, \sigma_2^*$, and $\sigma_3^*$ are components of the conductivity tensor $\sigma$, $\omega_c = \omega_c(\mu_0) = \mu_0/m_e$ is the electron cyclotron frequency, $n_e$ is the electron density, $e$ is the charge of electron, $m_e$ is the mass of electron and $\mu_0$ is the permeability of free space. The + sign represents a left-hand circularly polarized wave (L wave) and the - sign represents a right-hand circularly polarized wave (R wave). Thus, the propagation constant $\gamma$ in each cell is calculated using the electron density distribution $n_e$ obtained by plasma particle simulation. If wave reflection is disregarded,
the R wave is given by
\[ E_x = E_0 \exp(\alpha z) \cos(\kappa z + \omega t) \] (2-5)
\[ E_y = -E_0 \exp(\alpha z) \sin(\kappa z + \omega t) \] (2-6)

The whole curve in the discharge space can be obtained for the microwave electric field by connecting the amplitudes and the phases at the cell boundaries from the right wall. The wave numbers at arbitrary positions in the cell are obtained by a Cubic spline function. Numerical parameters for the following calculations are shown in Table 1.

3. Results and discussion
3.1 Characteristics of microwave damping with constant plasma density
First, the characteristics of microwave damping with constant plasma density has to be clarified. Figure 3.1 shows the numerical results for the electric field amplitude damping of an R wave in the z direction using the dispersion relation Eq. (2-4). The damping of the L wave at the left wall was very small (about 1% of the input electric field amplitude at the right wall). Therefore, the L wave is not considered in the following simulations because it seems that it is not important for plasma production. Figure 3.1a shows the R wave damping curves at different gas pressures in the plasma with a density of \( 1 \times 10^{17} \text{ m}^{-3} \). The R wave at the lower gas pressure is dampened near the ECR point due to resonance damping, while that at the higher gas pressure is dampened earlier due to the effect of collision damping. Figure 3.1b shows the R wave damping curves with different plasma densities at a gas pressure of 0.67 Pa. The R wave with the higher plasma density is dampened more sharply than that with the lower one.

3.2 Interaction of plasma and microwave
Figure 3.2 shows the simulation results when combining a particle model and a microwave damping model for the plasma density and the microwave damping distribution, where the gas pressure was changed to 0.13, 0.67, 9.3 Pa. As the plasma density is not constant, the microwave damping curve of the simulation differs from the results (Fig. 3.1b) obtained when density remained constant. The plasma density distribution shows very different curves for each pressure. The maximum moves towards the right side (upstream of the microwave) from the ECR point as the gas pressure increases. The plasma distribution for lower gas pressures becomes symmetrical about the center line. This is due to the higher diffusion coefficient and the sharper absorption of microwaves near the ECR point which is set as the center line.

References
Solid wall Solid wall

Resonance Damping Dispersion

Electric field $E_x$, $E_y$, $E_z$

Cell

Magnetic field

Magnetic induction

0.0875
0.0875

0.5H Z position H

Fig. 2.1 Schematic view of calculation model

Table 1 Numerical parameters

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<tr>
<td>Ion</td>
<td>400000</td>
</tr>
<tr>
<td>Timestep</td>
<td>$\Delta t = 1.02 \times 10^{-11}$ s</td>
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Fig. 3.1 Characteristics of microwave damping with the constant plasma density distribution

Fig. 3.2 The simulation results with the different gas pressures
Abstract.

We have used a ray tracing method in order to describe the propagation and absorption of the ordinary and the extraordinary modes, of an electromagnetic wave, from an emission point outside the plasma, till the electron cyclotron zone, for several angles of incidence. The plasma is confined in a flaring magnetic field. The electronic temperature is 1-5 eV and the density $10^{11}$ to $10^{12}$ cm$^{-3}$. Temperature effects and electron-neutral collisions have been taken into account in the dielectric tensor.

Introduction.

In recent years, plasma heating by Electron Cyclotron Resonance (ECRH) has found a wide field of applications. The plasma source of ERIC, a device for stable isotopes separation by ion cyclotron resonance [1], is also based on this principle. The ERIC apparatus consists of the microwave ECRH plasma source, the ion cyclotron resonance excitation zone and finally the isotope collector (Fig. 1).

The source region has a cylindrical shape of a 30 cm diameter and about 60 cm length, situated in the flaring magnetic field zone. Plasma ignition takes place by ECRH in Argon (or Krypton) atmosphere at $10^{-4}$ Torr using cw 1.5 kW klystrons at 10, 18 or 29 GHz. A horn type microwave antenna couples the electromagnetic wave to the plasma. The rare gas ions initialize sputtering on a metal plate biased to -2 kV and containing the isotopes to be separated. The plasma is thus composed of metal and gas atoms, electrons,
metal and gas ions. Using experimental values of the electronic density and temperature spatial distributions, we study here the absorption of different rays penetrating the plasma at various angles with respect to the magnetic field direction.

Ray tracing calculation.

The cold dispersion relation for the extraordinary and ordinary modes of an electromagnetic wave with frequency $\omega/2\pi$ and wave vector $k$ propagating in a plasma with electronic density $n_e$ and refractive index $N$, confined in a magnetic field $\mathbf{B}$, writes [2]:

$$D - N^2_1 + N^2_2 - 1 + X \pm \sqrt{\left(1 - N^2_2\right)^2 y^2 + 4 N^2_2 (1 - X) \pm Y (1 + N^2_2)} = 0$$

$$Y = \left(1 - N^2_2\right)^2 y^2 + 4 N^2_2 (1 - X) \pm Y (1 + N^2_2)$$

$$D = \frac{X Y}{2 (1 - X - Y^2)}$$

where: $X = \left(\frac{\omega_p}{\omega}\right)^2$ with $\omega_p = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$ the plasma frequency and $Y = \left(\frac{\omega_c}{\omega}\right)$ with $\omega_c = \frac{|\mathbf{e}|}{m_e}$ the electron cyclotron frequency. Also, $N^2 = N^2_1 + 2 N_2 \frac{|k|^2 c^2}{\omega^2}$ and $N_2 = \frac{\mathbf{B}}{1 B}$

Using (1), the ray trajectories are determined by [3,4]:

$$\frac{dr}{dt} - v_g = - \frac{\partial D}{\partial k}$$

$$\frac{dk}{dt} = - \frac{\partial D}{\partial r}$$

In our case, the components of the flaring magnetic field can be expressed analytically in cylindrical coordinates $(r, \theta, z)$ as follows:

$$B_r = \frac{2 \pi r^2}{Z (r_0 + z^2 / Z)} B_0, \quad B_\theta = 0, \quad B_z = \frac{r_0^2}{(r_0 + z^2 / Z)^2} B_0$$

$z$ being the axis of the magnetic coil and $B_0$ the homogeneous zone magnetic field whose maximum intensity is 3 Tesla. $Z = 340 \text{ cm}$, $r_0 = 15 \text{ cm}$, $r$ and $z$ in centimeters. The radial profile of the electronic density and temperature has been determined experimentally using Langmuir probes:

$$G(r) \simeq \left(1 - \left(\frac{r}{a}\right)^2\right)^{3/2} G(0)$$

where $G = T_e$ or $n_e$, $a = 7.8 \text{ cm}$ and $r$ in cm. A ray trajectories example at $10^\circ$, $20^\circ$ and $25^\circ$ degrees emission angles with respect to the magnetic field direction, for $\omega/2\pi = 29 \text{ GHz}$, $T_e(0) = 5 \text{ eV}$, $n_e(0) = 5 \cdot 10^{11} \text{ cm}^{-3}$ and $B_0 = 2 \text{ T}$. $\omega_e = 5 \cdot 10^{11} \text{ cm}^{-3}$ and $T_e(0) = 5 \text{ eV}$. Figure 2 shows the ray trajectories at $10^\circ$, $20^\circ$ and $25^\circ$ degrees emission angles.
The ordinary mode absorption is extremely low. The relationship between the wave polarization and the extraordinary mode has been discussed elsewhere [5]. At a given point $s$ of the trajectory, the power absorption is given by:

$$P(s) = P_0 \exp\left(-2 \int_0^s \text{Imag}(k_1) ds\right)$$

The quantity $\text{Imag}(k_1)$ is determined by resolving the worm dispersion relation expressed in terms of the dielectric tensor components $\varepsilon_{ij}$ assuming a maxwellian electron distribution function [2,6]:

$$N(\varepsilon_{ij}(1 - X_{33}) + X_{13}(2\varepsilon_{ii} + X_{13}) + N^2_{\omega}X_{33}) - N^2_{\omega} \times \left(\varepsilon_{12}^2 (1 - X_{33}) + (\varepsilon_{11} - N^2_{\omega}) \left((\varepsilon_{11} - N^2_{\omega})(1 - X_{33}) + X_{13} + \varepsilon_{33}^2 + (N_{\omega}X_{13})^2\right) - 2i\varepsilon_{12}X_{13}(N_{\omega}X_{13})\right)$$

$$+ \varepsilon_{33}^2 (\varepsilon_{11} - N^2_{\omega})^2 + \varepsilon_{12}^2 = 0$$

with $\varepsilon_{13} = N_{\omega}X_{13}$ and $\varepsilon_{33} = \varepsilon_{33}^0 + N^2_{\omega}X_{33}$, where [6]:

$$\varepsilon_{33}^0 = 1 - X_{13}, \quad \varepsilon_{11} = 1 - \frac{1}{2} X_{0}(Z(\xi_1) + Z(\xi_2)), \quad \varepsilon_{12} = - \frac{1}{2} X_{0} \left(\varepsilon_{11} \varepsilon_{33}^0 + X_{13}\right)$$

$$\varepsilon_{33} = \frac{X_{0}}{2Y} \left(\varepsilon_{11} \varepsilon_{33}^0 + X_{13}\right) Z(\xi_1) + Z(\xi_2)$$

$$\nu_e = \sqrt{\frac{2 T_e \nu_e}{m_e}}$$

being the electron mean velocity and $Z(\xi_p)$ the Fried and Conte function:

$$Z(\xi_p) = \exp(-\xi_p^2) \left(1 - \frac{\nu_e \omega}{\nu_e \omega}ight)$$

With $p = 0, \pm 1$

In fact, the electrons move within a neutral gas of density $n_n = 10^{12}$ to $10^{13}$ cm$^{-3}$. In order to study the influence of electron - neutral elastic collisions we have used in (8) $Z$ functions with imaginary arguments [7]:

$$\xi_p = \frac{c}{N_{\omega} \nu_e} \left(1 - p Y + \frac{i \nu_{e,n}}{\omega}ight)$$

Of course, $\nu_{e,n} = \sigma_{e,n} n_e v_e$ is the electron - neutral elastic collision frequency and $\sigma_{e,n} \approx 10^{-15}$ cm$^2$ [8] the corresponding cross section. In fact, in our experimental conditions $\nu_{e,n}$ is typically of the order $10^4$ s$^{-1}$.

The neutral gas influence upon the power absorption of the extraordinary mode along the ray trajectory abscissa until the electron cyclotron zone ($\omega_{ce}$) is presented in Fig. 3.

An increase of the thickness of the absorption region is observed. As a conclusion, we can calculate the microwave power absorption distribution near the electron cyclotron

![Fig. 3. Influence of the electron - neutral elastic collisions on the ray absorption.](image)
zone in a flaring magnetic field at low electronic temperatures. The
electron - neutral elastic collisions have no significant influence except a
slight increase of the thickness of the absorption zone.

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CURRENT AND GEOMETRY DEPENDENCE OF MASS
AND ENERGY FLUXES IN CAPILLARY, ABLATIVE CRITICAL FLOWS

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There is growing interest in high pressure discharges in ablative capillaries which are also essential components of Electrothermal Launchers.¹-⁵ In these devices, the out flow of the plasma through the hollow electrode is balanced by the ablation of the tube walls that are made of low atomic weight material, such as polyethylene. This ablation is caused by energy losses (especially radiative ones) from the plasma to the wall and can yield steady state operation conditions.

In this work we investigate theoretically the current and geometry dependence of the mass and energy fluxes in the capillary ablative discharges described above. For this, we solve numerically radius-averaged one-dimensional steady state continuity, momentum and energy equations for a cylindrical ablative capillary discharge plasma. The following main features of the work should be mentioned:

1. The spatial (z) dependence of the ionization degree of the plasma, Z, as well as of the correction factor to the Spitzer conductivity, β, are taken into account by using for Z(z) and β(z) best fits to numerical results obtained in a separate, parametric ionization study providing, Z(Î̇ρ, Î̇T) and β(Î̇ρ, Î̇T) (Î̇ρ and Î̇T are the normalized mass density and temperature, respectively).
2. A non-ideal equation of state is used to calculate the thermodynamical quantities entering the problem. Thus, from the SESAME-Los Alamos library⁶ one extracts discrete numerical values and subsequently, by best fit techniques one obtains continuous functions for the pressure and internal energy required by the integration method of the equations.
3. Self-consistent boundary conditions are obtained and used for the solution. This is of major importance since we are faced with a closed physical system whose parameters (current, radius, length) are kept fixed during the experiment; the axial profiles of the plasma characteristics (mass density, temperature, pressure and flow velocity), including their values at the ends of the capillary are determined in a self consistent manner by the physical processes involved.

BASIC EQUATIONS.

The radius-averaged steady state fluid equations for a viscous conductive and radiative one-dimensional flow in a cylindrical capillary can be written.¹,²
\[
\frac{d}{dz} \left( \rho \frac{dV_z}{dz} \right) + \frac{d}{dz} \left( V_x \frac{d\rho}{dz} \right) = \rho \dot{a} ,
\]
(1)

\[
\frac{d}{dz} \left( \rho V_x \frac{dV_z}{dz} \right) + \frac{d}{dz} \left( \frac{dP}{dz} \right) = \frac{4}{3} \frac{d}{dz} \left( \eta \frac{dV_z}{dz} \right) ,
\]
(2)

\[
\frac{d}{dz} \left( \frac{dV_x}{dz} \right) + \frac{d}{dz} \left( \rho V_x \frac{d\rho}{dz} \right) = \frac{4}{3} \eta \left( \frac{dV_z}{dz} \right) + \frac{d}{dz} \left[ \left( \frac{K_{\text{rad},e} + K_{\text{cond},e}}{dz} \right) + L_J \right] .
\]
(3)

In eqs. (1)-(3), \( \rho, V_x, T, P \) and \( \varepsilon \) are the mass density, axial velocity, temperature, pressure and the internal energy respectively; \( \eta, K_{\text{rad},e} \) and \( K_{\text{cond},e} \) are respectively, the viscosity, radiative heat conduction and thermal heat conduction coefficients; \( \dot{a} \) and \( L_J \) represent, respectively, the ablation and Joule heating terms.\(^1\)\(^3\)

For convenience, we here use the normalizations:

\[
y = z/\ell_0 , \quad \dot{\rho} = \rho/\rho_0 , \quad V = V_x/V_0 , \quad T = T/T_0 , \quad \tilde{P} = P/P_0 , \quad \varepsilon = \varepsilon/\varepsilon_0 .
\]
(4)

For length, mass density and temperature we chose the following reference values

\[
\ell_0 = 24\text{cm} , \quad \rho_0 = 1.36 \times 10^{-3}\text{g/cm}^3 , \quad T_0 = 4\text{eV} .
\]
(5)

For pressure and internal energy we use as reference the corresponding expressions holding for ideal gases, namely

\[
P_0 = k \left( 1 + \frac{Z}{2} \right) T_0 \rho_0 / \bar{n}, \quad \varepsilon_0 = k \left( 1 + \frac{Z}{2} \right) T_0 / \bar{n}(\gamma - 1) .
\]
(6)

Here, \( \gamma \) is the adiabatic coefficient, \( \bar{n} \) the average mass, \( k \) Boltzmann’s constant and \( \frac{Z}{2} \), the average ion charge in the uniform plasma case. For the polyethylene considered here, we take \( \bar{n} = 5m_p \) (\( m_p \) being the proton mass) and \( \frac{Z}{2} = 1 \).

For the flow velocity we take as reference value the sound speed, namely

\[
V_0 = \left[ \gamma k \left( 1 + \frac{Z}{2} \right) T_0 / \bar{n} \right]^{1/2} = 1.47 \times 10^6\text{cm/s} .
\]
(7)

To eqs. (1)-(7) we add the (normalized) expressions for the mass flux, \( F_m \), energy density, \( E \) and energy flux, \( F_E \), namely

\[
F_m = \tilde{\rho} V , \quad F_E = F_m E = \tilde{P} / \gamma \tilde{\rho} + \varepsilon / (\gamma - 1) + 0.5 V^2 .
\]
(8)

**SOLUTION METHOD AND RESULTS**

A. For a fixed set of physical parameters (namely current \( I \) and capillary radius, \( a \)) one solves eqs. (1)-(3) for different sets of arbitrarily chosen boundary conditions \( \dot{\rho}(y = 0) \) and \( \tilde{T}(y = 0) \); in
all cases, one uses the obvious condition \( V(y=0) \). For each set of b.c. one finds specific axial profiles \( \tilde{\rho}, \tilde{T}, \tilde{V} \) as well as a corresponding critical (optimal) axial distance, \( y_{\text{opt}} \), at which the flow velocity reaches the local sound velocity. The analysis of these results reveals the following important features (see Fig.1): 1. The critical flow characteristics \( \tilde{\rho}_{\text{min}}(y_{\text{opt}}), \tilde{T}_{\text{min}}(y_{\text{opt}}), \tilde{V}_{\text{max}}(y=0) \) and \( \tilde{P}_{\text{min}}(y_{\text{opt}}) \) as well as \( \tilde{P}_{\text{max},\text{y}=0}(y_{\text{opt}}) \) are almost linear functions of \( y_{\text{opt}} \); 2. To a particular \( y_{\text{opt}} \) -value always correspond the same values \( \tilde{\rho}(y_{\text{opt}}), \ldots, \tilde{P}_{\text{max},\text{y}=0}(y_{\text{opt}}) \) independent of the pair of boundary values \( \tilde{\rho}(y=0) \) and \( \tilde{T}(y=0) \).

B. From the results given in Fig.1, (which are based on arbitrary b.c. at \( y=0 \)) it is now possible to obtain consistent boundary conditions.

Thus, consider a capillary length \( l_i \) \((l_i/l_0=\gamma_i)\); from the results given in Fig.1, obtain the maximum pressure corresponding to \( y_{\text{opt}}=\gamma_i \), namely \( P_{\text{max},y=0}(y_{\text{opt}}=\gamma_i) \). Now, solution of eqs.(1)-(3) at \( y=0 \) together with the best fit values for \( \tilde{\omega}, \tilde{\beta}, \tilde{\rho} \) and \( \tilde{T} \) indicated above, as well as the \( \tilde{P}_{\text{max},\text{y}=0} \)-value indicated by Fig.1 determine the values of \( \tilde{\rho} \) and \( \tilde{T} \) at \( y=0 \); these, together with the natural condition \( V(y=0)=0 \) represent the consistent boundary conditions for the interpretation of the eqs.(1)-(3).

Using this method we carried out a systematic investigation of the effect which various parameters - such as, electrical current, \( I \), capillary radius, \( a \), and capillary length, \( l \), have on the plasma characteristics, namely energy flux, \( F_E \) and mass flux, \( F_m \). The results are presented in Fig. 2 and table 1.

Thus, Figs. 2a, 2b and 2c give, respectively, the axial profiles \( F_E(y) \) and \( F_m(y) \) for several values of \( I, a \) and \( l \) (as indicated on the curves); Figs. 2d, 2e and 2f give, respectively, the corresponding axial profiles of the Joule heating term, \( L_j \), the ionization degree, \( Z \), and
the ablation contribution, \( L_p \). Thus, within the parameter range considered here, the critical (exit mass and energy fluxes \( F_m \) and \( F_E \)) increase almost linearly with the increase of the current (\( I \)) and capillary length (\( l \)); they decrease with increasing capillary radius (\( a \)). This is so because increasing \( I \) means increasing the energy supply to the discharge, which affects \( F_m \) as well as the ablation rate; as a result, \( F_E \), is enhanced. Next, since the total ablated mass depends on the capillary length, also the \( l \)-dependence is simply understood. Finally, the decrease (for constant current) of \( F_m \) and \( F_E \) with increasing radius is a result of the decrease of the energy supplied to the unit volume of plasma and its consequences.

Table 1. Dependence of the exit values of \( F_m \), \( F_E \) and \( Z \) on \( I \).

<table>
<thead>
<tr>
<th>( I )</th>
<th>( F_m = F_m(I)/F_m(I_0) )</th>
<th>( F_E = F_E(I)/F_E(I_0) )</th>
<th>( Z = Z(I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>1</td>
<td>0.886</td>
</tr>
<tr>
<td>1.111</td>
<td>1.122</td>
<td>1.200</td>
<td>1.012×0.886</td>
</tr>
<tr>
<td>1.255</td>
<td>1.287</td>
<td>1.416</td>
<td>1.023×0.886</td>
</tr>
</tbody>
</table>

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Fig 1: Dependence of mass density velocity, temperature and pressure values at \( y=y_{opt} \) as well as of the pressure at \( y=0 \) on \( y_{opt} \) (where choking occurs), as obtained by solving eqs. (1)-(3) for a large number of pairs of \( \tilde{\rho} \) and \( \tilde{T} \)-values used as boundary conditions at \( y=0 \). In all cases, \( V(y=0)=0 \); \( \tilde{P}_1, \tilde{P}_2 \) and \( \tilde{P}_3 \) correspond to \( \tilde{T}=1/I_0=1.000, 1.111 \) and 1.255 , \( I_0=9000A, a=0.2cm \); \( \tilde{P}_a - a=0.18cm \).  

Fig 2: Results of integration of eqs. (1)-(3), upon consistent boundary conditions at \( y=0 \), the \( \tilde{\rho} \) and \( \tilde{T} \)-values obtained by the method described above.
New Method of Ion Extraction from Plasma by Using Radio-frequency Electric Field

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1. Introduction

Ion extraction from plasmas has become of considerable importance for such applications as plasma processing, an ion source or atomic vapor laser isotope separation (AVLIS). A conventional ion extraction is carried out with an electrostatic field. Ion flux, extracted by the electrostatic field, however, is limited to the space charge limited current at the ion sheath according to the Child-Langmuir law, because the electric field cannot penetrate the plasma due to its shielding effect. Although the ion flux increases with the applied voltage, the voltage is limited by an electric discharge or breakdown.

A new ion extraction method has been developed with the use of a radio-frequency (RF) electric field. In it, the frequencies of the RF field are chosen to excite eigen modes of the plasma in a low magnetic field. The RF field in the plasma makes ions drift to the electrodes. This paper presents the principle of the method and the results of its experimental proof.

2. Principle

Figure 1 shows a simple model for a plasma and RF electrodes system. A uniform slab plasma is sandwiched by a pair of parallel plate electrodes and the magnetic field $B$ is applied perpendicular to the electrodes. The impedance of the electrodes is written by

$$Z = i \left( \frac{1}{\omega C_v} + \frac{1}{\omega C_p} \right),$$

(1)

where,

$$C_v = \frac{\varepsilon_0 S}{d - p}, \quad C_p = \frac{\varepsilon_p p S}{p},$$

and $\varepsilon_0$, $\varepsilon_p$, $S$, $d$ and $p$ are the dielectric permittivity of vacuum, the plasma permittivity, the electrode area, the distance between the electrodes, and the plasma thickness, respectively. If the resistance of the plasma is negligible, a series resonance appears in the circuit where $Z = 0$. The RF field in the plasma is expected to become a maximum at the resonance. When the resistance is taken into account, the Q value of the resonance becomes low and the RF field in the plasma becomes small. The plasma permittivity which satisfies the resonance condition is obtained by Eq. 1 as
Here we analyze a low magnetic field case, where Larmor radii \( \rho_i, \rho_e \) and \( d \) satisfy \( \rho_i \gg d, \rho_e \ll d \). The frequency dependence of \( \varepsilon_p \) under these conditions was calculated by using a dielectric permittivity for electrostatic waves for an infinite uniform plasma \(^4, 5\)). An example of \( \varepsilon_p \) in the frequency range of \( \omega_{ci} \ll \omega \ll \omega_{ce} \) is shown in Fig. 2. Parameters used are listed in the figure caption. Resonances, where \( \text{Re}(\varepsilon_p) \) satisfies Eq. 2, exist at 2.6 MHz and 12.0 MHz. At these resonant frequencies, the large electric field is expected to be excited in the plasma. Then, ions are oscillated and they drift away to the electrodes with the drift velocity depending on the initial phase between ions and the RF field.

3. Experimental Setup
To prove the existence of the predicted resonances, a preliminary ion extraction experiment has been carried out by using Xe plasma in low B field \((5 \times 10^{-3} \text{T})\). An experimental apparatus is shown schematically in Fig. 3. The Xe plasma is generated by the RF electric discharge. The RF frequency and the power for the plasma generation are 13.56 MHz and about 700 W, respectively. The Xe gas pressure in the plasma generation region is \(10^{-2} \text{Torr} \). The plasma is transferred along the B field to the extraction test region. The electron density and electron temperature there are \(5 \times 10^{15} \text{m}^{-3} \) and 7.5 eV, respectively, as measured by a double probe method. The test region is constructed by the parallel plate electrode (length: 300 mm or 150 mm, gap...
distance: 42 mm) and the RF circuits. The RF field frequency is swept in the range from 1 MHz to 30 MHz. The RF voltage and the RF current at the electrode are measured by the voltage and current probe. The electric field strength perpendicular to the B field is obtained by measuring the difference of floating potential at two points in the plasma using a differential FET probe. The ion flux is measured by a Faraday cup with two grids, which is set behind the electrode.

4. Results and Discussion

The frequency dependencies of the electric field strength in the plasma and the ion flux are measured when the RF power is constant (about 1 W). The results (electrode length: 300 mm) are shown in Fig. 4. The electron density and temperature are almost constant in Fig. 4. The electric field strength in Fig. 4 is the ratio of the measured electric field strength to the RF voltage applied on the electrodes. It is shown that the electric field strength has a sharp peak at 4.2 MHz and a broad peak at 12 MHz, and the ion flux also has two peaks near these frequencies. The calculated resonant frequencies are 2.6 MHz and 12 MHz for this experimental condition and these are well agree with the experimental values.

When the electrode length is 150 mm (half of that in Fig. 4), the experimental resonant frequencies are found to be 5 MHz and 15 MHz. On the other hand, the calculated values are 5 MHz and 24 MHz. Although there is the quantitative discrepancy between the experiment and the calculation for the higher resonant frequencies, the tendency for the resonant frequency to increase with decreasing electrode length is the same for both.

The RF voltage dependencies of the electric field strength and the ion flux at the resonant frequency are studied (Fig. 5). The electric field strength is observed to be proportional to the RF voltage. The ion flux is also proportional to the RF voltage beyond about 8 V. The results indicate that the ion extraction efficiency increases with the RF voltage.

4. Conclusion

A new method to extract ions from the plasma, by using the RF field, has been developed and tested experimentally. It is shown in the experiments that the electric field in the plasma becomes resonantly large at the frequencies, which are predicted by the calculation, and the resonant ion flux to the electrodes is observed around the resonant frequencies.

Reference
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Fig. 3 Schematic diagram of experimental apparatus.

Fig. 4 The frequency dependences of the electric field strength and the ion flux. The electrode length is 300 mm.

Fig. 5 The RF voltage dependences of the electric field strength and the ion flux at the resonant frequency. The frequency is 4.2 MHz and the electrode length is 300 mm.
The ion cyclotron resonance process (ICR) designed to separate isotopes has been studied at Saclay on the ERIC device which has been described elsewhere /1-3/. In order to achieve the separation in a cylindrical plasma column confined by an homogeneous magnetic field (B) of about 1 Tesla, an external helicoidal antenna generates an electromagnetic field with a frequency matched to the ion cyclotron frequency of a selected isotope which is progressively heated (up to a few hundred eV). On the contrary the other non-resonant isotopes are only slightly heated (up to a few eV, i.e. a few times their initial temperature). The separation itself is done geometrically at the end of the plasma column: the resonant species with a greater Larmor radius are collected on parallel blades (with respect to B), while the other non-resonant species are collected on an end plate.

The separation factor $\alpha$ is very sensitive to the plasma source parameters as the transverse ionic temperature that must be minimized /1/. So, in order to optimise $\alpha$, a laser absorption diagnostic has been developed and tested on the ERIC device with a barium plasma. The barium atoms produced by a Joule oven are ionized by the electrons heated by microwaves at their cyclotron frequency, namely 10 GHz.

**Theory:**

The intensity of a parallel beam transmitted at the frequency $\nu$ through an homogeneous absorption cell of length $l$ can be written according the well known relation /4/:

$$I_\nu = I_0 \exp(-k_\nu l)$$

(1)

where $k_\nu$ is the absorption coefficient and $I_0$ the incident intensity.

Following the Einstein theory of radiation, one gets:

$$k_\nu = \frac{2\pi\Delta\nu}{m_e c^2} \frac{e^2}{N_1} f_{12} \exp\left(\frac{-\nu - \nu_0}{\Delta\nu_{0}}\right)^2$$

(2)

where $\Delta\nu_{0} = 2\nu_0 (2\ln 2 T_i/m_i c^2)^{1/2}$ is the Doppler width, $T_i$ being the temperature of the ions with mass $m_i$, $\nu_0$ the frequency at the center of the line, $f_{12}$ the absorption oscillator strength of the transition $2 \rightarrow 1$ from the level with density $N_2$ to the level with density $N_1$, providing that: $N_2 \ll N_1$.

When a magnetic field is present, the degeneracy of the levels is revealed by a Zeeman structure on the absorption profile (fig.1). The density of the lower level can be written as follows:
where $\gamma_i$ is the ratio of the intensity of a component $i$ of the multiplet relative to its total intensity $/5/$, $I_{\nu_i}$ is the intensity at the center of the component $i$, $g_1$ and $g_2$ are the degeneracy of the lower and upper levels respectively and $A_{21}$ the transition probability between these levels.

Experimental set-up

The different parts of the laser absorption diagnostic are represented on the fig.2. We use a 3800 SPECTRA-PHYSICS dye laser (Rh 660) pumped by a 164 SPECTRA-PHYSICS $\lambda^2$ laser (4 Watts). The laser is monomode and frequency stabilized, its line-width is 1-2 MHz, the band-width is about 54 nm, the frequency sweeping for each record is about 30 GHz and the output power is laying between 30 and 200 mW. The control in wavelength is made by a 0.6 m JOBIN-YVON spectrometer, within 90 GHz with an iodine lamp. The detection is done with photodiodes. The parasite emission due to the laser and to the plasma is suppressed by using a synchronous detector analyser ATNE ADS2.

Note that this diagnostic currently used in non-magnetized plasmas /6/ is utilized here for the first time to our knowledge in a plasma confined by a strong magnetic field.

Experimental results

The laser beam is directed transversally to the magnetic field so that both the $\Pi$ and $\sigma$ components of the Zeeman structure appear. The selected line is Ba II 6141.72 $\AA(2d^2D_{5/2} \rightarrow 6p^2P_{3/2})$ and its lower metastable level has an energy equal to 5675 cm$^{-1}$. Its experimental record of the $\Pi$ components of the Zeeman structure is represented on the fig.3, while the theoretical structure is shown on the fig.4.

For this case, the density of the metastable level at 5675 cm$^{-1}$ is found to be equal to $1.2 \pm 0.6 \times 10^{11}$ cm$^{-3}$, while the transverse ionic temperature is $T_i = 0.38 \pm 0.01$ eV.

As it is found that the optimum operating condition for the isotopic separation corresponds to the higher temperature of the Joule oven, 750°C ($T_i$ minimal and $N_i$ maximal), a separation experiment of $^{132}$Ba has been performed with these conditions /3/. The effect of the radio-frequency (RF) heating (at the ion cyclotron frequency) on the ionic temperatures at the collector side is represented on the fig.5.

The rather high separation factor $\alpha = 8.5$ for $^{132}$Ba, justifies the efficiency of the absorption diagnostic used here to optimize the ICR process.
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Fig.1: Absorption profile of a Zeeman triplet

Fig.2: Experimental set up for laser absorption diagnostic
Fig. 3: Π components of a Zeeman pattern of Ba II 6141 Å on ERIC

Fig. 4: Theoretical Zeeman pattern of Ba II 6141 Å

Fig. 5: Laser absorption measurements of the line Ba II 6141.72 Å
THE PLASMA CHEMICAL REACTIONS IN NEGATIVE CORONA DISCHARGES

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1. INTRODUCTION

The sulfur hexafluoride SF₆, either pure or mixed with other gases, is widely used as an insulator gas in high voltage equipment or as a reactive gas for dry etching of materials used for VLSI integrated technologies. The SF₆ gas is known to be chemically inert gas under the normal conditions. However, in gas discharges the decomposition of SF₆ was observed /1/. If pure oxygen or oxygen containing compounds are added into the SF₆, the decomposition of gaseous SF₆ by negative corona discharge leads to formation of oxyfluorides, predominantly to thionyl tetrafluoride SOF₄ /1/. The concentration of neutral by-products formed by corona discharge has a back effect on the properties of discharge /2/. The oxyfluorides have a similar influence like ozone produced by negative corona in oxygen containing gas mixtures /3/. All these species cause the formation of new ions different from the primary ions. Because the processes of O₃ and SOF₄ generation are initiated by atomic oxygen, the activities of mentioned species will be dependent on the amount of oxygen in the mixture. The purpose of this work is to study the generation of ozone by negative corona discharge in mixtures of oxygen with sulfur hexafluoride.

2. EXPERIMENTAL

The mixture of oxygen with SF₆ flew down the cylindrical discharge tube at constant flow rate 1 cm.s⁻¹ what corresponds to the retention time 100 sec. The tube contains a central wire electrode and is divided to five insulated parts allowing the parallel measurements of currents in each individual part of discharge tube. The mixture of gases leaving the discharge tube passed through the cell of UV spectrometer. From measured transparency of gas mixture the concentration of ozone in the mixture was estimated. The temperature and the pressure of gas mixture were equal to ambient values.
3. EXPERIMENTAL RESULTS AND DISCUSSION

The dependence of ozone concentration in the mixture of gases leaving the discharge tube on the mean energy \( \eta \) consumed by discharge is shown in Fig. 1. The mean energy \( \eta \) is defined like a ratio of discharge power \( P = U.I \) [W] and the flow rate \( Q \) [cm\(^3\).s\(^{-1}\)]. The decrease of ozone concentration at constant value of \( \eta \) which is evidently caused by the increase of SF\(_6\) concentration can be explained in the following way.

In the pure oxygen an electron-impact dissociation
\[
e + O_2 \rightarrow O + O + e \quad (1)
\]
is a very effective source of oxygen atoms needed for generation of ozone. If molecules of sulfur hexafluoride are added into the oxygen, they can be also dissociated very effectively by electron impact
\[
e + SF_6 \rightarrow F + SF_5 + e \quad (2)
\]
The intermediate by-products O and SF\(_5\) can react between themselves or with the other components of mixture
\[
O + O_2 + M \rightarrow O_3 + M \quad (3)
\]
\[
O + SF_5 \rightarrow F + SOF_4 \quad (4)
\]
The electrons which are generated in the ionization region in the vicinity of thin wire electrode undergo also the collisions in which they are attached both with oxygen and sulfur hexafluoride molecules. Because the detachment of electrons from negative ions is a very slow process in investigated gas mixtures, thus this can be neglected. The removal of free electrons from a gas under electrical stress by formation of negative ions can be formally expressed by equation
\[
e + X \rightarrow X^- + \text{energy} \quad (5)
\]
The equation includes all attachment processes of electrons i.e. dissociative and three-body attachment. Because the electron-impact dissociation processes (1) and (2) belong to the reactions having rather high energy thresholds, the main part of collisions resulting to dissociation of O\(_2\) and SF\(_6\) molecules is realized in the region of high electric field near the wire electrode. Thus in the equation (5) predominantly the dissociative attachment of electrons is active.

The system of equations characterizing the main processes in
the ionizing region of negative corona discharge in mixture of oxygen with sulfur hexafluoride, was used for estimation of the relationship between the production of ozone and production of oxyfluoride SOF$_4$. The gas mixture O$_2$ + SF$_6$ was considered at normal pressure 101.3 kPa and normal temperature 273.15 K. The reaction of electrons with oxygen and sulfur hexafluoride and consequent reactions of by-products were simulated by solving of system of differential equations

\[
\begin{align*}
\frac{d[O]}{dt} &= 2k_1[e][O_2] - k_3[O][O_2] - k_4[O][SF_5] \quad (I) \\
\frac{d[O_2]}{dt} &= -k_1[e][O_2] - k_3[O][O_2] \quad (II) \\
\frac{d[O_3]}{dt} &= k_3[O][O_2] \quad (III) \\
\frac{d[SOF_4]}{dt} &= k_4[O][SF_5] \quad (IV) \\
\frac{d[SF_5]}{dt} &= k_2[e][SF_6] - k_4[O][SF_5] \quad (V) \\
\frac{d[SF_6]}{dt} &= -(k_2 + k_5)[e][SF_6] \quad (VI) \\
\frac{d[SF_5^-]}{dt} &= k_5[e][SF_6] \quad (VII) \\
\frac{d[e]}{dt} &= -k_5[e][SF_6^-] \quad (VIII)
\end{align*}
\]

The system was solved for following parameters:

\[k_1 = 6.2E(-9), k_2 = 3.1E(-8), k_3 = 2.8E(-14), k_4 = 2.0E(-12), k_5 = 3.1E(-10)\]

The unit for all rate constants is cm$^3$·s$^{-1}$. The initial value of electron density $[e] = 1.0$ E(14) cm$^{-3}$ is convenient value in ionization region of negative corona discharge. The dependence of calculated equilibrium values [SOF$_4$]/[O$_3$] on the initial concentration of SF$_6$ in the mixture is shown in fig. 2. From this the negligible effect of reaction (4) on the ozone formation is evidently clear if the concentration of SF$_6$ is below 1%.

4. REFERENCES

Concentration of SF$_6$ [%]

- - - 0  - 5  - 10  - 25  - 50

Fig. 1

Fig. 2

$- - - k = 3.1 \times 10^{-10}$
Plasma parameter profiles and fluctuation properties of an ECR process plasma source for Si etching at LN2 temperatures.

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Recently ECR plasma is being utilized for various types of process devices in the ULSI semiconductor development and production lines such as 16 and 64 mega D-RAM’s due to its high ionization efficiency and resultant low pressure operation capability with higher plasma density and lower space potential compared to RF discharges. However, problems like unstable microwave coupling to the plasma, inhomogeneity and fluctuations in the plasma properties at the position of the sample remain as obstacles to overcome for semiconductor fabrication and other applications.

We designed and constructed an ECR plasma source operating at 2.45 GHz with emphasis on plasma diagnostics and parameter control to investigate these problems. Initially operational diagnostics include Langmuir probes and gridded ion energy analyzers, and measurements of basic plasma parameters such as plasma density and plasma potential in argon plasma have been performed. Additional diagnostics being prepared are: 70GHz microwave interferometer/scattering system for plasma density and fluctuation measurement and a laser induced fluorescence (LIF) measurement system. The LIF measurement is to monitor behaviours of radicals and products such as SiF2 from SF6/Ar etching of polysilicon, and to investigate possibilities of end-point detection from the measurement results. There are reports of observations [1] of SiF2 and similar radicals in CF4/O2 plasma, but no reports from SF6 or SF6/Ar plasma as far as we know. Probe beam utilizes 220 nm UV radiation from BBO second harmonic generator which is pumped by 440 nm radiation from Coumarine 450 pulsed dye laser (50 nsec, 100mJ, 1A linewidth tuned with a birefringent tuner). Fluorescence emission is observed with 1m monochromator and detected by a PM tube and a 1024 element gated photodiode array connected to an OMA. In addition a CARS spectrometer will join the measurement from flame/combustion research group at KRISS.

The machine has three coils for the magnetic field, two of which produce ECR field of 875 gauss. Current in each coil can be adjusted independently to change the resonance layer. The third coil is located in the downstream area at the position of the substrate holder, to change the magnetic field line shape and to investigate its effect on plasma and the etching process (Figure 1). Substrate mount for maximum 8” diameter wafer has been designed, constructed and tested for vacuum and cooling performances. With all the diagnostics and the holder installed, the base pressure is 4 x 10^{-8} mbarr (with LN2 circulating). Mounting surface was measured with Pt resistance thermometer to be cooled down to -100°C in 30 minutes, and to -180°C in 90 minutes. Space potential in the vicinity of the wafer can be adjusted by producing an RF plasma layer from 13.56
MHz, 1 kW RF generator connected to the mount. Test etching of "line-and-space" patterned polysilicon wafer at liquid nitrogen temperature (-100-140 °C) in the SF6 plasma is planned during May-June, and the initial result will be presented during the conference. In addition, as a supporting experiment, a double plasma[2] source is being equipped with a magnetron to investigate ECR plasma production scheme with surface field[3]. This DP machine will soon be expanded to a magnetized triple plasma source where microwave-plasma interaction and other wave-plasma interactions will be investigated.

Some of the measured profiles of the ECR plasma and their dependence on external parameters such as pressure and magnetic field shape at the substrate holder position are shown in figure 2. Te(r) profiles is very irregular, due to noisy probe signal. However, at all positions and field shapes, Te decreases with increasing pressure. Space potential data from ion analyzer is cleaner, and the profile becomes flatter when the 3rd magnet is turned on to make the field line parallel at the substrate holder position. Like Te, V decreases with increasing gas pressure, but when parallel field line is produced by the 3rd magnet, the decrease is much weaker. Ion temperature show much less dependence on pressure, with sudden increase around pressure value of ~ 4x10⁻³ mbarr, the reason of which is not understood yet but being investigated in relation to density fluctuation properties described in the next paragraph.

Fluctuations in plasma density was measured from electron saturation current of Langmuir probe and ion analyzer collector current. Fluctuation signals were compared with microwave reflected power level behaviour from crystal detector which consistently showed coherent fluctuation of 2 - 10 kHz when the magnetic field by the 3rd coil is weak and the field line is somewhat diverging. During normal operation conditions (Pargon = 4 x 10⁻⁴ mbarr, 200W microwave in, I= 50 Amperes to make B parallel and normal to the substrate holder), ion analyzer located on axis (r = 0 cm) and facing up along the field line showed periodic sawtooth-like modulation in the fluctuation (figure 3). This modulation becomes irregular in period and weaker in amplitude as the analyzer is pulled out. At the plasma boundary (r ~ 14 cm), modulation disappears and looks similar to random fluctuation of crystal signal. When facing down along the field line ion current shows no modulation at all radial position. However, the analyzer I-V characteristics (i.e. ion distribution) does not reveal any gross difference between the two directions. Furthermore, the fluctuation level increases suddenly as the gas pressure crosses 3 - 4 x 10⁻³ mbarr from below, and dependence on analyzer direction and location is more complicated than lower gas pressure case. To further investigate properties of this fluctuation such as relation to possible ion drift and to microwave coupling to the plasma, a correlation measurement with two ion analyzer is being performed, and a 70GHz heterodyne collective scattering system is being assembled and tested on bench.

REFERENCES
Figure 1. (a) Schematic diagram of the ECR plasma source. (b) Cooling and reheating of the holder surface.

Figure 2. (a) Radial profile and (b) pressure dependence of the plasma potential, (c) pressure dependence of central density.
Figure 2 (cont). (d) radial profile and (e) pressure dependence of electron temperature; (f) pressure dependence of ion temperature.

(a) Figure 3. (a,b) Ion analyzer collector current fluctuation signal compared to reflected microwave power fluctuation. (a) analyzer at \( r = 0 \) cm facing toward and away from the field line, (b) at \( r = 12 \) cm. (c) I-V curves at \( r = 6 \) cm.
MAGNET PLASMA DYNAMIC ACCELERATOR WITH ULTRA HIGH FREQUENCY RESONATOR

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With the purpose of service time increasing and plasma flow admixture decreasing, an experimental model accelerator with ultra high frequency (UHF) discharge is created. In given installation (Fig. 1) in distinction from Magnet Plasma Dynamic (MPD) accelerator [1,2], thermionic emitter cathode is absent, the discharge appearing at UHF power feeding into the cavity resonator from the magnetron, running at quasi pulse mode. The resonator is placed in axisymmetric magnetic field and has coaxial anode in the form of convergent-divergent nozzle for plasma streaming into outer space.

Anode hole diameter is chosen in the absence output condition of UHF radiation into outer space, that is, the anode is a piece of an unlimited wave-guide [3].

To define the cutoff root mean square (RMS) of high frequency electric force (HFEEF) and magnetron power of UHF ge-
erator necessary for plasma creation, the electron movement, in crossed E and B fields at its collisions with atoms and molecules, is examined. In the region of "throat" of the magnetic nozzle, where a discharge arises, a magnetic field created by solenoid is homogeneous and is directed along the symmetry axis of the accelerator, and HPEF also being homogeneous, is directed on the radius. The electron movement in the plane, perpendicular to the magnetic field, may be divided into cyclotron rotation and drift movement representing rotation on elliptical orbit with the frequency of high frequency field.

While evaluating the magnetic movement due to electron rotation on elliptical orbit, its initial movement conditions were chosen in such a way to that cyclotron rotation be absent. The electron movement equation solving, in the supposition of electric field harmonic dependence upon time $\vec{E}=\vec{E}\cos(wt+\gamma)$ enabled to find electron coordinates as time functions

$$X = \frac{qE}{m} (I_+ \frac{v_m^2}{w^2})^{1/2} \frac{1}{\Omega^2} \cos(wt+\gamma - \gamma_1 - \gamma_2)$$

$$Y = \frac{qE}{m} \frac{w_k}{w} \frac{1}{\Omega^2} \cos(wt+\gamma - \gamma_1 + \frac{\nu_1}{2})$$

where $\vec{E}$ is directed along X axis, $\vec{B}$ along Z-axis, $q, m$ - charge and mass of the electron, $w_k$ - electron cyclotron frequency, $\nu_1, \nu_2$ - electron elasticity collision frequency, $w$ - frequency high frequency field,

$$\gamma_1 = \arctg \left[ \frac{2w \cdot v_m}{(w_k^2 + v_m^2 - w^2)} \right]$$

$$\gamma_2 = \arctg \left( \frac{v_m}{w} \right), \Omega = \left[ (w_k^2 + v_m^2 - w^2)^2 + 4w^2 \cdot v_m^2 \right]^{1/4}$$

Expressions (1) present a written parametric equation of ellipse, the axes of which are turned relative to coordinate axes to the angle

$$\alpha = 0.5 \arctg \left[ 2 \cdot \frac{v_m \cdot w_k}{(w_k^2 - w^2 - v_m^2)} \right]$$

Magnetic current moment, carried by revolving electron on elliptical orbit, is directed against solenoid magnetic
field and equals
\[ \vec{J} = \frac{-q_E^2}{2m_3 \gamma} \vec{B} \]

Similar consideration of ion movement along with given data, results in the conclusion that in described system, high frequency electric field, perpendicular to the magnetic one, increases plasma diamagnetism, which in high frequency field absence is due to just a cyclotron revolution of charged particles.

Average high frequency field work in a period, done in time unit above the electron against friction force, defines average speed of increasing heat electron energy.

\[ \frac{dE}{dt} = \frac{q^2 E^2 \text{aver.}}{m} \left( \frac{\omega E}{m} \right)^2 + \frac{1}{2} \frac{\omega^2}{m} \]

where $E_{\text{aver.}} = E/\sqrt{2}$

RMS of HFEF.

The electron will take the energy needed for exciting working medium atom $I^+$, during $\gamma = I^+/dE$ if, certainly, it does not leave discharge region during this time due to diffusion. Break down possibility is defined by $\gamma = R^2/D$ condition, from which threshold RMS of HFEF is found

\[ E_{\text{aver.}} = \left[ I^+ \cdot \gamma^4 \cdot D \cdot \left( q^2 R^2 \cdot \frac{1}{m} \cdot \left( \frac{\omega^2}{m} + \frac{\omega^2}{m} \right)^{-1} \right) \right]^{1/2} \]

Here $D$ - electron diffusion coefficient across the magnetic field, in the investigated installation $W_n = 2 \times 10^{-10} \text{ c}^{-1}$,

$\nu = 10^8 \text{ c}^{-1}$, $\omega = 2 \times 10^{-10} \text{ c}^{-1}$, $R = 3 \text{ cm}$ - the radius of critical section of discharged chamber and the working medium - argon.

The power of the energy necessary for discharge combustion is defined by the extracting speed of Joule heat

\[ P = \frac{dE}{dt} = n_e V_k \sim 50 \text{ Vt}, \]

where $V_k$ - discharge chamber volume, $n_e$ - electron concentration, the peak value of which $(n_e^{\text{peak}} \sim 10^{17} \text{ m}^{-3})$ is calculated using electromagnet wave reflex condition from ionized gas region, where $n_e \gg \xi_0 \cdot \frac{m^2 \omega^2}{q^2}$.

In experiments use was made of 500 VT magnetron, opera-
ting in quasi momentum regime, which created electrical field in the resonator of $3 \cdot 10^4 \text{ V/m}$ intensity.

Local parameters of plasma flow $n_e$, $T_e$, directed ion speed $V_i$ and plasma potential $\psi$ were defined by directed probes [2], which were placed on the axis of source directly behind the anode nozzle and at a distance of 20 cm from it. The effective operation of the described installation was watched both at constant positive voltage absence on the anode and at potential difference availability between the anode and the cathode. Average electron concentration behind the nozzle cut, in both cases, roughly was $\sim 10^{17} \text{ m}^{-3}$ and decreased to $10^{16} \text{ m}^{-3}$ at 20 cm distance. The directed jet speed was $V_i \approx (5 - 10) \times 10^3 \text{ m/s}$.

Note, that the measured value of plasma jet directed speed is in agreement with the speed, obtained from the examination evaluation of elementary diamagnetic acceleration in divergent magnetic field. In the region of "throat" of the magnetic nozzle, where the magnetic field intensity is a maximum, the plasma with cyclotron resonance heating is being created. At plasma jet movement in the direction of magnetic field decreasing, the energy transformation of crossed movement of charged particles takes place into the directed movement energy along the axis of the system. The strength of high frequency pressure, arising while electromagnetic wave reflection from the plasma occupied region, does not greatly contribute into the jet acceleration due to negligible value of electric field intensity.

At positive voltage feeding on the anode, plasma jet intensity increasing was registrated due to the jet speed increase.

References
ION IRRADIATION STUDIES FROM DENSE PLASMA FOCUS FOR CHANGE OF PHASE OF THIN FILM OF CdS

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There has been a growing interest in the structural transformations in non-metallic solids induced by energetic ions. Structural transformations take the form of amorphization, crystallization and stoichiometric changes. Naguib and Kelly/1/ have reported the criteria for bombardment-induced structural changes in such solids. Tell and Gibson/2/ studied the Bi and Xe implanted CdS and observed that the chemical properties are obscured by the radiation damage effects. Their measurements were restricted to optical effects. Therefore, an extensive investigation was undertaken on Bi-ion implanted CdS. Eldridge et al./3/ found gross structural changes on the surface with the help of Reflection Electron Diffraction (RED). However, their studies were not sensitive enough to indicate the type and extent of radiation damage in CdS specimen. Subsequently, Govind and Fraikor/4/ studied radiation damage of Bi-ion implanted CdS single crystal qualitatively with the help of transmission electron microscopy. They observed black spots and interpreted them to be interstitial clusters. However, they did not observe amorphization of CdS crystals. The study of CdS thin film is of interest because of its applications as solar cells/5/ piezo electric transducers/6/ and photo activated light valves.

Due to above mentioned applications, we have investigated in the present paper, the change of phase of Argon-ion implanted amorphous and crystalline thin films of CdS having thickness $\approx 0.6 \mu m$. The film is as-grown by vacuum evaporation technique and irradiated by the energetic Argon-ions of Dense Plasma Focus (DPF). The energetic ions from DPF have already been used by Rawat et al./7/ for the crystallization of amorphous thin film of Lead Zirconate Titanate. The thin films of CdS have been analysed before and after irradiation with the help of X-ray diffractometer (Rigaku Rotaflex), UV-visible spectrophotometer (Shimadzu-260), and Scanning electron microscope (Shimadzu - JSM 840).

Mather type DPF device of 3.3 kJ energy as shown in fig.1 is used to produce plasma of very high densities ($\approx 10^{25} - 10^{26} \text{m}^{-3}$) and having
temperature in the range of 1-2 keV. DPF is a source of X-rays, energetic ions/8/ and electrons/9/. The sub-systems of DPF are shown in fig.1. The sample is being placed on a mount which moves vertically along the axis of the chamber from the tip of the anode. In this way, the energetic ions can be bombarded on the sample at different distances from the anode. At first, the amorphous film of CdS is analysed by X-ray diffractometer and then irradiated by Ar-ions present in DPF. The irradiated film is again analysed with the help of X-ray diffractometer. The parent sample and the irradiated sample are also studied with the help of UV-visible spectrophotometer and Scanning electron microscope. In the similar manner, the crystalline thin film of CdS is irradiated and studied.

Fig.1 Schematic of the Experimental Set-up

![Fig.1 Schematic of the Experimental Set-up](image)

Fig.2 X-ray diffraction spectrum of as-grown amorphous film of CdS

![Fig.2 X-ray diffraction spectrum of as-grown amorphous film of CdS](image)

Fig.3 X-ray diffraction spectrum of as-grown amorphous film of CdS after irradiation.

![Fig.3 X-ray diffraction spectrum of as-grown amorphous film of CdS after irradiation](image)
X-ray diffraction spectrum for initially amorphous and crystalline CdS films as well as irradiated by energetic ions of DPF are shown in fig.2 - fig.5. Fig.2 shows the pattern for amorphous CdS film. When this film is irradiated by Ar-ions, a peak is recorded in the X-ray diffraction pattern as shown in fig.3. This peak indicates the change in phase of the CdS film. On the other hand, for the as-grown crystalline CdS film the spectrum is as shown in fig.4.

Fig.4 X-ray diffraction spectrum of as-grown crystalline film of CdS.

This film is then irradiated by Ar-ions of DPF and the X-ray diffraction pattern of such a film is shown in fig.5. This figure exhibits the amorphization of the CdS film.

These results lead us to conclude that Ar-ion bombardment is responsible for the change of phase of CdS films. Structural changes have also been observed in the scanning electron micrographs, taken for the irradiated thin films of CdS. The transmission and absorption characteristics are also found to change due to irradiation.

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/7/ R.S. Rawat, M.P. Srivastava, S. Tandon, A. Mansingh, Communicated for publication.
Influence of Metastable Atoms on the Characteristics of Surface Wave Produced Plasmas

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Plasmas generated by surface waves (ω/2π = 60 MHz – 2.45 GHz) in many cases exhibit a rather homogeneous structure of electric rf field strength /1/, simplifying modelling. Dominance of the axial component of the electric field |E_z| and its radial homogeneity are assured when the "thin cylinder approximation" is fulfilled, whereas axial constancy of |E_z| is achieved when dominant diffusion losses (≈ \( n_e D_{eff} / \Lambda_{eff}^2 \)) are balanced by ionization purely from the ground state (\( n_e \nu_{ig} (|E_z|) \)); the continuity equation enforces |E_z| as well as (non-Maxwellian /1/) electron energy distribution functions ("temperature") to be constant axially in spite of axially decreasing \( n_e \) ("diffusion regime").

However, ionization via excited states (\( n_e \nu_{im} \)) complicates and slightly modifies this regime, since the population density of excited states \( N_m \) is generally itself dependent on the electron density \( n_e \). Balancing processes such as collisional excitation from the ground state, depopulation by superelastic collisions (and by ionization), all linear in \( n_e \), compete with radiative depopulation not dependent on \( n_e \) (corona situation). The importance of radiative losses may be reduced – and simultaneously that of step-ionization increased – by optical thickness (effectively lengthening the lifetime of excited states) and in particular by metastability; in the latter case the lifetime, however, is limited by diffusion of metastables. Thus \( N_m \) should generally be assumed to depend on \( n_e \), though tending to saturation at large \( n_e \). Now the continuity equation reads:

\[
\frac{n_e D_{eff}}{\Lambda_{eff}^2} = n_e \nu_{ig} (|E_z|) + n_e \nu_{im} (|E_z|, n_e)
\]

With axial decreasing \( n_e \) according to (1) inevitably \(|E_z|\) has to increase slightly and the electron energy distribution function has to change.

This can be supported by numerically solving simultaneously the electron and ion continuity and momentum equations with the Boltzmann equation (including electron-electron and electron-ion collisions /2/), coupled with a collisional-radiative model describing processes of excited states; the solutions have to satisfy Bohm's boundary criterion. In this treatment radial variations are accounted for in more detail. Self-consistent numerical results are presented for a simple He model including all states of main quantum number \( n=2 \) and their interactions and two \( n=4 \) states.
As to be expected the total ionization frequency $\nu_{|\nu_{|E}|} + \nu_{im}$ tends to stay reasonably constant with (axially decreasing) $n_e$, whereas $\nu_{im} (|E_z|, n_e) / \nu_{|E|} (|E_z|)$ in Fig.1 changes substantially. Penning effects are included which mitigate the changes up to a factor of 2. In Fig.2 the predicted change of the self-consistent $|E_z|$ is depicted. A variation of, for instance, 10% when $n_e$ (averaged over the discharge cross section) changes a factor 10 axially, is of modest size and may seem almost imperceptible, but it suffices to regulate the large variation of inelastic processes. Therefore the point is not so much the quantitative deviation from $|E_z| = \text{const}$, rather than the resultant consequences for ionization and excitation rates and spectroscopic observations, influenced by $n_e$ directly and indirectly through varying $|E_z|$.

To transform dependencies on $n_e$ into those on the axial coordinate $z$, Maxwell's equations have to be involved which ensure a self-consistent power flow. Only a minor adjustment due to metastables is necessary as compared to the regime $|E_z| = \text{const}$. This can easily be seen from the simple relation /3/:

$$\frac{dn_e}{dz} = -\frac{1}{2 + \frac{\nu}{a}} \frac{\omega \nu}{c^2} \frac{\epsilon_0 m_0}{f}$$

The usual notations are used, with $\nu$ the elastic collision frequency and $a$ the plasma radius; $f$ is a slowly varying function essentially due to the cylindrical geometry, in thin cylinder approximation $\approx 0.16$ in the range $\omega/\omega_p \approx 0.1 - 0.3$ and $\nu/\omega < 1$, slightly smaller in the presence of glass tubes and decreasing for smaller $\omega/\omega_p$. $s$ stems from
the relation: \( n_e \propto |E_z|^{2s} \). Here \( s \) is negative and \( |s| \) about 10, e.g., when the above mentioned large \( n_e \)-drop is accompanied by a minor increase of \( |E_z| \). Thus basically a steepening of the slope of \( n_e(z) \) can be concluded, but the factor -0.5 almost imperceptibly changes to -0.53, for instance. This can also be ascertained by integrating an alternative expression /4,5/:

\[
\frac{dn_e}{dz} = -\frac{2n_e \alpha}{1 - \frac{n_e}{\alpha} \frac{d\Theta}{dn_e}} + \frac{n_e}{\Theta} \frac{d\Theta}{dn_e} \quad (3)
\]

\( \alpha \) is the absorption coefficient of the dispersion relation /6/, \( \Theta \) the energy absorbed per electron (from self-consistent \( |E_z| \) and distribution functions). Since \( d\Theta/dn_e \) gives small negative contributions again basically a steepening as well as an usually immeasurable effect on \( n_e(z) \) can be concluded. This is demonstrated in Fig.3 by an example. When electron-electron collisions are included the effect is almost imperceptibly increased. The – virtually unchanged – and almost linear profiles predicted by (2) and (3) and Fig.3 are consistent with observed ones to be reported elsewhere.

The effect of metastables and step-processes really shows up in excitation rates and observed line intensities, for instance in the intensity ratio of the He singulet 504.7 nm line to the triplet 471.3 nm line, often referred to as a measure of temperature. The results of Fig.4 give witness to the influence of axially varying \( n_e \), directly entering the collisional-radiative model as well as causing changes in \( |E_z(n_e)| \) and thus a changing distribution function (“temperature”). In this sense the results testify – only to a smaller extent – to an axially changing “electron temperature”, caused originally also by changing \( n_e \). It should also be emphasized that the trends such as in Fig.4 can be understood properly only if the correct self-consistent non-Maxwellian distribution functions (with “depressed” tail in the energy range of inelastic processes) are used /7/. The remaining differences between calculations and observations can easily be accounted for by uncertainties of cross-sections. Moreover the collisional-radiative model used is incomplete. Also errors in the \( n_e \)-determination give rise to small horizontal displacements, and the interferrometrically determined \( T_{gas} \), carries
some inaccuracies. Analysis of all these effects leaves essentially untouched the trends showing up in calculations as well as in observations.

Finally as an obvious check on the model used, in Fig.5 calculated and observed (via absorption measurements) population densities of the $^2S$ state for 200 mTorr are compared, yielding agreement actually better than to be expected. It should be noted that the $n_e$-dependence for higher pressures is weaker, but then the contributions of the resonance states ($^2P$ in particular) to $\nu_{im}$ become more noticeable.

In summary it may be concluded that the influence of metastables lead to interesting, but seemingly small modifications in the characteristics of the diffusion regime, i.e. small changes of the constancy of $|E_z|$ with only imperceptible steeping of $n_e(z)$. The resultant influence on inelastic collision rates, however, may be large and has to be taken into account in order to properly understand spectroscopic data. In doing so the use of non-Maxwellian distribution functions is essential.

\begin{itemize}
\end{itemize}
Use of Cold Plasmas in Film Deposition and Structuring

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1. INTRODUCTION

In general chemical compounds decompose during evaporation and sputtering. The highly volatile reactive components oxygen and nitrogen, disappear rapidly within the pumping system and compound films formed on the substrates are therefore often non-stoichiometric. Direct evaporation as well as direct sputtering of Al2O3 results in oxygen deficient films [1] and the same occurs with TiO2 and other compounds. To overcome this difficulty, Auwärter [2] and Brinsmaid [3] suggested that the deposition should be carried out in the presence of oxygen or generally, depending on the compound processed, of other reactive gases. In the meanwhile reactive deposition has developed into a powerful technology.

In recent developments considerable attention has been paid to techniques to improve all optical and mechanical film properties besides stoichiometry and residual absorption.

These techniques avoid high substrate temperatures and supply the required energy to achieve proper optical, chemical and mechanical film characteristics by the use of high energy of coating material atoms and working or reactive gas atoms. The increase of the average particle-energy in the growing film by energetic ion processes appears to be beneficial. Ion-assisted reactive evaporation is therefore a very effective technology, along with reactive sputtering variants and reactive ion plating.

Sputter etching is often included in sputtering systems and used mainly to clean substrate surfaces (prior to film growth) to promote adhesion and to structure substrate-surface or films.

2. REACTIVE AND ACTIVATED REACTIVE EVAPORATION

The properties of thin films deposited on various substrates by conventional evaporation are generally found to be different from those of the corresponding bulk materials.

Oxygen loose, which usually occurs during evaporation of many oxides may cause optical absorption in the films. The effect could be appreciably corrected in the reactive deposition process [2] by adding oxygen to the residual gas. For all that, however, oxide coatings produced in this way are often still partially non-stoichiometric and slightly absorbing.

Heating the substrates to about 300 °C during reactive evaporation improved a number of film properties including stoichiometry, purity, density, refractive index and adhesion. Therefore substrate heating became the standard procedure in this process although it favours generally undesirable coarser film microstructure and surface roughness.

Reactive evaporations performed under standard conditions, but with additional activation and partial ionisation of the reactive gas, further improved film quality [4]. Better stoichiometry and rather low absorption values have been obtained when films were prepared by activated reactive evaporation (ARE) [5]. These coating experiments with ions and excited molecules clearly showed that chemical reactivity is enhanced in the presence of a
gas-discharge plasma. To activate oxygen for ARE cold glow discharge ion sources with a hollow cathode inside a quartz tube are used [5].

3. ION ASSISTED DEPOSITION

Most properties of evaporated films are highly determined by the lack of mobility of the condensing atoms and molecules due to their low thermal energies of about 0.1 eV. Controlled bombardment of a growing film by argon or/and oxygen ions with energies up to several hundred electron volts has been shown to improve density and stoichiometry [6, 7]. Adding energy to the film growth process appears to improve also adhesion [8] and to modify stress [9]. Ion beam techniques can be applied, however, also before and after film deposition. Substrates can be ion beam cleaned with inert or reactive gas ions, depending upon substrate materials involved, and freshly deposited films can be bombarded by ions for further improvement of their properties.

Hot-cathode Kaufman-type [10] ion guns are used usually in ion beam assisted deposition processes (IBAD), but also other installations have been applied. It is impossible to discuss and compare here all the variants of this technique. Considerable efforts have been directed to the analysis of plasma conditions and their correlation with deposited film properties. The mechanism responsible for densification seems to be momentum transfer as was shown recently by molecular-dynamics calculation [11].

4. GAS DISCHARGE AND ION BEAM SPUTTERING

During sputtering atoms or molecules are ejected from a target by momentum transfer processes caused by noble gas ions, e.g. Ar⁺ bombardment. The coating material atoms with energies between 1 and 40 eV pass through the gas phase, loose energy by collisions and condense on substrates which, in case of gas discharge sputtering, are arranged opposite the target at a small distance.

Ions stem from a dc or rf gas discharge or from special ion guns. In the first case the typical working gas pressure is in the 10⁻² to 10⁻³ mbar range whereas in ion beam sputtering lower background pressures and therefore larger target to substrate distances can be applied. With all variants [12] of sputter processes reactive gas procedures can be performed, but conventional gas discharge sputtering is a rather slow process. The deposition rate is increased by magnetrons, in which in addition to the electric field a magnetic field increases the number of ionising collisions by keeping the electrons in trajectories close above the target surface. These procedures are used for film deposition on large flat substrates, e.g. architectural glass and plastic foils [13, 14].

In ion beam sputtering (IBS) the target erosion is caused by a mono-energetic argon ion beam. This beam is usually produced by a Kaufman-type ion gun but also high frequency ion sources should be considered [15]. In dual ion beam sputtering a second ion gun is used, whose beam aims directly towards the growing film similar to ion assisted deposition [16]. With the IBS techniques dense, well adherent and stoichiometric films of low optical absorption can be obtained. In all variants care must be taken to avoid unwanted sputtering of plant installations by high energetic neutral atoms which causes film contamination.

5. STRUCTURING BY SPUTTER-ETCHING AND REACTIVE ION-ETCHING

Although it is not common, sputter etching can be used to etch patterns into thin films (or into the substrates themselves) first covered by a pattern-defining mask. The patterning mask may be either mechanical or an developed photoresist mask as used in
exhibits a monotonical increase. On the other hand for 2.7 Pa O₂ where the plasma density is lower, it is notified that δΔ exhibits a peak at discharge time of 100 s (P).

The results of Fig.3 and Fig. 4 show that oxygen pressure change leads to the plasma density change and affects the time evolution of relative phase shift. δΔ is a function of a refractive index and a film thickness /6/. On the ignition of the plasma, the film thickness starts to increase and at the same time the refractive index decreases rapidly. The time evolution of δΔ can be determined by a competition between the refractive index and the film thickness. Very rapid decrease in the refractive index accompanying film growth leads to the peak in the δΔ versus discharge time /5/. For the plasma oxidation at 2.0 Pa O₂ the peak was not observed (Fig. 3). This is because the growth rate of the silicon oxide is high because of high plasma density at this pressure (a weak shoulder seen at around 50 s (S in Fig. 4) may be related to the rapid decrease of the refractive index for the higher plasma density).

In conclusion, the change rate of δΔ was observed by ISDP RE immediately after starting the RF discharge as being strongly dependent on O₂⁺ density. In the thin oxide film region, the oxidant species of O₂⁺ and/or O atoms were suggested to readily react on silicon surface, and the time evolution of δΔ was sensitive to the plasma density.

REFERENCES

on the plasma density. To observe chemical species in oxygen plasma, the OES measurements were performed. Very sharp lines of O atoms were observed at 436.8 nm (3P→3S0), 532.9 nm (5D0→5P), 615.8 nm (5D0→5P), and 777.5 nm (5P→5S0) /3,7/. Somewhat broader lines from molecular oxygen ions (O2+) at 525.1, 559.7, 597.3, and 635.1 nm were also observed. All lines are assigned to the b4Σg−→a4Πu transition /3,8/. Figure 3 shows that the sum of the O2+ ions emission spectrum intensity increases quite linearly as RF power increases. The O atoms emission intensity also increased with respect to the RF power. Furthermore, plasma potential (Vp) was larger by 13-16 eV than floating potential (Vf), therefore suggesting that negative ions generated in the plasma does not dominate plasma oxidation directly. The present results indicate that positive ions and/or O atoms predominate in the plasma oxidation immediately after starting the plasma discharge. Further investigation on the sheath area is needed for understanding more detailed reaction mechanism.

Figure 4 shows δΔ versus discharge time at RF power of 300 W at an O2 pressure of 2.7 Pa and 2.0 Pa. The change in δΔ at 2.0 Pa O2 is larger than that at 2.7 Pa O2. For 2.0 Pa O2, δΔ...
3. RESULTS AND DISCUSSION

Change in $\Psi$ was very small. Figure 1 shows the relative change in $\Delta$, $\delta \Delta = \Delta_0 - \Delta$ ($\Delta_0$ is a value of $\Delta$ at starting the discharge), monitored by ISDP RE as a function of discharge time at RF power of 200 W, 300 W, and 500 W at an $O_2$ pressure of 2.0 Pa. $\delta \Delta$ exhibits an increase with a high speed for a short time and a slow speed for a long time.

The change rate of $\delta \Delta$ ($\delta \Delta' = \delta \delta \Delta/\delta t$) immediately after starting the RF discharge was determined as $1.1 \times 10^{-1}$, $1.7 \times 10^{-1}$, and $2.0 \times 10^{-1}$ degree/s for respective RF powers of 200, 300, and 500 W. In contrast, there was not such a strong correlation between $\delta \Delta'$ and RF power $1.0 \times 10^4$ s after starting the discharge. The main process in this stage may be the drift motion of oxygen ions across the oxide film /1/.

With the exception of the plasma density, which increased with respect to RF power increase, the plasma parameters are independent of RF power. Figure 2 shows the relation of $\delta \Delta'$ immediately after starting the RF discharge and the plasma density. $\delta \Delta'$ linearly increases as the plasma density increases between $6.4 \times 10^7$ and $2.2 \times 10^8$ cm$^{-3}$. This shows the initial oxidation rate strongly depends

![Fig. 1. Time dependence of relative change in $\Delta$ measured by in-situ during process rapid ellipsometry at an $O_2$ gas pressure of 2.0 Pa and RF power of a) 200 W, b) 300 W, and c) 500 W, respectively. The film thickness was estimated to be about 1.4 nm after a plasma oxidation for 16000 s at 2.0 Pa $O_2$ and 300 W RF power /5/.

![Fig. 2. Change rate ($\delta \Delta'$) immediately after starting the RF discharge versus plasma density obtained by the Langmuir probe method at an $O_2$ gas pressure of 2.0 Pa. Solid line shows least squares curve fitting.]
EXPERIMENTAL STUDY OF PLASMA EFFECT ON OXIDATION OF SOLID SURFACE

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1. INTRODUCTION

Characterization of plasma is one of the most important issues for understanding mechanism of plasma-surface interaction and also the plasma processing for semiconductor. The plasma oxidation is a current technique being used for growing insulator films on semiconductor surfaces at lower temperatures than when using thermal oxidation/1-3/. However, there is no conclusive interpretation for the oxidation mechanism, and little research has been performed on effects of plasma on the oxidation rate. This paper reports detailed experimental study on the oxidation of silicon surface in thin film region in an oxygen plasma. We measured parameters and chemical species of the plasma using a Langmuir probe (LP) and optical emission spectroscopy (OES), and compared the results with those of in situ during process (ISDP) rapid ellipsometry (RE) to investigate effect of plasma on oxidation reaction kinetics on solid surface.

2. EXPERIMENTAL

Samples used were p-Si(100) with resistivity of 1x10⁻⁴-3x10⁻⁴ Ωm. It was immersed in an HF solution for 5 min, rinsed with water, and annealed at 873 K in vacuum of about 10⁻⁷ Pa. The plasma oxidation of silicon was performed in a UHV chamber of 0.15 m in a diameter and 1.5 m in a length. An oxygen plasma was generated by RF discharge at 13.56 MHz, with its power of 100 to 600 W. The electrically isolated substrate was set up 1 m apart from the RF coil. Oxygen pressure ranged 1 to 3 Pa.

Electron temperature, plasma density, and other plasma parameters were measured by the LP method. The probe was placed 2 cm from the substrate, which is over 100 times greater than the Debye length (≈ 0.1 cm) /4/. The growth of silicon oxide films was monitored by ISDP RE (1s resolution). Ellipsometric parameters (Δ) and (Ψ) were measured in situ by a photo elastic modulator dual lock-in method ellipsometer (JASCO PME-30p) using He-Ne laser (632.8 nm), having a 0.1 s minimum resolution and interval /3/.
with many applications.

It is impossible to discuss and compare all the ion and plasma techniques currently used for optical film deposition. In this paper all reactive PVD techniques were mentioned but plasma assisted CVD, plasma polymerisation and plasma etching and the non-reactive energetic PVD techniques have been omitted intentionally. Because of their importance it should be mentioned here that some fluorides, particularly AlF₃, CeF₃ and LaF₃, have also been successfully deposited by ion assisted deposition and low-voltage ion plating. Many rare earth metal fluorides as those of Sm, Gd, Tb, Ho, Er and Y have been successfully processed by IBAD [21]. Deposition of MgF₂ is more critical because of slight impact dissociation. Only few results are available on plated sulphide films. ZnS-films on Ge substrates deposited by dc ion plating [22,23] improved considerably humidity resistance and adhesion compared to films obtained by conventional evaporation, but Clearly further work is required on this topic.

REFERENCES

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Reactive PVD coating Technologies

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7. DISCUSSION AND CONCLUSION

The possibilities to improve film performance in conventional reactive and even in activated reactive evaporation is rather limited. However, the process is effective, special with recently developed ion processes with higher particle energies as ion assisted deposition, sputtering, ion beam sputtering and reactive low voltage ion plating. Many of the property modifications observed using concurrent ion bombardment during film deposition are a combination of physical and structural changes as well as chemical changes in the film.

Important features of coatings made by these techniques are: dense homogeneous microstructure resulting in relatively high refractive indices, low optical losses, insensitivity to changes in humidity, high hardness and abrasion resistance and stability at high temperatures.

Bombardment of growing films with noble or reactive gas ions in ion assisted deposition and generation of coating material atoms with high energies in ion beam sputtering are powerful tools for basic parameter investigations in ion process developments and optimisation studies. The techniques are very effective for the deposition of low-loss dielectric films on small to medium size substrates.

Up scaling of directed ion beam processes for production purposes is difficult and requires for IBAD and IBS large size ion guns and highly efficient pumping systems. In film production the deposition time is important. Particularly with ion beam sputtering the low deposition rate of 1.5-20 minutes for a quarter wave thick film (visible range) is extremely time consuming in multilayer depositions. The new low-voltage reactive ion plating technology is an optimised and fast process for high volume production and it seems that RLVIP becomes an extremely useful production technique for high quality optical coatings.
semiconductor and integrated device fabrication.

Then sputter etching proceeds to bombard the substrate surface removing both substrate and the resist mask (there is no preferential etching of the substrate compared to the mask as in chemical etching for sputtering is a physical process). Therefore the resist must be thick enough to last long enough till the substrate is etched to the desired depth.

The key advantage of Sputter etching compared to chemical etching is that there is extremely little undercutting and the exact pattern of the resist mask can be maintained in the substrate. This is vital for very high resolution Very Large Scale Integration circuitry with patterns of under 3 μ dimensions. Etch rates are also well controlled and there are no chemical residues. Unfortunately the sputter etching process is slow compared to chemical etching and the polymeric photoresist materials are burned by high energy sputter etching.

The answer to occurring problems is the reactive ion etching (RIE) which is a form of the rather new plasma etching. In RIE gas is introduced into the system which produces chemically reactive etching species in the plasma. Combined with the sputter etching process this provides higher etch rates, but still relatively vertical etching walls. Plasma etching and RIE are actively pursued for fine line lithography.

6. REACTIVE ION PLATING

In the ion plating process [17] evaporation is performed in the presence of an argon gas discharge. In collisions and electron impact reactions ions of coating material are formed and accelerated in the electric field of biased substrates, so that condensation and film formation take place under the influence of ion bombardment. The bombardment involves ions of the working gas, of the film material vapour or of a mixture of both. In addition, neutral atoms of vapour and gas with a higher average energy are important for the deposition process too. This complex action is typical for ion plating. A large number of process variants is possible and different components can be combined to more complex triode and tetrode ion plating systems. It is furthermore remarkable that also high-speed sputtering cathodes are used as vapour sources in special ion plating arrangements. Bias sputtering and some types of plasma CVD also fulfil the definition of ion plating. In conventional ion plating the degree of ionisation is low and the accelerating voltage must be rather high, in general between 3 and 5 keV, to supply the necessary energy for the formation of well adherent and dense films. The small number of ions is also disadvantageous for a reactive gas process. To overcome this problems a new form of reactive ion plating has been developed [18].

The new reactive low voltage ion plating process (RLVIP) [19] can be used for the deposition of single and multilayer dielectric coatings on unheated substrates. Evaporations are made by two special 270°-type electron beam evaporators. The starting materials form electrically conducting melts. Very effective ionisation and activation of the evaporating coating material atoms and the admitted reactive gas molecules occur by a low-voltage high-current argon plasma beam (hot cathode type) directed to the crucibles (anode). The substrate holder is electrically insulated. In contact with the formed plasma cloud the substrates receive a relatively high negative self-biasing potential of 15 - 20 V with respect to the plasma, which acts as accelerating voltage for positive ions. The total pressure in the plant is in the low 10⁻³ mbar range. The film deposition is started and stopped by opening or closing the shutters in front of the electron beam evaporators. Deposition rate is controlled by an oscillating quartz crystal monitor [20], and the film thickness is determined by a quartz crystal monitor or by optical thickness monitoring. The RLVIP-system can be tooled for both reactive ion plating and conventional reactive evaporation allowing alternating processes without requiring any changes in the plant.
LANGEVIN-EQUATION FOR CHARGED PARTICLES
IN MICROTURBULENT PLASMA

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Microinstabilities seem to be rather important for momentum and energy transport in nearly collision-free plasmas. They play a role in anomalous particle diffusion, heating or acceleration. Driving forces of microinstabilities are usually strong currents and drifts, but also magnetic stresses and plasma anisotropy. Microinstabilities generate turbulent field fluctuations interacting in turn with the plasma particles, so that the free energy causing the instability will be reduced.

A special problem is the relation of microturbulence to magnetic reconnection, the restructuring of magnetic field lines, which can be connected with a transformation of large amounts of magnetic energy into kinetic and thermal energy [1]. Here one question is, whether reconnection may be caused by microturbulence even before the magnetic field is compressed strong enough, so that electrons move chaotically. Although the most favoured for reconnection electrostatic instabilities seem to generate too narrow diffusion regions or are saturated in the actual reconnection region with its high plasma-β, the idea of remote scattering [2] and the observation of magnetic noise bursts in the rather high-β region of the Earth's magnetotail [3] gives some hints, that microturbulence seems to contribute to reconnection in some cases.

The motion of the plasma particles (of type a) in a microturbulent system can be described by the Langevin-equation, which is found adding in the equation of motion of the particles the time-dependent force of the waves

\[ \frac{d\vec{r}}{dt} = \vec{v}, \quad m_a \frac{d\vec{v}}{dt} = q_a \left( \vec{E}_a(\vec{r}) + [\vec{v} \times \vec{B}_a(\vec{r})] \right) + \vec{F}_a(\vec{x}, t), \quad \{\vec{x}\} = \{\vec{r}, \vec{v}\} \]  

(1)

(\vec{E}_a(\vec{r}) - external electric field, \vec{H}_a(\vec{r}) = \vec{B}_a(\vec{r})/\mu_0 - external magnetic field). This wave-force

\[ \vec{F}_a(\vec{x}, t) = \langle \vec{F}_a(\vec{x}) \rangle + \delta \vec{F}_a(\vec{x}, t) \]  

(2)

is as usual divided in the ensemble average \( \langle \vec{F}_a(\vec{x}) \rangle \) and a stochastic force \( \delta \vec{F}_a(\vec{x}, t) \) with vanishing mean value \( \langle \delta \vec{F}_a(\vec{x}, t) \rangle = 0 \). Assuming Gaussian stochastic processes with a stochastic-force distribution

\[ f_{\delta \vec{F}_a}(\delta \vec{F}_a) = \exp \left\{ -\frac{(\delta \vec{F}_a)^2}{2\sigma^2} \right\} / \sqrt{2\pi \sigma} \]  

(3)

all correlation functions of the stochastic force of odd order are zero, correlation functions of even order can be expressed by functions of order \( n = 2 \),

\[ \langle \delta F_{ai}(\vec{x}, t_1) \delta F_{aj}(\vec{x}, t_2) \rangle = 2\delta_{ij}\delta(t_1 - t_2)D_a^{(2)}(\vec{x}). \]  

(4)
Components of the stochastic force in different directions and at different instants of time are uncorrelated. Usually the mean force $\langle F_{ai} \rangle$ is expressed by a friction term $-\gamma_a(\vec{x})v_i$. But here, according to an idea of [4], a second yet unknown term is added,

$$
\langle F_{ai}(\vec{x}) \rangle = -\gamma_a(\vec{x})v_i + \frac{\partial R_{ii}(\vec{x})}{\partial v_i}.
$$

Instead of solving the Langevin-equation (1) for given initial conditions $\vec{v}_0 = \vec{v}(t_0)$, $\vec{r}_0 = \vec{r}(t_0)$ and to determine the probabilities of all its solutions it is also possible to find an equation of motion for the one-particle distribution function $f_a(\vec{x}, t)$. Thus, now $\partial R_{ii}(\vec{x})/\partial v_i$ will be found regarding consistency between the Langevin-equation (1) and the Fokker-Planck-form of a corresponding kinetic equation.

Suggesting that some inverse effective wave-particle collision frequencies $1/\nu_{eff}$ are much smaller than the dissipation time of the plasma process $t_d$, $1/\nu_{eff} \ll t_d$, this process can be considered as Markovian during time intervals $1/\nu_{eff} < \Delta t < t_d \sim 1/\gamma_w$ ($\gamma_w$ growth rate of the wave generated by the instability). That means, the distribution function at time $t$, $f_a(\vec{x}, t)$, can be found knowing the distribution $f_a(\vec{x}^*, t^*)$ and the transition probability $p(\vec{x}, t \mid \vec{x}^*, t^*)$ from the state $\vec{x}^*$ at $t^*$ to $\vec{x}$ at $t = t^* + \Delta t$, $\Delta t \to 0$. Than the Master-equation yields,

$$
\frac{\partial}{\partial t} f_a(\vec{x}, t) = \int [w(\vec{x}, t \mid \vec{x}^*, t)f_a(\vec{x}^*, t) - w(\vec{x}^*, t \mid \vec{x}, t)f_a(\vec{x}, t)]d\vec{x}^*.
$$

$$
w(\vec{x}, t \mid \vec{x}^*, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} p(\vec{x}, t^* + \Delta t \mid \vec{x}^*, t^*)
$$
is the transition rate. For Gaussian processes [5] with

$$
p(\vec{x}, t \mid \vec{x}^*, t^*) = (2\pi \text{const}_1 \Delta t)^{-1/2} \exp\left(-\frac{(\vec{x} - \vec{x}^*)^2}{2\text{const}_1 \Delta t}\right)
$$

only the first two moments of the transition rate

$$
\alpha^{(i)}_1(\vec{x}, t) = \int (x_i^* - x_i)w(\vec{x}, t \mid \vec{x}^*, t)d\vec{x}^*,
$$

$$
\alpha^{(i,j)}_2(\vec{x}, t) = \int (x_i^* - x_i)(x_j^* - x_j)w(\vec{x}, t \mid \vec{x}^*, t)d\vec{x}^*
$$

are non-vanishing. Expanding in (6) $f_a(\vec{x}, t)$ at $\vec{x}$ in a Taylor-series (Kramers-Moyal-series) one obtains

$$
\frac{\partial}{\partial t} f_a(\vec{x}, t) = -\sum_i \frac{\partial}{\partial x_i} [\alpha^{(i)}_1(\vec{x}, t)f_a(\vec{x}, t)] + \frac{1}{2} \sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} [\alpha^{(i,j)}_2(\vec{x}, t)f_a(\vec{x}, t)]
$$

$$
= -\vec{\nu} \frac{\partial}{\partial \vec{r}} f_a(\vec{x}, t) - \frac{q_a}{m_a} \left( \vec{E}_a(\vec{r}) + [\vec{v} \times \vec{B}_0(\vec{r})] \right) \frac{\partial}{\partial \vec{v}} f_a(\vec{x}, t) + \frac{1}{m_a} \frac{\partial}{\partial \vec{v}} (\gamma_a(\vec{x})\vec{v} f_a(\vec{x}, t))
$$
+ \frac{1}{m_a^2} \sum_i \frac{\partial}{\partial v_i} D^n_{ii}(\vec{x}) \frac{\partial}{\partial v_i} f_a(\vec{x},t) + \sum_i \frac{\partial}{\partial v_i} f_a(\vec{x},t) \frac{\partial}{\partial v_i} \left( \frac{D^n_{ii}(\vec{x})}{m_a^2} - \frac{R^n_i(\vec{x})}{m_a} \right).

From (10) it is clear that $D^n_{ii}(\vec{x})$ should be equal to $m_a R^n_i(\vec{x})$ to develop from (1) a kinetic equation in usual form. In [4] it was found $m_a R^n_i(\vec{x}) = D^n_{ii}(\vec{x})/2$ obtaining the Fokker-Planck-equation by Stratonovich-method. But there the mean stochastic force was $(\partial D^n_{ii}(\vec{x})/\partial v_i)/2$.

If one generalizes the kinetic equation for microturbulent isotropic plasma in polarization approximation [6] to anisotropic plasmas with electromagnetic wave fields $\delta \vec{E}$, $\delta \vec{B}$ (taking into account also the influence of the magnetic field $\delta \vec{B}$ of the waves on the density fluctuations) and compares the result with (10), the friction coefficient

$$\gamma_{\alpha\beta} = \frac{q^2}{2\pi^2} \int \omega^3 \delta(\omega - \vec{k}\cdot\vec{v}) \left( \frac{\nu_{\alpha\beta} \epsilon_{\alpha\beta}}{1 \cdot \epsilon_{\alpha\beta} - c^2 k^2 \epsilon_{\alpha\beta} + c^2 k_{\alpha} k_{\beta}} \right) \frac{1}{\omega} \left( k_r v_r + k_s v_s \right) \omega \left( \epsilon_{\alpha\beta} - c^2 k^2 \epsilon_{\alpha\beta} + c^2 k_{\alpha} k_{\beta} \right) d\omega d\vec{k}$$

(11)

$$+ \frac{\nu_{\alpha\beta} \epsilon_{\alpha\beta}}{\omega} \left( \epsilon_{\alpha\beta} - c^2 k^2 \epsilon_{\alpha\beta} + c^2 k_{\alpha} k_{\beta} \right) \frac{1}{\omega} \left( \epsilon_{\alpha\beta} - c^2 k^2 \epsilon_{\alpha\beta} + c^2 k_{\alpha} k_{\beta} \right) d\omega d\vec{k}$$

(11)

can be expressed by the tensor of the dielectric function $\epsilon_{\alpha\beta}(\omega, \vec{k})$, and the velocity-diffusion tensor ($\alpha \neq \beta$, $\epsilon_{\alpha\beta} = \{x, y, z\}$)

$$D^n_{\alpha\beta} = \frac{q^2}{16\pi^3} \int \delta(\omega - \vec{k}\cdot\vec{v}) \left( \delta F_{\alpha}(\omega) \delta F_{\beta}^* \right) \omega \left( \epsilon_{\alpha\beta} - c^2 k^2 \epsilon_{\alpha\beta} + c^2 k_{\alpha} k_{\beta} \right) d\omega d\vec{k}$$

(12)

$$\delta F_{\alpha} = \delta E_{\alpha} \left( 1 - \frac{k_r v_r + k_s v_s}{\omega} \right) + \delta E_{\alpha} \left( \frac{v_s k_{\alpha}}{\omega} \right)$$

by the space-time spectral density of the energy of the waves ($\{\gamma; \delta\} = \{x, y, z\}$)

$$\langle \delta E_{\alpha} \delta E_{\beta} \rangle_{\omega} \delta \vec{k} = \frac{16\pi^2 \omega^2 \sum_{\alpha} q^2 \nu_{\alpha} 2\pi \int \nu_{\gamma} \nu_{\delta} \delta(\omega - \vec{k}\cdot\vec{v}) f_a(\vec{x},t) d\vec{v}}{\omega^2 \epsilon_{\gamma\delta} - c^2 k^2 \epsilon_{\gamma\delta} + c^2 k_{\gamma} k_{\delta}}$$

(13)

As a consequence of the fact that stochastic forces in different directions are uncorrelated (eq.(4)) in (10) only diagonal components of the diffusion tensor appear. But according to (12-13) the velocity-diffusion tensor in polarization approximation $D^n_{\alpha\beta}$ is also a symmetric tensor in the case of inhomogeneous plasma. Thus it is always possible to find a corresponding diagonal tensor

$$D^n_{\text{diagonal}} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} D^n = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix},$$

(14)

$$\vec{b}_1 = \vec{x}_1 / | \vec{x}_1 |, \quad \vec{b}_2 = (\vec{x}_2 - (\vec{x}_2 \cdot \vec{b}_1) \vec{b}_1) / | \vec{x}_2 - (\vec{x}_2 \cdot \vec{b}_1) \vec{b}_1 |, \quad \vec{b}_3 = (\vec{x}_3 - (\vec{x}_3 \cdot \vec{b}_1) \vec{b}_1 - (\vec{x}_3 \cdot \vec{b}_2) \vec{b}_2) / | \vec{x}_3 - (\vec{x}_3 \cdot \vec{b}_1) \vec{b}_1 - (\vec{x}_3 \cdot \vec{b}_2) \vec{b}_2 |.$$
\[ \vec{b}_3 = (\vec{x}_3 - (\vec{x}_3 \vec{b}_1) \vec{b}_1 - (\vec{x}_3 \vec{b}_2) \vec{b}_2) / | \vec{x}_3 - (\vec{x}_3 \vec{b}_1) \vec{b}_1 - (\vec{x}_3 \vec{b}_2) \vec{b}_2 |. \]

\( \vec{x}_l \{l = 1; 2; 3\} \) is the eigenvector of the matrix of the diffusion tensor \( D_{ij} \) belonging to the zero \( d_1 \) of the characteristic polynomial of \( D_{ij} \). The coordinates of \( \vec{x}_l \{x_{i1}, x_{i2}, x_{i3}\} \) are the non-trivial solutions of the homogeneous, linear system of equations

\[
\begin{align*}
(D_{xx}^a - d_1)x_{i1} + D_{xy}^a x_{i2} + D_{xz}^a x_{i3} &= 0, \\
D_{xy}^a x_{i1} + (D_{yy}^a - d_1)x_{i2} + D_{yz}^a x_{i3} &= 0, \\
D_{xz}^a x_{i1} + D_{yz}^a x_{i2} + (D_{zz}^a - d_1)x_{i3} &= 0,
\end{align*}
\]

\( d_1 = u + v + \frac{r}{3}, \quad d_2 = \epsilon_1 u + \epsilon_2 v - \frac{r}{3}, \quad d_3 = \epsilon_2 u + \epsilon_1 v - \frac{r}{3}, \)

\[
\epsilon_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}, \quad u = \left( -\frac{q}{2} + \sqrt{H} \right)^{1/3}, \quad v = \left( -\frac{q}{2} - \sqrt{H} \right)^{1/3}, \quad H = \frac{p^3}{27} + \frac{q^2}{4}, \quad p = s - \frac{r^2}{3}, \quad q = \frac{2s^3}{27} - \frac{rs}{3} + w,
\]

\[ r = -D_{xx}^a - D_{yy}^a - D_{zz}^a, \quad s = D_{xx}^a D_{yy}^a + D_{xy}^a D_{xz}^a + D_{yy}^a D_{zz}^a - D_{xy}^a D_{yz}^a - D_{xz}^a D_{yz}^a - D_{xx}^a D_{yy}^a, \]

\[ w = D_{xz}^a D_{yz}^a + D_{xy}^a D_{xx}^a + D_{yy}^a D_{xz}^a - D_{xz}^a D_{xz}^a D_{yy}^a D_{xx}^a - 2D_{xy}^a D_{xz}^a D_{yz}^a. \]

In the simple case of the ion-sound instability \( D_{xx}^a = D_{xy}^a = D_{yy}^a, D_{xz}^a = D_{yz}^a, \) so that

\[
D_{\text{diagonal}}^a = \begin{pmatrix} 2D_{xx}^a + Y & 0 \\ 0 & D_{zz}^a - Y \end{pmatrix},
\]

\[
Y = 2D_{xx}^a (2g - 2D_{xx}^a - D_{zz}^a) / ((g - D_{zz}^a)^2 + 2D_{xx}^a),
\]

\[ g = D_{xx}^a + \frac{D_{zz}^a}{2} + \sqrt{D_{xx}^a + D_{zz}^a + 2D_{xx}^a / 4 - D_{xx}^a D_{zz}^a + 2D_{zz}^a}. \]

Now the Gaussian-distributed stochastic force \( \delta F_\alpha (\vec{x}, t) \) can be estimated by its intensity \( \sigma = (2D_{\text{diagonal},ii}^a)^{1/2}. \)

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